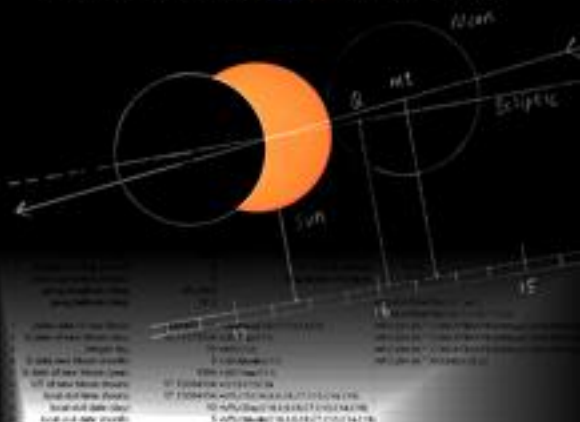


Fourth Edition

Practical Astronomy with your Calculator or Spreadsheet

Peter Duffett-Smith and Jonathan Zwart



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Practical Astronomy with your Calculator or Spreadsheet

Fourth Edition

Now in its fourth edition, this highly regarded book is ideal for those who wish to solve a variety of practical and recreational problems in astronomy using a scientific calculator or spreadsheet.

Updated and extended, this new edition shows you how to use spreadsheets to predict, with greater accuracy, solar and lunar eclipses, the positions of the planets, and the times of sunrise and sunset. With clear, easy-to-follow instructions, shown alongside worked examples, this handbook is essential for anyone wanting to make astronomical calculations for themselves. It can be enjoyed by anyone interested in astronomy, and will be a useful tool for software writers and students studying introductory astronomy.

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- Covers orbits, transformations and general celestial phenomena, for use anywhere, worldwide.
- High-precision spreadsheet methods for greater accuracy are available at www.cambridge.org/practicalastronomy.

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Fourth Edition

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To our friends and
colleagues at MRAO

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Preface to the fourth edition

Practical Astronomy with your Calculator or Spreadsheet has been written for those who wish to calculate the positions and visual aspects of the major heavenly bodies and important phenomena such as eclipses, either for practical purposes or simply because they enjoy making predictions. We present recipes for making calculations, where we have cut a path through the complexities and difficult concepts of rigorous mathematics, taking account only of those factors that are essential to each calculation and ignoring corrections for this and that, necessary only for very precise predictions of astronomical phenomena. Our simple methods, suitable for use with a pocket calculator, are usually sufficient for all but the most exacting amateur astronomer, but they should not be used for navigational purposes. For example, the times of sunrise and sunset can be determined to within 1 minute and the position of the Moon to within one fifth of a degree. But new to this fourth edition are spreadsheets which offer much higher precision (see below).

The second edition included much more material in response to letters and requests from readers of the first edition. Many errors were also corrected. The third edition continued the same process, adding four new sections on generalised coordinate transformations, nutation, aberration and selenographic coordinates, improving the sunrise/set and moonrise/set calculations so that they worked properly everywhere in the world, including a rigorous method of calculating precession, taking account of the J2000 astronomical system where appropriate, and correcting mistakes or clarifying obscurities wherever they were found in the second edition.

The fourth edition has also been updated considerably; however the major change is that we have included, for the first time, a spreadsheet for nearly every calculation. Each spreadsheet illustrates the calculation, making it easier to get the right answer. But we have also written a library of powerful functions which can carry out many of the calculations for you with much higher precision, so those people who wish to use their computers can do so and obtain the benefits of greater accuracy. For example, use the simple recipes and your calculator to find the times of moonrise and moonset to within a precision of 10 minutes or so, or use the spreadsheet functions to obtain the results correct to within 1 minute. You will need to visit our website (see page 209) to download the spreadsheets to your computer; the library of functions will come automatically with the spreadsheets.

We are most grateful to those kind people who have taken the trouble to write in with their suggestions, criticisms and corrections, in particular to Mr E. R. Wood, who kindly scanned the manuscript of the third edition for errors, Mr S. Hatch, Mr S. J. Garvey, who supplied the nomogram for the solution of Kepler's equation, and Mr Anthony Ehrlich of Pittsburgh, Pennsylvania, who developed a rudimentary scheme for calculating the circumstances of sunrise/set and moonrise/set into one that actually worked

(superseded in this edition). We would also like to thank and acknowledge those authors whose books we have read and whose ideas we have cribbed, mentioning particularly Jean Meeus (*Astronomical Formulae for Calculators*) and W. Schroeder (*Practical Astronomy*). We have made extensive use of *The Explanatory Supplement to the Astronomical Ephemeris* and the *American Ephemeris and Nautical Almanac*, as well as the *Astronomical Almanac* and its predecessors.

Our thanks are also due to Dr Anthony Winter, who suggested writing the first edition of the book, to Mrs Dunn who typed it, to Dr Guy Pooley who read the manuscript and made many helpful suggestions, and to Dr Simon Mitton for taking so much trouble over the production of the book. Thanks for particular help with the fourth edition go to William Lancaster, Sehar Tahir and our editor Vince Higgs.

We hope you have as much fun with these recipes and spreadsheets as we have had! Please let us know when you find an error. You can contact us via the book's website (see page 209).

About this book and how to use it

How many times have you said to yourself, ‘I wonder whether I can see Mercury this month?’ or ‘What will be the phase of the Moon next Tuesday?’ or even ‘Will I be able to see the solar eclipse in Boston?’ Perhaps you could turn to your local newspaper to find the information, or go down to your local library to consult the *Astronomical Almanac*. You may even have an astronomical journal containing the required information, or perhaps some computer software or a website that might do the trick. But you would not, we suspect, think of sitting down and calculating it for yourself. Yet even though you may not find mathematics particularly transparent, you can still do this for yourself. You can quite easily find the answer to many astronomical questions using this book of calculation recipes. You use it just as you would a recipe book in the kitchen – follow the recipe and produce a delicious dish! All you need in addition is a calculator, a piece of paper, a ruler and a pencil. (For those of us with access to a computer, we can use that instead of the calculator and carry out all the calculations in a spreadsheet program as further described below.)

Your calculator does not have to be a very sophisticated device costing a great deal of money; on the other hand it should be a little better than a basic four-function machine. At a minimum, it must have buttons for the trigonometric functions sine, cosine and tangent. It should also be able to find square roots and logarithms. Such calculators generally describe themselves as ‘scientific calculators’. Features other than these are not essential but can make the calculations easier. For example, having a number of separately-addressable memories in which you can store intermediate results would be useful. If you have a programmable calculator, you can write programs to carry out many of the calculations automatically with a subsequent saving of time and effort.

When choosing a calculator, don’t be led astray by arguments about whether ‘reverse Polish notation’ (RPN) or ‘algebraic notation’ (AN) is the better system. Each has its advantages and the same complexity of calculation may be made using either. It is important, however, to read the instructions carefully and to get to know your calculator thoroughly, whatever system it uses. Make sure that you like the ‘feel’ of the keypad, and that pressing a key once results in just one digit appearing in the display. Look out for special functions that can help you, like a key that gives you π (the constant 3.141 592 654), a key that converts between times or angles expressed as hours or degrees, minutes, and seconds, and their decimal equivalents, a key that takes any angle, positive or negative, and returns its equivalent value reduced to the range 0° to 360° , and a key that converts between rectangular and polar coordinates (very useful for removing the ambiguity of 180° on taking the inverse tangent of an angle).

When you go through the worked examples given with each calculation, do not be alarmed if your figures do not match ours exactly. There are several reasons why they may not, including rounding errors

and misprints. You should try to work with at least seven or eight significant figures. If you write your own programs to carry out any of the calculations on a computer, make sure that you use variables having sufficient resolution. Use double precision (eight-byte precision) everywhere if possible.

Having gathered together your writing materials, calculator and book, how do you proceed? Let us take as an example the problem of finding the time of sunrise. Turn to the index and look up 'sunrise'; you are directed to page 112 where you will find a paragraph or two of explanation and a list of instructions with a worked example in the form of a table. We have kept things brief on purpose and have made no attempt to provide mathematical derivations. We have also simplified the calculations. As you work through each step, write down the step number and the result in a methodical fashion. Take care here and it will save you a lot of time later!

Many calculations require you to turn back and forth between different sections. For example, step 1 of 'sunrise' directs you to another section to calculate the position of the Sun. Make the calculations in that section, and then turn back to carry on with step 2. You will find it useful to keep several slips of paper handy as bookmarks.

This book is not intended to match the precision of the results found in the *Astronomical Almanac*. As we have already mentioned, the calculations have deliberately been simplified although they are good enough for most purposes. If you have your own computer, you can use the methods to write programs displaying the evolving Solar System with a precision that is better than the resolution of the computer screen. But those of us with simple pocket calculators can find great satisfaction in simply being able to work out the stars for ourselves and to predict astronomical events with almost magical precision.

A word about spreadsheets – what are they?

In 1979, when the first edition of *Practical Astronomy with your Calculator* was published, very few people had access to a computer. Although home computers were beginning to appear in the high street, they were not the commonplace household accessory we see today. Calculations were made using a calculator, the sophistication of which ranged from the simple four-function device to the versatile programmable reverse-Polish scientific machine. You may already own a calculator that would be suitable for the recipes given here, but you might also own a computer and wish to make the calculations using that instead. If you are good at programming, you could consider using the methods described in this book as a basis for writing your own astronomical software. But most of us don't want to embark on such a project. How then can we use our computers to make astronomical calculations?

One answer is to use a **spreadsheet** program such as Microsoft's *Excel*, or OpenOffice *Calc*. The latter is available at no cost, and described as fully compatible with the former, so if you do not already own a commercial spreadsheet program, then *Calc* might be a good way to go. Once you have loaded the software on to your machine, open the spreadsheet program. The screen display should then look something like Figure I. (Here and throughout the book, toolbars, sidebars and many other features have been removed from the spreadsheet views.)

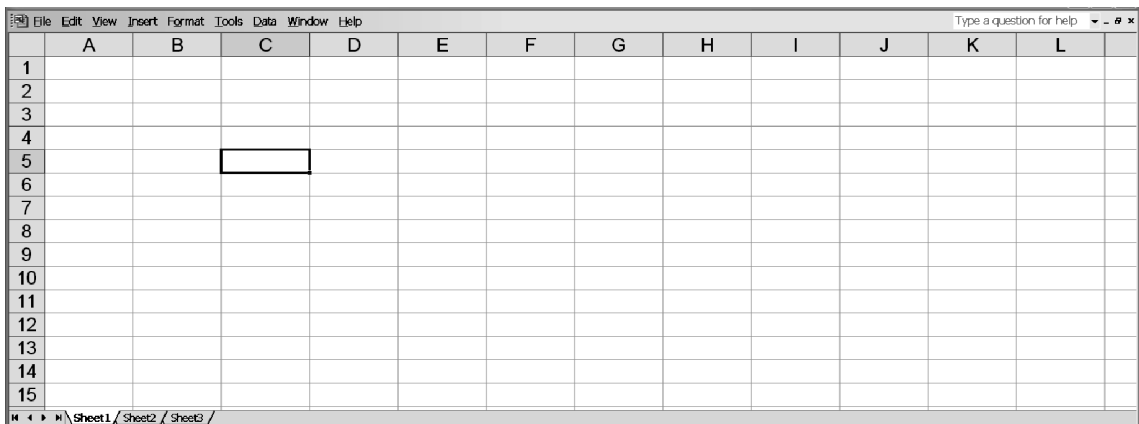


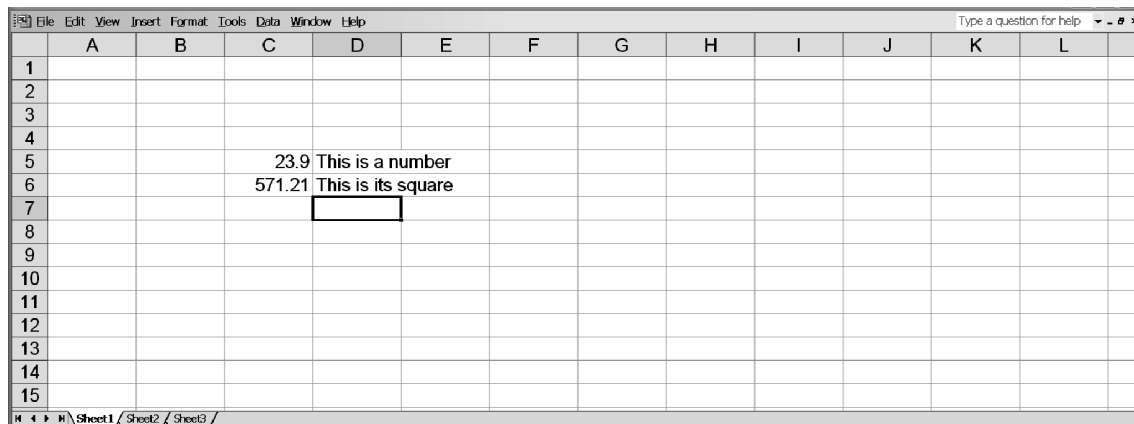
Figure I. An empty spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5			23.9	This is a number								
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												

Figure II. Cell C5 carries the number 23.9, and cell D5 carries the label This is a number.

The spreadsheet consists of an array of cells, labelled A, B, C etc. across the top (these are the column labels) and 1, 2, 3 etc. down the left-hand side (these are the row labels). Each individual cell is labelled by its column letter and its row number, e.g. A1, B25 etc. The cell with the thick border around it in Figure I is cell C5. You can write some text or numbers in any cell. In Figure II, the number 23.9 has been placed in cell C5, and the label This is a number has been placed in cell D5. (Since cell E5 is empty, the program has allowed the label to overwrite the space allocated to E5, although the entire content This is a number remains in D5, and E5 remains empty.) The spreadsheet knows that something placed in a cell is a label (i.e. text) if you begin the entry with a single apostrophe symbol ('). If you want to enter a number as a number, just type it in. If you want the spreadsheet to treat the number as a label, put the apostrophe in front of it.

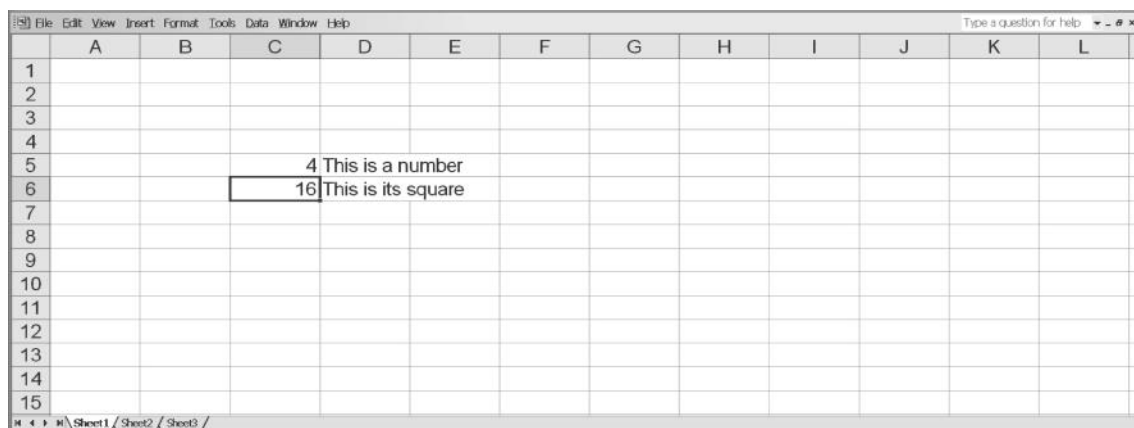
We can obviously put labels and numbers in any of the cells, but the real power of the spreadsheet comes from using formulas. A formula is a calculation which can use the contents of other cells. The result of the calculation is displayed in the cell carrying the formula, so you are not usually aware of the calculation that has gone on in the background since what is displayed is the result rather than the formula itself. A formula is placed in a cell by typing the equals sign (=) followed by the formula. The spreadsheet knows from the equals sign that it is to calculate the formula and display the result. For example, in Figure III, cell C6 carries the entry =C5*C5. You will see that C6 now displays the result of multiplying the number in cell C5 by itself (the star symbol * means 'multiply'), i.e. the square of the number 23.9, which is the number 571.21. We have also placed the label This is its square in cell D6.



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5			23.9	This is a number								
6			571.21	This is its square								
7												
8												
9												
10												
11												
12												
13												
14												
15												

Figure III. Cell C6 carries the formula $=C5*C5$ and hence displays the square of 23.9.

Let's see what happens if now we change the number in cell C5 without making any other change to the spreadsheet. In Figure IV the number in C5 has been changed to the number 4.0 and, hey presto, the square of 4 (i.e. 16) is displayed in cell C6. You can begin to see that complex calculations can be performed for you automatically with a spreadsheet program. With the right formulas placed in order in the spreadsheet, the results can be calculated for any set of starting values. That is just what we want to do in this book. We can hide the complications of the calculation of, say, the time of sunrise within the formulas and just enter a date and geographical location in the correct cells at the top to obtain the result immediately.



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5			4	This is a number								
6			16	This is its square								
7												
8												
9												
10												
11												
12												
13												
14												
15												

Figure IV. Cell C5 now carries the number 4 and so cell C6 displays the number 4 multiplied by 4 which is 16.

We don't need to explain much more about spreadsheets here, although we will note various techniques as we go along. If you want to learn more about their powerful capabilities we suggest buying a book about spreadsheets (see the Bibliography on page 208 for a suggestion). In this book, we have supplied you with the spreadsheet and formulas for most calculations, so all you have to do is to type in the labels, numbers and formulas as shown. The spreadsheet will then do its work automatically and give you the answer for

any starting values you enter. (We have provided the spreadsheets ready-made on our website. Please look in the section “A useful website” on page 209 for details.)

The layout of spreadsheets in this book

All of the spreadsheets in this book conform to the same general format (see Figure V). At the top, in cell A1, is the title of the spreadsheet (in this case Converting decimal hours to hours, minutes and seconds). It is best to use a slightly larger font size for this and to make it boldface as here. We have used Arial 16 point for the title. Row number 2 is left blank (i.e. none of the cells has anything in it). In row 3, we have written the label *Input* in A3 (Times New Roman font, italic face, 10 point) to remind us that the input values for the spreadsheet are entered to the right of this cell. In the case shown in Figure V, there is only one input value, the decimal hours (name label in B3, Arial font, bold face, 14 point), and it is entered in cell C3 (also Arial font, bold face, 14 point). In spreadsheets which have more than one input value, the others have their name labels in cells B4, B5 etc. and their corresponding values in C4, C5 etc.

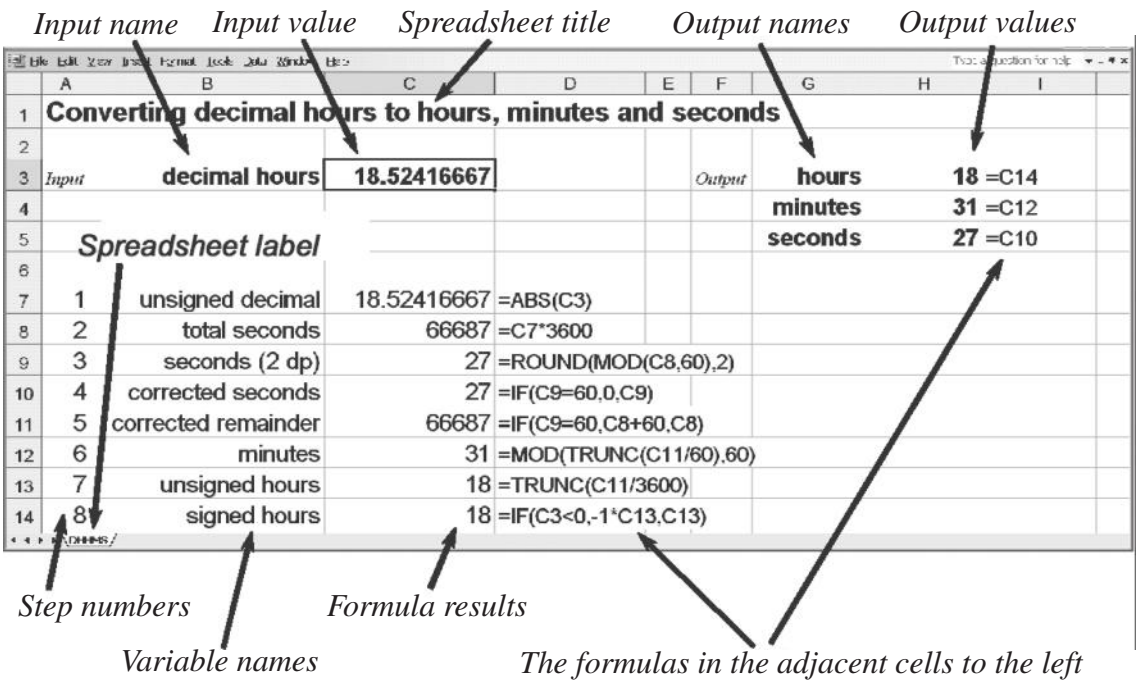


Figure V. The layout of a spreadsheet.

The results of the calculations, i.e. the output values, are provided to the right of cell F3. We have written the label *Output* in F3 (Times New Roman font, italic face, 10 point) to remind us that the output values calculated by the spreadsheet appear to the right of this cell. In the case shown in Figure V there are three output values, called hours, minutes, seconds. Their name labels appear in cells G3, G4, G5 (Arial font, bold face, 14 point) and their values in H3, H4, H5 (also Arial font, bold face, 14 point) respectively. Just to the right of the three output values, in column I, are shown the formulas (written as labels, i.e. with an

apostrophe in front of the equals sign to stop the program calculating the formula) that are actually in the output value cells. Thus cell H3 actually contains the formula =C14 (i.e. it will display the value of the cell C14) and you will need to enter =C14 in the cell H3. Wherever you see a formula (anything beginning with the equals sign) enter exactly that formula in the cell immediately to its left. In this case you would put =C14 in cell H3, =C12 in cell H4, and =C10 in cell H5.

The calculations carried out by the spreadsheet begin on row 7 in Figure V. Each row corresponds to one step in the calculation, in this case the calculation method of Section 8. In the method table shown in that section there are just two steps, whereas in the spreadsheet there are eight. There is only a rough correspondence between method steps and spreadsheet steps. This is partly because the spreadsheet calculations do not have the benefit of human intelligence to assist them! For example, if you used your calculator to carry out the steps of Section 8, and you found that the result was, say, 6h 35m 60s, you would automatically write this as 6h 36m 0s. The spreadsheet would, however, quite happily report the result in the first format. We get over the problem in the spreadsheet by first stripping out the sign, then converting to seconds, then finding the seconds, minutes and hours in that order, and finally putting back the sign.

In the example shown in Figure V, you would enter the labels and formulas exactly as shown. Thus on row 7 you place the label '1 in A7 (this is text, and the apostrophe tells the spreadsheet so), the label 'unsigned decimal in B7 and the formula in C7 shown immediately to its left, i.e. =ABS(C3). Do this for each calculation row (7 to 14 in this case). Finally, rename the spreadsheet on the tab at the bottom (DHHMS in this case). (You can probably do this by pointing at it with the mouse, pressing the right-hand mouse button, and selecting the 'rename' option.)

Although the labels in columns A and B make no difference to the calculations, we recommend that you put them in as they make the spreadsheet much easier to understand. This becomes more important if you return to a spreadsheet some time after you constructed it.

One other note about spreadsheets that you might find useful concerns column widths. If the column width is too narrow to display the content of a cell, you may just see something like ##### displayed instead. You can adjust the column width by placing the mouse pointer on the division between the label (A, B, C etc.) of the column you want to alter and the label of the column immediately to its right, holding down the left-hand mouse button, and 'dragging' the column width left or right as needed.

Calculations involving multiple sheets

Some of the spreadsheet calculations, as in the example just given, use just one sheet. Most, however, use several. For example, suppose that a first spreadsheet calculation results in a number expressed in decimal hours but the answer has to be in the form hours, minutes and seconds. The first sheet passes its answer (in decimal hours) to a second sheet which carries out the conversion and passes the converted result back again to the first sheet.

A concrete example is illustrated by a spreadsheet for Section 14, reproduced in Figure VI. You will see that there are three tabs in the bottom left-hand corner, corresponding to three sheets labelled GSTLST, HMSDH and DHHMS. Only the top sheet, GSTLST is visible in the figure with the other two lying 'underneath' it. The input values to the calculation include the Greenwich sidereal time (GST) expressed in hours, minutes and seconds (cells C3, C4 and C5). These must first be converted to the GST expressed in decimal hours, a calculation covered in Section 7. The spreadsheet for that section, labelled HMSDH, must

	A	B	C	D	E	F	G	H	I
1	Conversion of GST to LST								
2									
3	<i>Input</i>	GST hours	4		<i>Output</i>	LST hour	0	=C12	
4		GST mins	40			LST min	24	=C13	
5		GST secs	5.23			LST sec	5.23	=C14	
6		geog long	-64						
7									
8	1	GST	4.668119444	=HMSDH!H3					
9	2	offset	-4.266666667	=C6/15					
10	3	LST (hours)	0.401452778	=C8+C9					
11	4	LST (hours)	0.401452778	=C10-(24*INT(C10/24))					
12	5	LST hour	0	=DHHMS!H3					
13	6	LST min	24	=DHHMS!H4					
14	7	LST sec	5.23	=DHHMS!H5					

Figure VI. A spreadsheet with multiple sheets.

	A	B	C	D	E	F	G	H	I
1	Converting hours, minutes and seconds to decimal hours								
2									
3	<i>Input</i>	hours	4	=GSTLST!C3	<i>Output</i>	decimal hours	4.668119444	=C10	
4		minutes	40	=GSTLST!C4					
5		seconds	5.23	=GSTLST!C5					
6									
7	1	A	0.087166667	=ABS(C5)/60					
8	2	B	0.668119444	=(ABS(C4)+C7)/60					
9	3	C	4.668119444	=ABS(C3)+C8					
10	4	D	4.668119444	=IF((C3<0)+(C4<0)+(C5<0),-C9,C9)					

Figure VII. Illustrating cross-references between sheets.

be included in this spreadsheet file as an additional sheet – the tab HMSDH in Figure VI. Figure VII shows the spreadsheet with the HMSDH sheet on top so it is visible.

The link between the sheets is accomplished by using the sheet name, followed by an exclamation mark (!) and the cell reference. In Figure VII, the input value of hours (C3) is obtained from cell C3 of sheet GSTLST by using the formula =GSTLST!C3. Similarly, the input value of minutes is obtained using the formula =GSTLST!C4 in cell C4 of HMSDH, and the input value of seconds is obtained by using the formula =GSTLST!C5 in cell C5. The result of the calculation by this sheet, the decimal hours, appears in cell H3

of Figure VII. This is passed back to sheet GSTLST in cell C8 of Figure VI, which contains the formula =HMSDHIH3.

Similarly, the result of the calculation of GSTLST, expressed in decimal hours, appears in cell C11 of Figure VI. This needs to be converted to the format hours, minutes and seconds and it is passed to sheet DHHMS (see Figure VIII) by using the formula =GSTLST!C11 in cell C3 of that sheet. Sheet GSTLST then extracts the results from sheet DHHMS (H3, H4 and H5 of Figure VIII) using the formulas DHHMS!H3, DHHMS!H4 and DHHMS!H5 respectively in cells C12, C13, and C14 of Figure VI.

File Edit View Insert Format Tools Data Window Help									
Type a question for help									
C3	A	B	C	D	E	F	G	H	I
1	Converting decimal hours to hours, minutes and seconds								
2									
3	Input	decimal hours	0.401452778	=GSTLST!C11		Output	hours	0	=C14
4							minutes	24	=C12
5							seconds	5.23	=C10
6									
7	1	unsigned decimal	0.401452778	=ABS(C3)					
8	2	total seconds	1445.23	=C7*3600					
9	3	seconds (2 dp)	5.23	=ROUND(MOD(C8,60),2)					
10	4	corrected seconds	5.23	=IF(C9=60,0,C9)					
11	5	corrected remainder	1445.23	=IF(C9=60,C8+60,C8)					
12	6	minutes	24	=MOD(TRUNC(C11/60),60)					
13	7	unsigned hours	0	=TRUNC(C11/3600)					
14	8	signed hours	0	=IF(C3<0,-1*C13,C13)					

Figure VIII. Illustrating cross-referencing between sheets.

Now you can proceed in this way if you wish, using multiple sheets to carry out specific calculations as just described, but the result can be quite confusing when you have a complicated calculation requiring many sheets. A better way to proceed is for us to define our own functions and use these instead to carry out the calculations. This is the approach that we have adopted here.

Using our own functions

Microsoft *Excel* and OpenOffice *Calc* both come with an internal programming language called BASIC. We don't need to go in to any of the details of what this is and how it works, but suffice it to say that we have written functions to carry out most of the calculations described in this book. All you have to do is to use the functions in your spreadsheet exactly as if they were formulas. This has the advantage that you now need only one sheet for any calculation with no cross-linking to multiple sheets, making the whole thing easier to comprehend. Another advantage is that we have provided functions with much higher accuracy than the simplified calculations of many of the sections. For example, you can use the method of Section 46 to calculate the Sun's ecliptic longitude approximately, or you can use the function SunLong to calculate it much more precisely.

Let us illustrate the use of functions instead of multiple sheets using the example above. Figure IX shows the spreadsheet of Section 14 using functions instead of multiple sheets. Compare this with Figure VI. You can see that in Figure IX there is now only one sheet, labelled GSTLST.

	A	B	C	D	E	F	G	H	I
1	Conversion of GST to LST								
2									
3	<i>Input</i>	GST hours	4		<i>Output</i>	LST hour	0	=C12	
4		GST mins	40			LST min	24	=C13	
5		GST secs	5.23			LST sec	5.23	=C14	
6		geog long	-64						
7									
8	1	GST	4.668119444	=HMSDH(C3,C4,C5)					
9	2	offset	-4.266666667	=C6/15					
10	3	LST (hours)	0.401452778	=C8+C9					
11	4	LST (hours)	0.401452778	=C10-(24*INT(C10/24))					
12	5	LST hour	0	=DHHour(C11)					
13	6	LST min	24	=DHMin(C11)					
14	7	LST sec	5.23	=DHSec(C11)					

Figure IX. Illustrating the use of functions instead of multiple sheets.

The results of the calculation, contained in cells H3, H4 and H5 in both Figures VI and IX, are identical, but in place of the cross-references between sheets at C8, C12, C13 and C14 of Figure VI there are formulas in the corresponding cells of Figure IX. In cell C8, for example, the formula =HMSDH(C3,C4,C5) converts the hours, minutes and seconds (in cells C3, C4 and C5) to decimal hours, with the result shown in cell C8. The contents of C3, C4 and C5 are passed to the function HMSDH as the references contained within the brackets after the function. When the spreadsheet program sees a formula, in this case =HMSDH(C3,C4,C5), it first looks through a list of its own formulas, and then checks to see if the function has been written in BASIC. If it has, the spreadsheet then runs the BASIC program corresponding to the function, passing the contents of the cells in the reference list to the BASIC program, in this case the contents of cells C3, C4 and C5. The result of the calculation is then passed back to the spreadsheet where it appears in the same cell as the function (C8).

Functions like this have been provided for most of the calculations in this book, and are described in the corresponding sections. You will need to download the spreadsheets from the Cambridge University Press website in order to obtain the functions (which are included invisibly with each sheet). Please look in the section “A useful website” on page 209 for details.

Time

Astronomers have always been concerned with time and its measurement. If you read any astronomical text on the subject you are sure to be bewildered by the seemingly endless range of times and their definitions. There's universal time and Greenwich mean time, apparent sidereal time and mean sidereal time, ephemeris time, local time, mean solar time and atomic time, to name but a few. Then there's the sidereal year, the tropical year, the Besselian year and the anomalistic year. And be quite clear about the distinction between the Julian and Gregorian calendars! (See the Glossary for the definitions of these terms.)

All these terms are necessary and have precise definitions. Happily, however, we need concern ourselves with but a few of them as the distinctions between many of them become apparent only when very high accuracy is required.

1 Calendars

A **calendar** helps us to keep track of time by dividing the year into months, weeks and days. Very roughly speaking, one **month** is the time taken by the Moon to complete one circuit of its orbit around the Earth, during which time it displays four phases, or quarters, of one week each, and a **year** is the time taken for the Earth to complete one circuit of its orbit around the Sun. In the Gregorian calendar, generally adopted in the West, we assume the convention that there are seven days in each week, between 28 and 31 in each month (see Table 1) and 12 months in each year. (Note that there are many other calendars, such as the Chinese calendar, with different definitions or rules.) By knowing the day number, and the name of the month, we are able to refer precisely to any day of the year.

January	31	July	31
February	28 (or 29 in a leap year)	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

Table 1. The number of days in each month.

The problem with this method of accounting for the days in the year lies in the fact that, whereas there is always a whole number of days in the civil year, the Earth actually takes about 365.2422 days to complete one circuit of its orbit around the Sun. (This is the **tropical year**; see the Glossary for its definition.) If we were to take no notice of this fact and adopt 365 days for every year, then the Earth would get progressively more out of step with the civil calendar at a rate of 0.2422 days per year. After 100 years the discrepancy would be about 24 days; after 1500 years the seasons would have been reversed so that summer in the northern hemisphere would be in December. Clearly, this system would have great disadvantages.

Julius Caesar made an attempt to put matters right by adopting the convention that three consecutive years have 365 days followed by a **leap year** of 366 days, the extra day being added to February whenever the year number is divisible by four. On average, his civil year has 365.25 days in it, a better approximation to the tropical year of 365.2422 days. Indeed, after 100 years the discrepancy is less than one day. This is the **Julian calendar** and it worked very well for many centuries until, by 1582, there was again an appreciable discrepancy between the seasons and the calendar date. Pope Gregory XIII (1502–1585) then improved on the system by abolishing the days 5 October to 14 October 1582 inclusive so as to bring the civil and tropical years back into line, and by missing out three days every four centuries. In his reformed calendar, the years ending in two zeroes (e.g. 1700, 1800, 1900 etc.) are only leap years if they are also exactly divisible by 400. Thus the year 2000 was a leap year, whereas 1700, 1800 and 1900 were not.

This system is called the **Gregorian calendar** and is the one in most general use today. According to it 400 civil years contain $(400 \times 365) + 100 - 3 = 146\,097$ days, so that the average length of the civil year is $146\,097/400 = 365.2425$ days, a very good approximation indeed to the length of the tropical year. In fact, this calendar will not get out of step with the tropical year for many millions of years!

2 The date of Easter

Easter Day, which always occurs on a Sunday, is the day to which such moveable feasts as Whitsun and Trinity Sunday in the Christian calendar are fixed, and is defined in *The Explanatory Supplement to the Astronomical Almanac* (1992) as follows:

In the Gregorian calendar, the date of Easter is defined to occur on the Sunday following the ecclesiastical full moon that falls on or next after March 21st.

The problem is that the ecclesiastical full Moon is not the same as the astronomical full Moon. The former is based on a set of tables which do not take into account the complexity of the Moon's motion. As a fair guide, we may say that Easter Day is usually the first Sunday after the fourteenth day after the first new Moon after 21 March. Several authors have provided algorithms for calculating the date of Easter. You can, for example, use the methods and tables given in the *Book of Common Prayer* (1662) or that given in the *Explanatory Supplement*. Here we describe a method devised in 1876 which first appeared in *Butcher's Ecclesiastical Calendar*, and which is valid for all years from 1583 onwards. It makes repeated use of the result of dividing one number by another number, the integer part being treated separately from the remainder. A calculator displays the result of a division as a string of numbers either side of a decimal point. The numbers appearing before (i.e. to the left of) the decimal point constitute the integer part; the decimal point and the numbers after (i.e. to the right of) the decimal point constitute the fractional part. The remainder may be found from the latter (including the leading decimal point) by multiplying it by the divisor (i.e. the number that you divided by) and rounding the result to the nearest integer value. For example, $2000/19 = 105.263\ 1579$. The integer part is 105, and the fractional part is 0.263 1579. Multiplying the latter by 19 gives 5.000 000 100 so the remainder is 5.

We shall illustrate the method by calculating the date of Easter Day in the year 2009. This will give us practice for the sort of calculation we will be carrying out in the rest of this book.

Method		Example	
	<i>Integer part</i>	<i>Remainder</i>	
1.	Divide the year by 19.	a	$\frac{2009}{19} = 105.7368421$ $a = 14$
2.	Divide the year by 100.	b	$\frac{2009}{100} = 20.090000$ $b = 20$ $c = 9$
3.	Divide b by 4.	d	$d = 5$ $e = 0$
4.	Divide $(b + 8)$ by 25.	f	$f = 1$
5.	Divide $(b - f + 1)$ by 3.	g	$g = 6$
6.	Divide [†] $(19a + b - d - g + 15)$ by 30.	h	$(19a + b - d - g + 15) = 290$ $h = 20$
7.	Divide c by 4.	i	$i = 2$ $k = 1$
8.	Divide $(32 + 2e + 2i - h - k)$ by 7.	l	$l = 1$
9.	Divide $(a + 11h + 22l)$ by 451.	m	$(a + 11h + 22l) = 256$ $m = 0$
10.	Divide $(h + l - 7m + 114)$ by 31.	n	$(h + l - 7m + 114) = 135$ $n = 4$ $p = 11$
11.	The day of the month on which Easter Day falls is $p + 1$. The month number is n (=3 for March, =4 for April). Therefore Easter Day 2009 is		$p + 1 = 12$ $n = 4$, so April 12 April

[†] $19a$ means 19 multiplied by a ($19 \times 14 = 266$ in this example).

The spreadsheet for this calculation, called DOE (the acronym for Date Of Easter), is shown in Figure 1. It makes repeated use of two spreadsheet functions, TRUNC and MOD. (These are all examples of built-in, or intrinsic, spreadsheet functions; we will make use of many of the useful ones throughout this book.) The former truncates the number at the decimal point, so gives you the integer part of the number. Thus TRUNC(23.445) is 23. In cell C8 of the spreadsheet, the formula =TRUNC(C3/100) takes the number from cell C3 (2009 in this case), divides it by 100, and returns the integer part of the result (20).

The MOD function has two arguments separated by a comma. (An argument is a number or a reference within the brackets immediately following the function name. Two or more arguments are separated by commas.[‡]) The first argument is divided by the second argument, and then the remainder of the result is returned. Thus MOD(13,5) is 3 since 5 goes into 13 twice ($2 \times 5 = 10$) leaving a remainder of 3 (i.e. $13 - 10$). In cell C7 of the spreadsheet, the formula =MOD(C3,19) takes the number from cell C3 (2009 in this case), divides it by 19, and returns the remainder (14).

We have used the spreadsheet function IF in cell H4 to replace the month number, 3 or 4, with its name equivalent, 'March' or 'April'. The IF function takes three arguments. The first is the test argument, which can be 'true' or 'false'. In this case, the test argument is C22=3, i.e. if the number in cell C22 is equal to 3 the result of the test is 'true', and if not it is 'false'. In this case, the number in cell C22 is 4 so the test returns 'false'. The IF function returns the second argument (March in this case) if the test returns 'true', or the third argument (April in this case) if the test returns 'false', as here.

[‡] Some spreadsheet programs use different separators; check yours.

	A	B	C	D	E	F	G	H	I	J	K
1	Date of Easter										
2											
3	<i>Input</i>	year	2009			<i>Output</i>	day	12	=C21		
4							month	April	=IF(C22=3,"March","April")		
5							year	2009	=C3		
6											
7	1	<i>a</i>		14 =MOD(C3,19)							
8	2	<i>b</i>		20 =TRUNC(C3/100)							
9	3	<i>c</i>		9 =MOD(C3,100)							
10	4	<i>d</i>		5 =TRUNC(C8/4)							
11	5	<i>e</i>		0 =MOD(C8,4)							
12	6	<i>f</i>		1 =TRUNC((C8+8)/25)							
13	7	<i>g</i>		6 =TRUNC((C8-C12+1)/3)							
14	8	<i>h</i>		20 =MOD((19*C7)+C8-C10-C13+15,30)							
15	9	<i>i</i>		2 =TRUNC(C9/4)							
16	10	<i>k</i>		1 =MOD(C9,4)							
17	11	<i>l</i>		1 =MOD(32+2*(C11+C15)-C14-C16,7)							
18	12	<i>m</i>		0 =TRUNC((C7+(11*C14)+(22*C17))/451)							
19	13	<i>n</i>		4 =TRUNC((C14+C17-(7*C18)+114)/31)							
20	14	<i>p</i>		11 =MOD(C14+C17-(7*C18)+114,31)							
21	15	day		12 =C20+1							
22	16	month		4 =C19							

Figure 1. Calculating the date of Easter Day 2009.

You can put any year after 1582 you like into cell C3 of the spreadsheet in place of 2009 and the date of Easter Day for that year will be calculated for you automatically. Try 2012. The answer should be 8 April.

3 Converting the date to the day number

In many astronomical calculations, we need to know the number of days in the year up to a particular date. We shall choose our starting point as 0 hours on 0 January, equivalent to the midnight between 30 and 31 December of the previous year. This might seem to be a peculiar choice, but you will see that it simplifies our calculations so is a good one to make. Midday on 3 January can then be expressed as January 3.5 since precisely three and a half days have elapsed since January 0.0. This is illustrated in Figure 2. Finding the day number from the date is then a simple matter. Proceed as follows:

1. For every month up to, but not including, the month in question add the appropriate number of days according to Table 1. These totals are listed in Table 2.
2. Add the day of the month.

For example, what is the day number of 19 June (not a leap year)? The answer is day number = $151 + 19 = 170$. If you own a programmable calculator, you may be able to use Routine R1 (at the end of this section) to write a program to make this calculation automatically. We can also use the method of the section on Julian day numbers (Section 4) as an alternative.

Later in this book we adopt the date 2010 January 0.0 as the starting point, or starting **epoch**, from which to calculate orbital positions. Days elapsed since this epoch at the beginning of each year (January 0.0) from 1990 to 2029 are tabulated in Table 3. To find the total number of days elapsed since the epoch, simply add the number of days elapsed to the beginning of the year since the epoch (Table 3) to the number of days elapsed since January 0.0 of the year in question (i.e. the result of the calculation of the previous paragraph). For example, the number of days elapsed since the epoch at 6 pm on 19 June 2009 is $-365 + 170 + 0.75 = -194.25$. The negative sign indicates that the epoch is after this date. The fraction of the day to 6 pm is $(18/24) = 0.75$ since 6 pm is 18 h on a 24-hour clock, and there are 24 hours in the day.

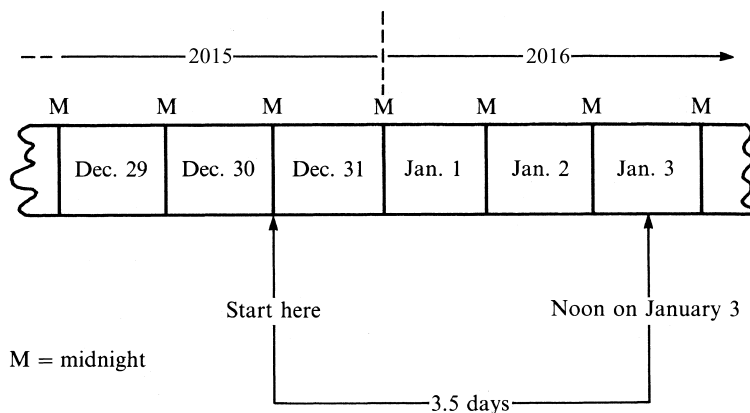


Figure 2. Defining the epoch.

	Ordinary year	Leap year
January	0	0
February	31	31
March	59	60
April	90	91
May	120	121
June	151	152
July	181	182
August	212	213
September	243	244
October	273	274
November	304	305
December	334	335

Table 2. The number of days to the beginning of the month.

1990	−7305	*2000	−3653	2010	0	*2020	3652
1991	−6940	2001	−3287	2011	365	2021	4018
*1992	−6575	2002	−2922	*2012	730	2022	4383
1993	−6209	2003	−2557	2013	1096	2023	4748
1994	−5844	*2004	−2192	2014	1461	*2024	5113
1995	−5479	2005	−1826	2015	1826	2025	5479
*1996	−5114	2006	−1461	*2016	2191	2026	5844
1997	−4748	2007	−1096	2017	2557	2027	6209
1998	−4383	*2008	−731	2018	2922	*2028	6574
1999	−4018	2009	−365	2019	3287	2029	6940

*Denotes a leap year.

Table 3. The number of days to the beginning of the year since the epoch 2010 January 0.0.

Routine R1: Converting the date to the day number.

1. Key in the month number (e.g. 11 for November).
2. Is it greater than 2?
 - If yes, go to step 8.
 - If no, proceed with step 3.
3. Subtract 1 from the month number.
4. Multiply by 63 in an ordinary year, or 62 in a leap year.
5. Divide by 2.
6. Take the integer part.
7. Go to step 12.
8. Add 1 to the month number.
9. Multiply by 30.6.
10. Take the integer part.
11. Subtract 63 in an ordinary year, or 62 in a leap year.
12. Add the day of the month. The result is the day number.

4 Julian dates

It is sometimes necessary to express an instant of observation as so many days and a fraction of a day after a given fundamental epoch. Astronomers have chosen this fundamental epoch as the Greenwich mean noon of 1 January 4713 BC, that is midday as measured on the Greenwich meridian on 1 January of that year. (You can look up the meaning of **meridian** and other technical terms in the Glossary at the back of the book starting on page 197.) The number of days that have elapsed since that time is referred to as the **Julian day number**, or **Julian date**[†]. It is important to note that each new Julian day begins at 12h 00m UT, half a day out of step with the civil day in time zone 0. (See Section 9, or the Glossary, for the precise meaning of UT.)

The term '**Before Christ**', or BC for short, usually refers to the chronological system of reckoning negative years. In this system, there is no year zero. The **Christian Era** begins with the year 1 AD (short for **Anno Domini**); the year immediately preceding this is 1 BC. Some authors have adopted different labels for the same things by referring to the Christian Era as the **Common Era** instead. They retain the same numeric values for the days, but use the label CE (Common Era) instead of AD, and BCE (**Before the Common Era**) instead of BC.

For astronomical purposes, we want to count the years logically without a gap. Thus the year immediately preceding 1 AD is designated 0; the other years BC are denoted by negative numbers, each of which has an absolute value (i.e. the number without its minus sign) which is one less than the BC value. Thus

[†]Sometimes the **modified Julian date**, MJD, is quoted. This is equal to the Julian date minus 2400000.5; MJD zero therefore begins at 0h on 17 November 1858.

10 BC corresponds to the astronomical year -9 , and 4713 BC corresponds to -4712 . We shall adopt the astronomical way of counting throughout this book. Where you see a BC (or BCE) year, subtract one from it and change its sign to negative before using it in any of the calculations. Similarly, if the result of a calculation is a negative year, remove the minus sign, add one to the year number, and append the letters BC (or BCE) after it.

The Julian date of any day in the Julian or Gregorian calendars may be found by the method given below. Here, and throughout the book, the expression TRUNC refers to the integer part of the number (i.e. the part preceding the decimal point). Thus TRUNC(22.456) is 22, and TRUNC(-3.914) is -3 . You will need to look carefully in the instruction book of your calculator to see what function is offered on your machine. On ours, this is called INT (short for integer). Note that computer languages offer several truncation functions such as INT, FIX, FLOOR and TRUNC. These do similar things with positive numbers, but beware what they do with negative ones. For example, INT on some machines returns the largest (most positive) integer whose value is less than or equal to the number. In this case, INT(-3.914) is -4 . Beware! You can avoid this worry by taking INT of the absolute value of the number, and then inserting a negative sign in front of the result for a negative number.

A further complication, but an important one, is to distinguish between the local date, i.e. the calendar date at your location, and the corresponding Greenwich date, i.e. the calendar date on longitude 0° with no daylight saving. These are often not the same. For example, if you live in Sydney, Australia, your time may be 10 or 11 hours ahead of the time at Greenwich depending on whether daylight saving time is in operation. If it is 03:45 in the early morning in Sydney, and the time-zone correction is +10 hours with daylight saving adding a further hour, the corresponding time at Greenwich is 11 hours behind, i.e. 16:45 the previous day. In this case, your local calendar date and the Greenwich date differ by 1 day. We therefore need to be precise about what we mean by the 'date'. Look to see whether it is the Greenwich date or the local date that is required in a given calculation.

As an example, we shall calculate the Julian date corresponding to the Greenwich calendar date of 2009 June 19.75 (i.e. 6 pm on 19 June).

Method	Example
1. Set y = year, m = month and d = day.	$y = 2009$ $m = 6$ $d = 19.75$
2. If $m = 1$ or 2 , set $y' = y - 1$ and $m' = m + 12$; otherwise $y' = y$ and $m' = m$.	$y' = 2009$ $m' = 6$
3. If the date is later than 1582 October 15 (i.e. in the Gregorian calendar) calculate: (a) $A = \text{TRUNC}(y'/100)$; (b) $B = 2 - A + \text{TRUNC}(A/4)$. Otherwise $B = 0$.	$A = \text{TRUNC}(2009/100)$ so $A = 20$ $B = 2 - 20 + \text{TRUNC}(20/4)$ so $B = -13$
4. If y' is negative calculate $C = \text{TRUNC}((365.25 \times y') - 0.75)$. Otherwise, $C = \text{TRUNC}(365.25 \times y')$.	$C = \text{TRUNC}(365.25 \times 2009)$ so $C = 733\,787$
5. Calculate $D = \text{TRUNC}(30.6001 \times (m' + 1))$.	$D = \text{TRUNC}(30.6001 \times 7)$ $D = 214$
6. Find $\text{JD} = B + C + D + d + 1720994.5$. This is the Julian date.	$\text{JD} = 2455\,002.25$

The Julian date corresponding to our adopted starting epoch of 2010 January 0.0 is 2455 196.5. We can easily find the number of days that have elapsed since the epoch by subtracting this number from the Julian date. Thus the number of days elapsed since the epoch to 2009 June 19.75 is $2455002.25 - 2455 196.5 = -194.25$, as found in the previous section.

The spreadsheet for the calculation of the Julian date is called CDJD (the acronym for Calendar Date to Julian Date conversion) and is shown in Figure 3. We have also provided a spreadsheet function of the same name, i.e. CDJD(GD,GM,GY), which takes three arguments GD, GM, and GY. These have exactly the same values as the input values to the spreadsheet CDJD, and represent, respectively, the calendar day, month and year at Greenwich. You could carry out exactly the same calculation as that shown in Figure 3 by deleting rows 7 to 16 entirely and replacing cell H3 with the formula =CDJD(C3,C4,C5). Why not try this for yourself (but save a copy of the full spreadsheet first)?

	A	B	C	D	E	F	G	H	I	J
1	Greenwich calendar date to Julian date conversion									
2										
3	Input	Greenwich day	19.75			Output	Julian date	2455002.25	=C16	
4		Greenwich month	6							
5		Greenwich year	2009							
6										
7	1	y	2009	=C5						
8	2	m	6	=C4						
9	3	d	19.75	=C3						
10	4	yd	2009	=IF(C8<3,C7-1,C7)						
11	5	md	6	=IF(C8<3,C8+12,C8)						
12	6	A	20	=TRUNC(C10/100)						
13	7	B	-13	=IF((C7>1582)+((C7=1582)*(C8>10))+((C7=1582)*(C8=10)*(C9>=15)),2-C12+TRUNC(C12/4),0)						
14	8	C	733787	=IF(C10<0,TRUNC((365.25*C10)-0.75),TRUNC(365.25*C10))						
15	9	D	214	=TRUNC(30.6001*(C11+1))						
16	10	JD	2455002.25	=C13+C14+C15+C9+1720994.5						

Figure 3. Finding the Julian date corresponding to the Greenwich calendar date of 6 pm on 19 June 2009.

5 Converting the Julian date to the Greenwich calendar date

It is sometimes necessary to convert a given Julian date into its counterpart in the Gregorian calendar, i.e. the calendar date at Greenwich. As mentioned in the previous section, the calendar date at Greenwich is not necessarily the same as the local calendar date where you are, but depends upon the local time, your time-zone correction, and the number of hours (if any) of daylight saving in operation. We will discuss this further in Section 9.

The method shown here works for all dates from 1 January 4713 BC[†]. For example, let us find the calendar date at Greenwich corresponding to the Julian date JD = 2 455 002.25.

Method	Example
1. Add 0.5 to JD. Set I = integer part and F = fractional part.	$JD + 0.5 = 2455002.75$ $I = 2455002$ $F = 0.75$
2. If I is larger than 2 299 160, calculate: (i) $A = \text{TRUNC}\left(\frac{I - 1867216.25}{36524.25}\right)$; (ii) $B = I + A - \text{TRUNC}(A/4) + 1$. Otherwise, set $B = I$.	$A = 16.0$ $B = 2455015.0$
3. Calculate $C = B + 1524$.	$C = 2456539.0$
4. Calculate $D = \text{TRUNC}\left(\frac{C - 122.1}{365.25}\right)$.	$D = 6725.0$
5. Calculate $E = \text{TRUNC}(365.25 \times D)$.	$E = 2456306.0$
6. Calculate $G = \text{TRUNC}\left(\frac{C - E}{30.6001}\right)$.	$G = 7.0$
7. Calculate $d = C - E + F - \text{TRUNC}(30.6001 \times G)$. This is the day of the month including the decimal part of the day.	$d = 19.75$
8. Calculate $m = G - 1$ if G is less than 13.5, or $m = G - 13$ if G is more than 13.5. This is the month number.	$m = 6$
9. Calculate $y = D - 4716$ if m is more than 2.5, or $y = D - 4715$ if m is less than 2.5. This is the calendar year.	$y = 2009$

Hence the date at Greenwich in the Gregorian calendar is 2009 June 19.75, or 6 pm on 19 June of that year. Figure 4 shows the spreadsheet for this calculation. The single input value is the Julian date, entered in cell C3, and the three output values, the day (including the fraction), month and year of the corresponding calendar date at Greenwich, appear in cells H3, H4 and H5 respectively.

The spreadsheet is called JDCD, corresponding to the acronym for Julian Date to Calendar Date conversion. We have also supplied spreadsheet functions to carry out the same calculations as formulas in a spreadsheet. There are three of them since a single function can only return a single value, and we need three, i.e. the day, the month and the year. The function names are respectively JDCDay(JD), JDCMonth(JD) and JDCYear(JD), and each takes the single argument JD which must be set equal to the Julian date. You can replace the calculation part of the spreadsheet shown in Figure 4 with these three functions by deleting rows 7 to 17 and replacing cells H3, H4, and H5 by the formulas =JDCDay(C3), =JDCMonth(C3) and =JDCYear(C3) respectively. Try it for yourself, but remember to save the spreadsheet first.

[†] See the previous section about the meaning of the term BC.

	A	B	C	D	E	F	G	H	I
1	Julian date to Greenwich calendar date conversion								
2									
3	<i>Input</i>	Julian date	2455002.25			<i>Output</i>	Greenwich day	19.75	=C15
4							Greenwich month	6	=C16
5							Greenwich year	2009	=C17
6									
7	1	I	2455002	=TRUNC(C3+0.5)					
8	2	F	0.75	=C3+0.5-C7					
9	3	A	16	=TRUNC((C7-1867216.25)/36524.25)					
10	4	B	2455015	=IF(C7>2299160,C7+1+C9-TRUNC(C9/4),C7)					
11	5	C	2456539	=C10+1524					
12	6	D	6725	=TRUNC((C11-122.1)/365.25)					
13	7	E	2456306	=TRUNC(365.25*C12)					
14	8	G	7	=TRUNC((C11-C13)/30.6001)					
15	9	d	19.75	=C11-C13+C8-TRUNC(30.6001*C14)					
16	10	m	6	=IF(C14<13.5,C14-1,C14-13)					
17	11	y	2009	=IF(C16>2.5,C12-4716,C12-4715)					

Figure 4. Finding the calendar date at Greenwich corresponding to the Julian date of 2 455 002.25.

6 Finding the name of the day of the week

It is sometimes useful to know on what day of the week a particular date will fall. For instance, you might want to know whether your birthday will be on Sunday next year, or – perhaps working out your holiday entitlement around Christmas – which day of the week corresponds to Christmas Day. This can be found easily from the Julian date using the following calculation in which we find the name of the day of the week corresponding to 19 June 2009 at Greenwich as an example.

Method	Example
1. Find the Julian date corresponding to midnight at Greenwich (§4).	2009 June 19.0 JD = 2455001.5
2. Calculate $A = \left(\frac{JD+1.5}{7} \right)$.	$A = 350714.714286$
3. Take the fractional part of A , multiply by 7, and round to the nearest integer. ^a This is the weekday number n as follows:	Fractional part = 0.714286 $n = 5$
Sunday $n = 0$	
Monday $n = 1$	
Tuesday $n = 2$	
Wednesday $n = 3$	
Thursday $n = 4$	
Friday $n = 5$	
Saturday $n = 6$	
	Friday

^aThis may be done by taking TRUNC(fractional part+0.5).

The spreadsheet for this calculation, FDOW (Finding the Day Of the Week; Figure 5) selects the name corresponding to the weekday number using a nested IF formula at row 7 (that is quite long and confusing to read!). The test argument of the first IF is the first argument, C6=0. If this is true, then the formula returns Sunday. If not, then a second IF statement takes the place of the third argument and the value of C6 is tested against 1 (Monday), and so on until all seven possible values of n have been tested. If the formula has still not been satisfied at that point, the text ** error is returned. This should never happen!

In row 5 we have used the INT function for finding the integer part of the argument. Since the Julian date is always positive, there is no issue here about exactly how the INT function deals with negative values. We make extensive use of INT throughout the book. Row 5 ensures that the fraction of the day after midnight is removed from the Julian date before proceeding with the calculation.

We have also supplied a spreadsheet function called FDOW(JD) which does the same calculation. It returns the text corresponding to the name of the day of the given Julian date (JD), or the text Unknown if the calculation suggests n lies outside of the range 0 to 6 inclusive. You can delete rows 5 to 7 of Figure 5 and replace cell H3 with the formula =FDOW(C3). Don't forget to save the spreadsheet first if you want to try this out.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Finding the name of the day of the week											
2												
3	Input	Julian date	2455001.5		Output	Day of the week	Friday =C7					
4												
5	1	JD	2455001.5	=INT(C3-0.5)+0.5								
6	2	n	5	=MOD(C5+1.5,7)								
7	3	day name	Friday	=IF(C6=0,\"Sunday\",IF(C6=1,\"Monday\",IF(C6=2,\"Tuesday\",IF(C6=3,\"Wednesday\",IF(C6=4,\"Thursday\",IF(C6=5,\"Friday\",IF(C6=6,\"Saturday\",\"** error\"))))))								

Figure 5. The Julian date 2455 001.5 fell on a Friday at Greenwich.

7 Converting hours, minutes and seconds to decimal hours

Most times are expressed as hours and minutes, or hours, minutes and seconds. For example, twenty to four in the afternoon may be written as 3:40 pm, or 3h40m pm, or on a 24-hour clock as 15h 40m. In calculations, however, the time needs to be expressed in decimal hours on a 24-hour clock. The method of converting a time expressed in the format hours, minutes and seconds into decimal hours is given below. Some calculators have special keys to do this for you automatically. As an example, let's convert the time 6h 31m 27s pm into decimal hours.

Method	Example		
1. Take the number of seconds and divide by 60.	27/60	=	0.450000
2. Add this to the number of minutes and divide by 60.	31.45/60	=	0.524167
3. Add the number of hours.	+6.0	=	6.524167
4. If the time has been given on a 12-hour clock, and it is pm, add 12.	+12.0	=	18.524167 hours

The spreadsheet corresponding to this calculation is shown in Figure 6, and is called HMSDH (Hours Minutes Seconds to Decimal Hours conversion). We have defined variable names *A*, *B*, *C* and *D* in column B rows 7 to 10 for convenience. They have no counterparts in the method table above. Note that the spreadsheet converts the time already expressed on a 24-hour clock, so be careful to add 12 hours, if appropriate, to the number you enter in cell C3.

We have also supplied the spreadsheet function HMSDH(H,M,S) which will carry out this conversion for you. The three arguments correspond to the hours, minutes and seconds parts of the time to be converted to hours. You can delete rows 7 to 10 of the spreadsheet shown in Figure 6 and insert the formula =HMSDH(C3,C4,C5) in cell H3. Save a copy of your spreadsheet first. Note that in many cases, as here, you can use the function to convert partially-converted times. Thus =HMSDH(18,31,27) will give the same result as =HMSDH(18,31.524167,0), where you have expressed the same time in hours and minutes format (no seconds).

	A	B	C	D	E	F	G	H	I
1	Converting hours, minutes and seconds to decimal hours								
2									
3	Input	hours	18			Output	decimal hours	18.52416667	=C10
4		minutes	31						
5		seconds	27						
6									
7	1	A	0.45	=ABS(C5)/60					
8	2	B	0.524166667	=(ABS(C4)+C7)/60					
9	3	C	18.52416667	=ABS(C3)+C8					
10	4	D	18.52416667	=IF((C3<0)+(C4<0)+(C5<0),-C9,C9)					

Figure 6. Converting a time expressed in HMS format into decimal hours.

8 Converting decimal hours to hours, minutes and seconds

When the result of a calculation is a time, it is normally expressed as decimal hours, and we need to convert it to hours, minutes and seconds. (This is the reverse of the calculation in Section 7.) The method of doing so is given below. Again, some calculators have special keys to carry out this function automatically. We express the time 18.524 167 h in hours, minutes and seconds format as our example.

Method	Example
1. Take the fractional part and multiply by 60. The integer part of the result is the number of minutes.	$0.524\,167 \times 60 = 31.450\,020$
2. Take the fractional part of the result and multiply by 60. This gives the number of seconds.	$0.450\,020 \times 60 = 27.001\,200$ 18h 31m 27s

The spreadsheet for this calculation is shown in Figure 7 and is called DHHMS (Decimal Hours to Hours Minutes Seconds conversion). It has more steps and slightly greater complexity than the method in the above table as it needs to deal automatically with cases in which the result of the calculation is an integer number of minutes and/or seconds exactly equal to 60, such as 10h45m60s. In such cases, you would increment the number of hours and/or minutes by 1, and set the number of minutes and/or seconds to zero. Thus 10h 45m 60s is better expressed as 10h 46m 0s. The spreadsheet also rounds the number of seconds to two decimal places using the spreadsheet intrinsic function ROUND in cell C9. The first argument of this function is the number you wish to round, and the second argument is the number of decimal places.

We have also supplied three spreadsheet functions to carry out this calculation. We need to have three as any function can only return one result, and so we need separate functions for the hours, minutes, and seconds. These are DHHour(H), DHMin(H), and DHSec(H) respectively, where the argument in each case is the time in decimal hours to be converted. Thus (having saved a copy first) you could delete rows 7 to 14 of the spreadsheet shown in Figure 7 and insert the formulas =DHHour(C3), =DHMin(C3) and =DHSec(C3) in cells H3, H4 and H5 respectively to get the same result.

	A	B	C	D	E	F	G	H	I
1	Converting decimal hours to hours, minutes and seconds								
2									
3	<i>Input</i>	decimal hours	18.52416667			<i>Output</i>	hours	18	=C14
4							minutes	31	=C12
5							seconds	27	=C10
6									
7	1	unsigned decimal	18.52416667	=ABS(C3)					
8	2	total seconds	66687	=C7*3600					
9	3	seconds (2 dp)	27	=ROUND(MOD(C8,60),2)					
10	4	corrected seconds	27	=IF(C9=60,0,C9)					
11	5	corrected remainder	66687	=IF(C9=60,C8+60,C8)					
12	6	minutes	31	=MOD(TRUNC(C11/60),60)					
13	7	unsigned hours	18	=TRUNC(C11/3600)					
14	8	signed hours	18	=IF(C3<0,-1*C13,C13)					

Figure 7. Converting a time expressed in decimal hours to HMS format.

9 Converting the local time to universal time (UT)

The basis of civilian time-keeping is the rotation of the Earth. **Universal time** (UT) is related to the motion of the Sun as observed on the **Greenwich meridian**, longitude 0°. The Earth is not a perfect time-keeper, however, and today a more uniform flow of time is available from atomic clocks. **International atomic time** (TAI) is the scale resulting from analyses by the Bureau International de l'Heure, in Paris, of atomic standards in many countries. A version of universal time, called **coordinated universal time** (UTC), is derived from TAI in such a manner as to be within 0.9 seconds of UT and a whole number of seconds different from TAI. (In June 2010, TAI–UTC = 34 s). This is achieved by including occasional leap seconds in UTC (at the end of June or December – usually the latter). UTC is the time broadcast by some national radio stations (the ‘time pips’) and by standard time transmission services such as DCF 77 (Mainflingen, Germany), MSF 60 (Anthorn, UK) and WWV (Colorado, USA). It is now the basis of legal time-keeping on the Earth. UTC is thus an atomic time standard (and hence as uniform as we know how to measure) but with discontinuities to keep it in line with the irregular rotation of our planet.

Another time in common use today is **GPS time**. This is an atomic time kept by the US Naval Observatory, and which is broadcast by the satellites of the global positioning system (GPS). GPS time was equal to UTC on 1980 January 6 0.0, but, unlike UTC, is not adjusted by the insertion of leap seconds. Hence GPS time is equal, in June 2010, to UTC + 15 seconds (kept to within a microsecond) and is the time you can extract from your GPS navigation device.

The amateur astronomer need not be too concerned by all this complexity. For our purposes, we can take UT = UTC = GMT without noticing the difference. (Note that in a pre-1925 definition **Greenwich Mean Time** (GMT) started at midday, so was 12 hours out with respect to UT. However, this distinction is usually overlooked and people refer to UTC and GMT as the same thing. For example, the BBC World Service gives UTC times as GMT.) Where we need greater accuracy, we will use terrestrial time (TT) for events after 1984 January 0.0, and ephemeris time (ET) before then. TT is equal to TAI + 32.184 seconds

and took over from ET at the beginning of 1984 (see Section 16). (Note that TT was called **terrestrial dynamic time**, TDT, until 1991, when it was renamed by the International Astronomical Union.) UT is used as the local civil time in Britain during the winter months, but 1 hour is added during the summer to form British summer time (BST) so that the working day fits more conveniently into daylight hours. Many other countries adopt a similar arrangement; sometimes the converted time is known as **daylight saving time**.

Countries lying on meridians east or west of Greenwich do not use UT as their local civil time. It would be impractical to do so as the local noon, the time at which the Sun reaches its maximum altitude, gets earlier or later with respect to the local noon on the Greenwich meridian as one moves east or west respectively. The world is therefore divided into time zones, each zone usually corresponding to a whole number of hours before or after UT, and small countries, or parts of large countries lying within a zone, adopt the zone time as their local civil time (see Figure 8).

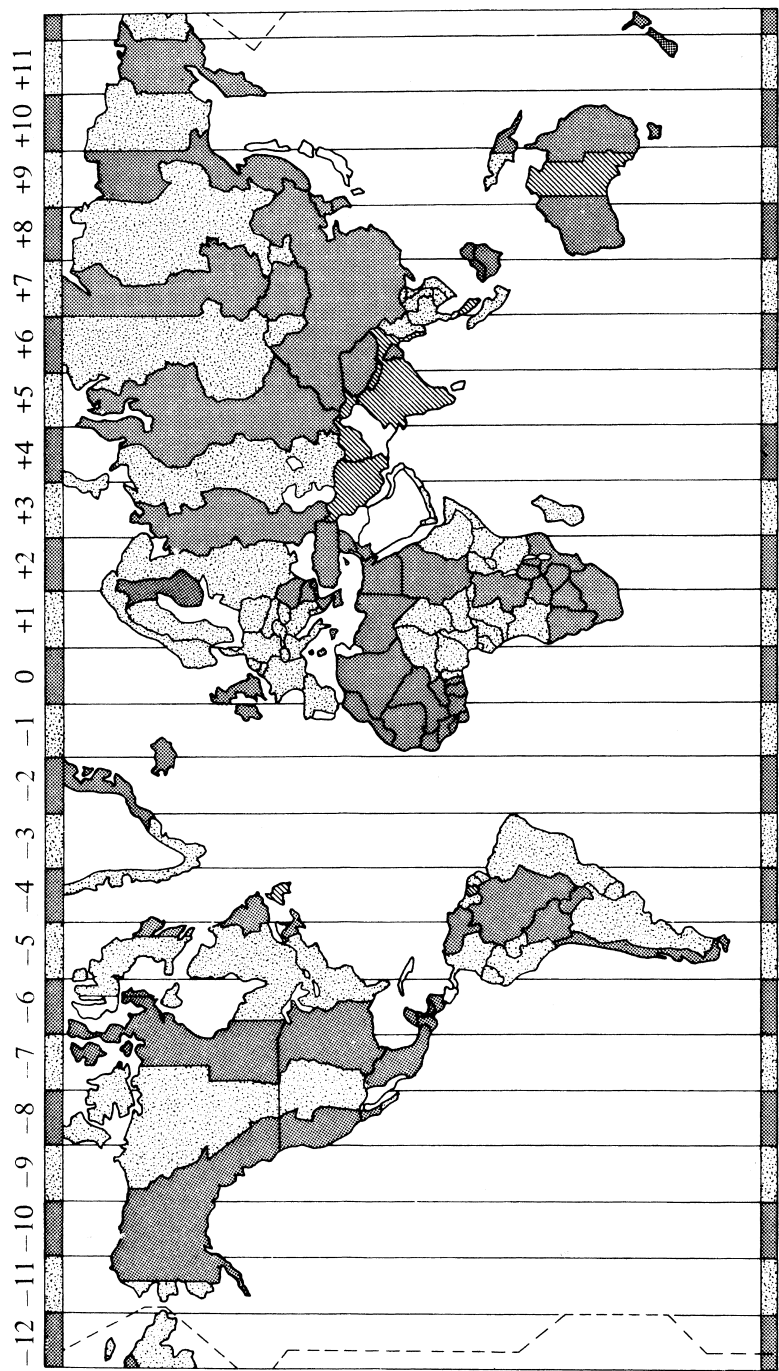


Figure 8. International time zones. This small-scale map can show only the general distribution of time zones around the world. If you are unsure of your own zone correction, you can check it by looking on the Internet, or by tuning your short-wave radio to the BBC World Service and comparing your watch with the time pips broadcast every hour from London.

The starting point for many astronomical calculations is often the local time and date, that is the time on your watch (assumed to be correct) on the date of the calendar on your wall. We will refer to your local time as the **local civil time**, and the local date as the **local calendar date**. However, the algorithms for calculating the positions of the heavenly bodies usually begin with the time on the Greenwich meridian, **universal time** (UT), and the Greenwich calendar date. We therefore need to be able to convert times and dates from your local position to Greenwich and vice-versa. For this you need to know your time-zone correction (hours ahead of UTC) and whether or not there is daylight saving in operation.

The following method converts your local time and date into UT and Greenwich calendar date. As an example we convert daylight saving time 3h 37m in time zone +4 hours on 1 July 2013.

Method	Example
1. Convert local civil time to zone time by removing the daylight saving correction, and convert to decimal hours (§7).	3h 37m – 1h = 2h 37m
2. Subtract the time-zone offset (time zones W are negative). This is UT.	Zone time = 2.616667 hours UT = 2.616667 – 4 = –1.383333 hours
3. Divide UT by 24 and add to the local calendar day. This is Greenwich calendar day.	G Day = 1 – (1.383333/24.0) hours = 0.942361 hours
4. Find the Julian date corresponding to the Greenwich calendar date (§4).	G cal date = 2013 July 0.942361 JD = 2456474.442
5. Convert the Julian calendar date back into the Greenwich calendar date (§5).	G Day = 30.942361 G Month = 6 G Year = 2013
6. The day of the Greenwich calendar date is TRUNC(G Day). Subtract this from G Day and multiply the result by 24 to obtain UT in the range 0 to 24 h. Convert to hours, minutes and seconds if required (§8).	GD = 30 UT = 0.942361 × 24 = 22.616667 = 22h 37m 0s G date = 2013 June 30

Steps 4 and 5 of the method table above may seem a bit unnecessary. What is the point of going through the lengthy conversion from Greenwich calendar date in step 4 only to be told in step 5 to convert back again? Actually, with your human mind carrying out this calculation you may be able to go directly from step 3 to step 6 because you will be able to see that G Day = 0.942361 is the same as G Day = 0 + 0.942361, and the day therefore corresponds to the previous day's date, i.e. 30 June, and the UT to 0.942361×24 . Note that you have made quite a complicated calculation in doing this, and of course the year might have changed as well. Steps 4 and 5, though cumbersome, take care of all of this, and are required in any case in the spreadsheet (Figure 9).

The spreadsheet is called LCTUT, following the acronym for Local Civil Time to Universal Time conversion. The spreadsheet functions corresponding to this calculation are LCTUT, LCTGDay, LCTGMonth and LCTGYear, returning the universal time, and the day, month and year of the Greenwich calendar date respectively. Each of them takes the same eight arguments: (H,M,S,DS,ZC,LD,LM,LY), in which H, M, S are the local time (hours, minutes, seconds), DS and ZC are the daylight saving offset and zone correction (hours), and LD, LM, LY are the day, month and year of the local calendar date.

	A	B	C	D	E	F	G	H	I	J
1	Conversion of Local Civil Time to Universal Time									
2										
3	Input	LCT hours	3		Output	UT hours	22	=DHHour(C19)		
4		LCT mins	37			UT mins	37	=DHMin(C19)		
5		LCT secs	0			UT secs	0	=DHSec(C19)		
6		daylight saving	1			Greenwich day	30	=INT(C16)		
7		zone correction	4			Greenwich month	6	=C17		
8		local day	1			Greenwich year	2013	=C18		
9		local month	7							
10		local year	2013							
11										
12	1	LCT	3.616666667 =HMSDH(C3,C4,C5)							
13	2	UT	-1.383333333 =C12-C6-C7							
14	3	G day	0.942361111 =C8+(C13/24)							
15	4	JD	2456474.442 =CDJD(C14,C9,C10)							
16	5	G day	30.94236111 =JDCDay(C15)							
17	6	G month	6 =JDCMonth(C15)							
18	7	G year	2013 =JDCYear(C15)							
19	8	UT	22.61666667 =24*(C16-INT(C16))							

Figure 9. Converting local time and date to universal time and Greenwich date.

Having saved a copy of the spreadsheet of Figure 9, you could delete rows 12 to 19 and insert these spreadsheet functions in cells H3 to H8 as follows:

```
=DHHour(LCTUT(C3,C4,C5,C6,C7,C8,C9,C10))
=DHMin(LCTUT(C3,C4,C5,C6,C7,C8,C9,C10))
=DHSec(LCTUT(C3,C4,C5,C6,C7,C8,C9,C10))
=LCTGDay(C3,C4,C5,C6,C7,C8,C9,C10)
=LCTGMonth(C3,C4,C5,C6,C7,C8,C9,C10)
=LCTGYear(C3,C4,C5,C6,C7,C8,C9,C10).
```

Note that the first three of these use nested functions, e.g. the function DHHour takes as its argument the result of running the function LCTUT. You can nest functions in this way almost indefinitely, although the resulting formula rapidly becomes unreadable as the nesting gets deeper.

10 Converting UT and Greenwich calendar date to local time and date

The result of an astronomical calculation can sometimes be a time and a date, usually the UT and calendar date at Greenwich. The following method will convert to the corresponding local civil time and calendar date appropriate to a point on the Earth in a given time zone, with or without daylight saving in operation. As in the previous section, the local date and the Greenwich date may not be the same, and we need to take account of differences in dates spanning month and/or year boundaries. Continuing with the previous example, what is the local civil time and local calendar date corresponding to 22h 37m UT when the Greenwich calendar date is 30 June 2013, in time zone +4 h and with daylight saving in operation?

Method	Example
1. Convert UT to decimal hours (§7).	22h 37m = 22.616667 hours
2. Add the time zone offset (time zones W are negative) and the daylight saving offset. This is the local civil time.	LCT = 22.616667 + 4 + 1 = 27.616667
3. Find the Julian date corresponding to the Greenwich calendar date (§4) and add (LCT/24).	LJD = 2465 473.5 + 27.616667/24 = 2465 474.651
4. Convert this local Julian date back into the local calendar date (§5). Take the integer part to get the local day number.	L Day = 1.150694 L date = 2013 July 1
5. Subtract the integer day from L Day and multiply the result by 24 to obtain the local civil time in the range 0 to 24 h. Convert to hours, minutes and seconds if required (§8).	LCT = 0.150694 × 24 = 3.616667 = 3h 37m 0s

A word here about rounding errors. In the method examples of both this and the previous section, you may have become aware of small differences in the last one or two decimal places between your calculated values and those shown in the method tables. For example, if we put 0.150694 into a calculator (step 5) and multiply by 24, we get 3.616656 instead of 3.616667 as shown. This is because of rounding errors, and there are two causes. First, the calculator maintains calculations accurate to about 11 or 12 significant figures, but in steps 3 and 4 we ‘use up’ seven of those in specifying the integer part of the Julian date, leaving only 4 or 5 for the fractional part. The calculator does its best, but the error on the last place creeps in and shows itself as a discrepancy. The spreadsheet calculation usually has much higher precision so does not suffer from this particular problem. We have shown full-precision results in the tables, rounded to six decimal places. Second, we have displayed the results of each calculation only to six decimal places. The truncation can make a small difference as here. With nine places of decimals, the value of LCT in step 5 is 0.150694444. Multiply this by 24 and round to six decimal places and you get 3.616667 as shown.

As in the method of the previous section, you may be able to leave out steps 3 and 4 which are included to make sure that the month and year boundaries are properly dealt with. You can see that the value of LCT in the second step, 27.616667 h, is equivalent to 1 day (24 h) plus 3.616667 h. The local civil time is therefore 3.616667 h = 3h 37m, and the local date is the Greenwich date plus one day, so 30 June 2013 becomes 1 July 2013.

The spreadsheet for this section is shown in Figure 10 and is called UTLCT (Universal Time to Local Civil Time conversion). Not having the advantage of the intelligence of the human brain, the program has to carry out the conversions to and from the Julian date (rows 15 and 16) for every calculation in order to deal properly with the month and year boundaries. In this case, without these steps, the spreadsheet would report the local date as 31 June 2013 – logically correct but not a recognised date for June which has only 30 days.

The corresponding spreadsheet functions are UTLCT, UTLCTDay, UTLCTMonth and UTLCTYear, which return respectively the local civil time in hours, the day, the month, and the year of the local calendar date. Each takes the same eight arguments (H,M,S,DS,ZC,GD,GM,GY) in which H, M and S are the universal time (hours, minutes, seconds), DS and ZC are the daylight saving adjustment and zone correction (both in hours), and GD, GM and GY are the day, month and year of the Greenwich calendar date.

	A	B	C	D	E	F	G	H	I	J
1	Conversion of Universal Time to Local Civil Time									
2										
3	Input	UT hours	22		Output	LCT hour	3	=DHHour(C20)		
4		UT mins	37			LCT Min	37	=DHMin(C20)		
5		UT secs	0			LCT sec	0	=DHSec(C20)		
6		daylight saving	1			day	1	=C17		
7		zone correction	4			month	7	=C18		
8		Greenwich day	30			year	2013	=C19		
9		Greenwich month	6							
10		Greenwich year	2013							
11										
12	1	UT	22.61666667	=HMSDH(C3,C4,C5)						
13	2	zone time	26.61666667	=C12+C7						
14	3	local time	27.61666667	=C13+C6						
15	4	local JD+local time	2456474.651	=CDJD(C8,C9,C10)+(C14/24)						
16	5	local day	1.150694444	=JDCDay(C15)						
17	6	integer day	1	=TRUNC(C16)						
18	7	local month	7	=JDCMonth(C15)						
19	8	local year	2013	=JDCYear(C15)						
20	9	LCT	3.616666663	=24*(C16-C17)						

Figure 10. Converting universal time and Greenwich date to local civil time and local date.

You can therefore delete rows 12 to 20 (save a copy first) and insert the following formulas in cells H3 to H8 respectively:

```
=DHHour(UTLCT(C3,C4,C5,C6,C7,C8,C9,C10))
=DHMin(UTLCT(C3,C4,C5,C6,C7,C8,C9,C10))
=DHSec(UTLCT(C3,C4,C5,C6,C7,C8,C9,C10))
=UTLCDay(C3,C4,C5,C6,C7,C8,C9,C10)
=UTLCMonth(C3,C4,C5,C6,C7,C8,C9,C10)
=UTLCYear(C3,C4,C5,C6,C7,C8,C9,C10).
```

Note that the first three of these use nested functions, e.g. the function DHHour takes as its argument the result of running the function UTLCT. You can nest functions in this way almost indefinitely, although the resulting formula rapidly becomes unreadable as the level of nesting increases.

11 Sidereal time (ST)

Universal time (UT), and therefore the local civil time in any part of the world, is related to the apparent motion of the Sun around the Earth. Roughly speaking, we may take 1 **solar day** as the time between two successive passages of the Sun across the meridian as observed at a particular place. Astronomers are interested, however, in the motion of the stars; in particular they need to use a clock whose rate is such that any star is observed to return to the same position in the sky after exactly 24 hours have elapsed according

to the clock. Such a clock is called a sidereal clock and its time, being regulated by the stars, is called **sidereal time** (ST). Solar time, of which UT is an example, is not the same as sidereal time because during the course of 1 solar day the Earth moves nearly 1 degree along its orbit round the Sun. Hence, the Sun appears progressively displaced against the background of stars when viewed from the Earth; turning that around, the stars appear to move with respect to the Sun. Any clock, therefore, which keeps time by the Sun does not do so by the stars.

There are about 365.25 solar days in the year[†], the time taken by the Sun to return to the same position with respect to the background of stars. During this period, the Earth makes about 366.25 revolutions around its own axis; there are therefore this many sidereal days in the year. Each sidereal day is thus slightly shorter than the solar day, 24 hours of sidereal time corresponding to 23h 56m 04s of solar time. Universal time and Greenwich sidereal time agree at one instant every year at the autumnal **equinox** (around 22 September). Thereafter, the difference between them grows in the sense that sidereal time runs faster than universal time, until exactly half a year later the difference is 12 hours. After 1 year, the times again agree.

The formal definition of sidereal time is that it is the hour angle of the vernal equinox (see Section 18).

12 Conversion of UT to Greenwich sidereal time (GST)

This section describes a simple procedure by which the UT may be converted into GST. It is accurate to better than one tenth of a second. For example, what was the GST at 14h 36m 51.67s UT on Greenwich date 22 April 1980?

<i>Method</i>	<i>Example</i>
1. Find the Julian date corresponding to 0h on this Greenwich calendar date (§4).	JD = 2444351.5
2. Calculate $S = JD - 2\,451\,545.0$.	$S = -7\,193.5$
3. Calculate $T = S/36\,525.0$.	$T = -0.196\,947$
4. Find $T0 = 6.697\,374\,558 + (2\,400.051\,336 \times T) + (0.000025862 \times T^2)$. Reduce the result to the range 0 to 24 by adding or subtracting multiples of 24.	$T0 = -465.986\,246 + 24 \times 20 = 14.013\,754$
5. Convert UT to decimal hours (§7).	UT = 14.614353
6. Multiply UT by 1.002737909.	$A = 14.654\,366$
7. Add this to $T0$ and reduce to the range 0 to 24 if necessary by subtracting or adding 24. This is the GST.	$+ 14.013\,754$ GST = 4.668120
8. Convert the result to hours, minutes and seconds (§8).	GST = 4h 40m 5.23s

The spreadsheet for this calculation is shown in Figure 11 and is called **UTGST** (an acronym for **UT** to **GST** conversion). The step of reducing to the range 0 to 24 is achieved, for example in row 14, by subtracting $(24 \times \text{INT}(C13/24))$ from C13. The INT function returns the whole number of times that 24 goes into the value of C13, and this is multiplied by 24 before being subtracted from the value in C13, just as is done in step 4 of the method table. This trick is used in many spreadsheets throughout the book.

We have also supplied the spreadsheet function **UTGST(H,M,S,GD,GM,GY)** which takes six arguments H, M, S (UT in hours, minutes and seconds) and GD, GM, GY (Greenwich calendar date as days, months, and years). It returns the GST in hours corresponding to the values of the arguments.

[†] See the definition of the year given in the Glossary.

	A	B	C	D	E	F	G	H	I
1	Conversion of UT to GST								
2									
3	<i>Input</i>	UT hours	14		<i>Output</i>	GST hours	4	=C19	
4		UT mins	36			GST mins	40	=C20	
5		UT secs	51.67			GST secs	5.23	=C21	
6		Greenwich day	22						
7		Greenwich month	4						
8		Greenwich year	1980						
9									
10	1	JD	2444351.5	=CDJD(C6,C7,C8)					
11	2	S	-7193.5	=C10-2451545					
12	3	T	-0.196947296	=C11/36525					
13	4	T0	-465.9862462	=6.697374558+(2400.051336*C12)+(0.000025862*C12*C12)					
14	5	T0	14.01375378	=C13-(24*INT(C13/24))					
15	6	UT	14.61435278	=HMSDH(C3,C4,C5)					
16	7	A	14.65436555	=C15*1.002737909					
17	8	GST	28.66811933	=C14+C16					
18	9	GST	4.668119327	=C17-(24*INT(C17/24))					
19	10	GST hours	4	=DHHour(C18)					
20	11	GST mins	40	=DHMin(C18)					
21	12	GST secs	5.23	=DHSec(C18)					

Figure 11. Converting universal time and Greenwich date to Greenwich sidereal time.

You can try this for yourself by deleting rows 10 to 21 (after saving a copy) and inserting the following formulas in cells H3, H4 and H5 respectively:

=DHHour(UTGST(C3,C4,C5,C6,C7,C8))

=DHMin(UTGST(C3,C4,C5,C6,C7,C8))

=DHSec(UTGST(C3,C4,C5,C6,C7,C8)).

13 Conversion of GST to UT

Here we deal with the reverse problem of the previous section, namely that of converting a given GST into the corresponding UT. The problem is complicated, however, by the fact that the sidereal day is slightly shorter than the solar day so that on any given calendar date a small range of sidereal times occurs twice. This range is about 3m 56s long, the sidereal times corresponding to UT 0h to 0h 3m 56s occurring again from UT 23h 56m 04s to midnight (see Figure 12). The method given here correctly converts sidereal times in the former interval, but not in the latter.

The accuracy of this method is the same as that of Section 12, namely better than one tenth of a second. Continuing our previous example, at GST = 4h 40m 5.23s on Greenwich date 22 April 1980, what was the UT?

Method	Example
1. Find the Julian date corresponding to 0h on this Greenwich calendar date (§4).	JD = 2 444 351.5
2. Calculate $S = JD - 2\,451\,545.0$.	$S = -7\,193.5$
3. Calculate $T = S/36\,525.0$.	$T = -0.196\,947$
4. Find $T_0 = 6.697\,374\,558 + (2\,400.051\,336 \times T) + (0.000\,025\,862 \times T^2)$. Reduce the result to the range 0 to 24 by adding or subtracting multiples of 24.	$T_0 = -465.986\,246 + 24 \times 20 = 14.013\,754$
5. Convert GST to decimal hours (§7).	GST = 4.668 119
6. Subtract T_0 and reduce to the range 0 to 24 if necessary by subtracting or adding 24.	$A = -9.345\,635$ $B = 14.654\,365$
7. Multiply B by 0.997 269 566 3. The result is the UT.	UT = 14.614 353
8. Convert the result to hours, minutes and seconds (§8).	UT = 14h 36m 51.67s

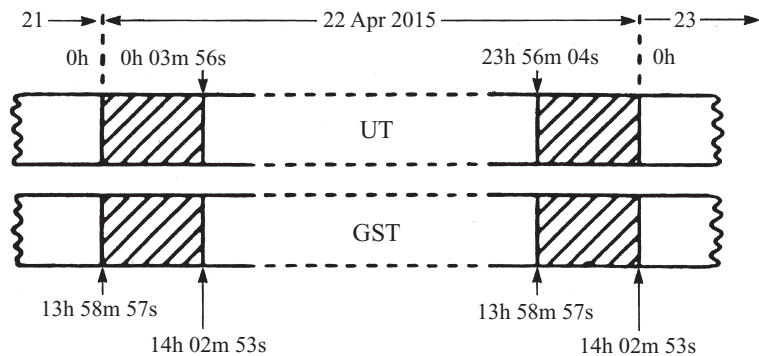


Figure 12. UT and GST for 22 April 2015. The hatched intervals of GST occur twice on the same day.

Figure 13 shows the corresponding spreadsheet, labelled GSTUT (an acronym for GST to UT conversion). It follows the method given in the table quite closely, but incorporates an extra step in row 22. This step tests to see whether the UT lies in the range 0h0m0s to 0h3m56s. If it does, it may not be the desired conversion since an equally-valid range of UT for this date corresponding to the given GST is 23h56m04s to 0h0m0s. There is insufficient information for us to be able to determine, on the GST and Greenwich calendar date alone, which is the desired result. The IF function in row 13 therefore issues a status flag, actually a word of text. This is OK if there is no ambiguity in the conversion, and Warning if there is.

The associated spreadsheet functions are:

GSTUT(H,M,S,GD,GM,GY) and

eGSTUT(H,M,S,GD,GM,GY),

where the arguments H, M, S represent the hours, minutes and seconds of the GST, and GD, GM, GY represent the day, month, year of the Greenwich calendar date. GSTUT returns the UT in hours corresponding with the argument values. You can use the other function, eGSTUT, to determine whether or not the conversion is ambiguous, and it returns status text OK or Warning as appropriate. Thus rows 10 to 22 can be

	A	B	C	D	E	F	G	H	I
1	Conversion of GST to UT								
2									
3	Input	GST hours	4		Output	UT hours	14	=C19	
4		GST mins	40			UT mins	36	=C20	
5		GST secs	5.23			UT secs	51.67	=C21	
6		Greenwich day	22			Status	OK	=C22	
7		Greenwich month	4						
8		Greenwich year	1980						
9									
10	1	JD	2444351.5	=CDJD(C6,C7,C8)					
11	2	S	-7193.5	=C10-2451545					
12	3	T	-0.196947296	=C11/36525					
13	4	T0	-465.9862462	=6.697374558+(2400.051336*C12)+(0.000025862*C12*C12)					
14	5	T0	14.01375378	=C13-(24*INT(C13/24))					
15	6	GST (hours)	4.668119444	=HMSDH(C3,C4,C5)					
16	7	A	-9.345634337	=C15-C14					
17	8	B	14.65436566	=C16-(24*INT(C16/24))					
18	9	UT	14.61435289	=C17+0.9972695663					
19	10	UT hours	14	=DHHour(C18)					
20	11	UT mins	36	=DHMin(C18)					
21	12	UT secs	51.67	=DHSec(C18)					
22	13	warning flag	OK	=IF(C18<(0.065574),"Warning","OK")					

Figure 13. Converting Greenwich date and Greenwich sidereal time to universal time.

deleted in the spreadsheet of Figure 13 (but save a copy first) and the following formulas inserted into cells H3 to H6 respectively:

```
=DHHour(GSTUT(C3,C4,C5,C6,C7,C8))
```

```
=DHMin(GSTUT(C3,C4,C5,C6,C7,C8))
```

```
=DHSec(GSTUT(C3,C4,C5,C6,C7,C8))
```

```
=eGSTUT(C3,C4,C5,C6,C7,C8).
```

Why not try this for yourself?

14 Local sidereal time (LST)

The Greenwich sidereal time discussed in the previous sections is the sidereal time correct for observations made on the Greenwich meridian, longitude 0° . It is in fact the local sidereal time (LST) for the Greenwich meridian. As you move west or east from longitude 0° , however, the local sidereal time gets earlier or later respectively because the hour angle of the vernal equinox, which defines the local sidereal time, changes. You can calculate your local sidereal time, given the Greenwich sidereal time, very easily as the difference between the two times in hours is simply the geographical longitude in degrees divided by 15. Longitudes west give local sidereal times earlier than GST, and longitudes east later. You should express longitudes E as positive numbers, and longitudes W as negative numbers. Take the example: what is the local sidereal time on the longitude 64° W when the GST is 4h 40m 5.23s?

<i>Method</i>	<i>Example</i>
1. Convert GST to decimal hours (§7).	GST = 4.668 119 h
2. Convert the geographical longitude in degrees to its equivalent in hours by dividing by 15. Note that longitudes W are negative.	-64° = $-4.266\,667$ h
3. Add this to the GST. Bring the result into the range 0 to 24 by adding or subtracting 24 if necessary. This is the local sidereal time (LST).	LST = 0.401 453 h
4. Convert LST to hours, minutes and seconds (§8).	LST = 0h 24m 5.23s

The spreadsheet labelled GSTLST (an acronym for GST to LST conversion) is shown in Figure 14. The corresponding spreadsheet function is GSTLST. This takes four arguments with the Greenwich sidereal time in the first three arguments expressed as hours, minutes and seconds, and the geographical longitude of the observer in decimal degrees in the fourth argument. It returns the local sidereal time in decimal hours. Hence rows 8 to 14 of the spreadsheet shown in Figure 14 could be deleted (after saving a copy) and the following spreadsheet formulas inserted consecutively into cells H3, H4 and H5:

```
=DHHour(GSTLST(C3,C4,C5,C6))
```

```
=DHMin(GSTLST(C3,C4,C5,C6))
```

```
=DHSec(GSTLST(C3,C4,C5,C6)).
```

	A	B	C	D	E	F	G	H	I
1	Conversion of GST to LST								
2									
3	Input	GST hours	4		Output	LST hour	0	=C12	
4		GST mins	40			LST min	24	=C13	
5		GST secs	5.23			LST sec	5.23	=C14	
6		geog long	-64						
7									
8	1	GST	4.668119444	=HMSDH(C3,C4,C5)					
9	2	offset	-4.266666667	=C6/15					
10	3	LST (hours)	0.401452778	=C8+C9					
11	4	LST (hours)	0.401452778	=C10-(24*INT(C10/24))					
12	5	LST hour	0	=DHHour(C11)					
13	6	LST min	24	=DHMin(C11)					
14	7	LST sec	5.23	=DHSec(C11)					

Figure 14. Converting Greenwich sidereal time to local sidereal time.

15 Converting LST to GST

This problem is the reverse of that treated in Section 14, namely, given the local sidereal time at a particular place, what is the corresponding Greenwich sidereal time? As an example, we shall calculate the GST when the LST on longitude 64° W is 0h 24m 5.23s.

Method	Example
1. Convert the LST to decimal hours (§7).	LST = 0.401453 h
2. Convert the geographical longitude in degrees to its equivalent in hours by dividing by 15. Note that longitudes W are negative.	-64° = -4.266667 h
3. Subtract this from the LST. Bring the result into the range 0–24 by adding or subtracting 24 if necessary. This is the GST.	GST = 4.668119 h
4. Convert GST to hours, minutes and seconds (§8).	GST = 4h 40m 5.23s

The spreadsheet labelled LSTGST (an acronym for LST to GST conversion) is shown in Figure 15. The corresponding spreadsheet function is also called LSTGST. This takes four arguments with the local sidereal time in the first three arguments expressed as hours, minutes and seconds, and the geographical longitude of the observer in decimal degrees in the fourth argument. It returns the Greenwich sidereal time in decimal hours. Hence rows 8 to 14 of the spreadsheet shown in Figure 15 could be deleted (after saving a copy) and the following spreadsheet formulas inserted consecutively into cells H3, H4 and H5:

```
=DHHour(LSTGST(C3,C4,C5,C6))
=DHMin(LSTGST(C3,C4,C5,C6))
=DHSec(LSTGST(C3,C4,C5,C6)).
```


	A	B	C	D	E	F	G	H	I
1	Conversion of LST to GST								
2									
3	<i>Input</i>	LST hours	0		<i>Output</i>	GST hours	4	=C12	
4		LST mins	24			GST mins	40	=C13	
5		LST secs	5.23			GST secs	5.23	=C14	
6		geog long	-64						
7									
8	1	GST	0.401452778	=HMSDH(C3,C4,C5)					
9	2	long (hours)	-4.266666667	=C6/15					
10	3	GST	4.668119444	=C8-C9					
11	4	GST	4.668119444	=C10-(24*INT(C10/24))					
12	5	GST hours	4	=DHHour(C11)					
13	6	GST mins	40	=DHMin(C11)					
14	7	GST secs	5.23	=DHSec(C11)					

Figure 15. Converting local sidereal time to Greenwich sidereal time.

16 Ephemeris time (ET) and terrestrial time (TT)

Universal and sidereal times are both tied directly to the period of the rotation of the Earth about its polar axis. The Earth is being used in effect as the balance wheel of a cosmic clock whose tick defines the length of the day. With the advent of extremely accurate atomic clocks, however, it has become apparent that the Earth's rotation is not strictly uniform but shows small erratic fluctuations which are not well understood. UT and ST, being reckoned by this irregular cosmic clock, are therefore not strictly uniform either. Astronomers need a system of time that *is* uniform since the theories of celestial mechanics assume that such a quantity exists. For example, two solid bodies in orbit about one another far away from any external influences should have an unchanging orbital period when measured on a regular clock. Before 1984, astronomers adopted **ephemeris time** (ET) for this purpose. It was calculated from the motion of the Moon and assumed to be uniform. Nowadays, atomic clocks provide the most uniform measure of time, and since 1984 **terrestrial time** (TT) has been used instead of ET. (In fact TT was called TDT, for **terrestrial dynamic time**, until renamed and slightly redefined by the International Astronomical Union in 1991.) TT is tied to the atomic time scale, TAI (see Section 9), by the equation:

$$TT = TAI + 32.184 \text{ s.}$$

The constant offset of 32.184 seconds was chosen to make TT equal to ET at the beginning of 1984. ET itself was chosen to agree as nearly as possible with the measure of universal time during the nineteenth century, and it is unlikely that TT will differ by more than a few minutes in the twenty-first.

The primary unit of ET was the length of the tropical year at 1900 January 0.5 ET which contained 31 556 925.974 7 ephemeris seconds. The primary unit of TAI, and hence TT, is the SI second, defined to be the duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium 133 atom. We need not be too concerned by all this since very high accuracy is not the aim of the book. In almost every case we can take $ET = TT = UT$ without noticing the difference. Only when calculating the motion of the Moon, and predicting eclipses (Section 71), will it pay us to take account of the difference between ET/TT and UT. In January 2010 this was 66.07 seconds, UT being behind TT; that is

$$TT - UT = \Delta T = 66.07 \text{ seconds.}$$

Figure 16 shows how ΔT has varied since 1620; we can predict that its value in the year 2020 might be around 70 seconds, but only direct observations at that time will confirm this.

Pulsars with very stable rates of spin have been discovered which appear to be even more precise than our best atomic clocks. TAI may well lose its place as the fundamental measure of time during this century, and be replaced by another scale based on the pulsars – GBT (galactic barycentric time) perhaps.



Figure 16. The variation of ΔT since 1620.

Coordinate systems

*To fix the position of any astronomical object, we must have a frame of reference, or coordinate system, which assigns a different pair of numbers to every point in the sky. The two numbers, or **coordinates**, usually refer to 'how far round' and 'how far up', just as do the longitude and latitude of an object on the Earth's surface. There are several such coordinate systems which you may meet, and we shall be concerned with four of these, namely the horizon system, the equatorial system, the ecliptic system and the galactic system. Each system takes its name from the fundamental plane which it uses as a reference; for instance, the ecliptic coordinate system makes all its measurements with respect to the plane of the ecliptic, the plane of the Earth's orbit about the Sun. In the next few sections, we shall find how to convert any position given in one system into the equivalent coordinates of another system. We shall also find how to describe positions on the surfaces of the Sun and Moon, how to deal with the problems of calculating the time of rising and setting, and with the effects of the Earth's precession, nutation, aberration, atmospheric refraction, and parallax on the apparent position of a celestial body.*

17 Horizon coordinates

The horizon coordinates, azimuth and altitude, of an object in the sky are referred to the plane of the observer's horizon (see Figure 17). Imagine an observer standing at point O; then his or her horizon is the circle NESW, where the letters refer to the north, east, south and west points of the horizon respectively. The direction north, by the way, relates to the direction of the north pole on the Earth's rotation axis and not to the magnetic north pole. You must imagine the stars as fixed on the surface of the hemisphere with the observer at the centre as in Figure 17; the whole sphere of which this hemisphere is part is called the **celestial sphere**. The point Z directly over the observer's head is called the **zenith**; the direction OZ is the direction defined by a plumb line held by the observer. Now consider a star X and imagine a great circle (i.e. a circle drawn on the surface of the sphere whose centre is the same as that of the sphere) going through Z and X; it meets the horizon at point B. The **altitude**, a , of the star is then the angle subtended at O by the points X and B. The **azimuth**, A , is the angle subtended by the points N and B. Hence, the altitude is 'how far up' in degrees (negative if below the horizon) and the azimuth is 'how far round' from the north direction, also measured in degrees. A increases from 0° to 360° as you go around in the sense NESW, N being 0° , S being 180° , etc.

The altitudes and azimuths of all heavenly bodies except geostationary satellites are continually changing with time as the Earth rotates. This coordinate system then, marvellous for setting the direction of your telescope, is not much good for fixing the positions of the stars. Another frame of reference, independent of the Earth's motion, is needed to do that. It is described in the next section.

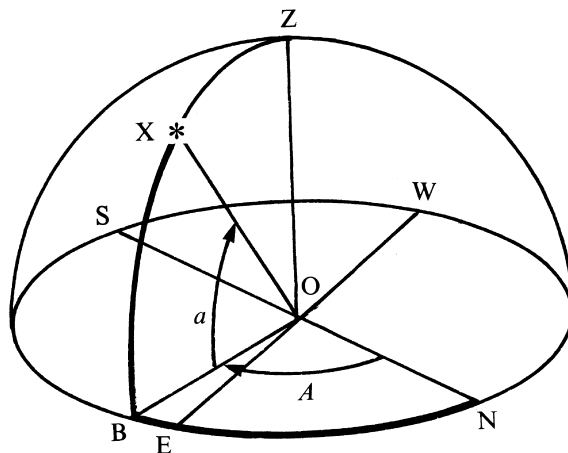


Figure 17. Horizon coordinates.

18 Equatorial coordinates

As their name suggests, these coordinates are referred to the plane of the Earth's equator (see Figure 18(a)). The observer (assumed to be in the northern hemisphere) is at O and the plane containing the circle NESW is again the horizon with Z the zenith point. You are to imagine now that the figure represents the view obtained at a vast distance from the Earth. The Earth, with the observer standing on it, has shrunk to a tiny dot at the centre of the diagram, but the plane of the equator has been extended to cut the celestial sphere along the circle E Υ RW. This is the equatorial plane and is inclined at the angle $90^\circ - \phi$ to the horizon, where ϕ is the observer's geographical latitude. For observations at latitude 52° N this angle is 38° . At right angles to the equatorial plane along the line OP lies the axis of rotation of the Earth; it intersects the celestial sphere at P, the **north celestial pole**, or **north pole** for short. Since this is the line about which the Earth spins, all the stars appear to describe circles in the sky about P.

Figure 18(b) shows the situation as seen by the observer O looking up into the sky. The south point, S, of

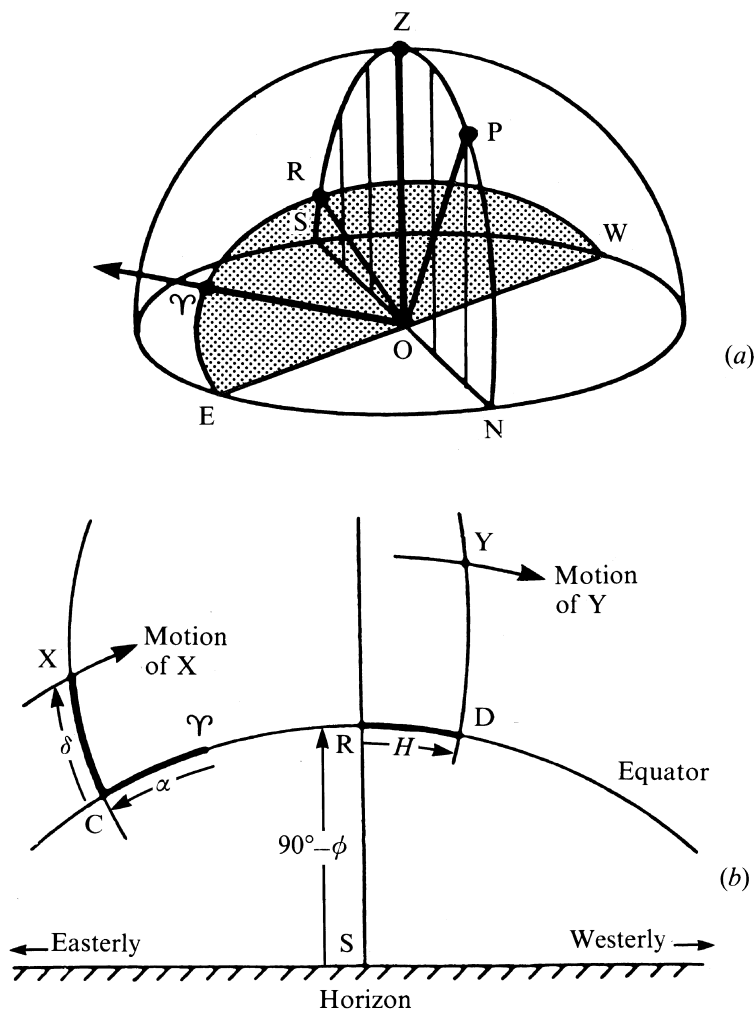


Figure 18. Equatorial coordinates: (a) on the celestial sphere, and (b) as seen from the ground.

the horizon is marked and so is the imaginary trace of the equator, $C\Upsilon RD$. The arc extending down through R and S is the great circle which goes through NPZRS in Figure 18(a). The arc extending down through XC is another great circle, not marked in Figure 18(a), which goes through PXC. Consider the star at X. The arc CX, or the angle subtended at O by the points C and X, is called the **declination**, δ , of X, defining ‘how far up’ from, or north of, the equator. The other coordinate, ‘how far round’, is defined with respect to a fixed direction in the sky, marked by the symbol Υ . This direction, called the **vernal equinox** or the **first point of Aries**, lies along the line of the intersection of the plane of the Earth’s equator with that of the Earth’s orbit around the Sun. But we needn’t worry about such definitions at the moment. All we need to know is that the direction Υ remains fixed with respect to the stars (except for the effects of **precession** and **nutation** – see Sections 34 and 35), and that we measure the other coordinate with respect to it. This coordinate is called the **right ascension**, α , and is the angle subtended at O by the points Υ and C.

Throughout the course of the day the star X moves steadily westwards along a circle centred on P, completing one revolution in 24 hours of sidereal time (see Section 11 for a description of sidereal time). Since this circle is a circle parallel to that of the equator the declination does not change. Furthermore, since the direction Υ is fixed in the heavens, it appears to move along the equator at exactly the same rate as X moves along the circle. Hence the right ascension does not change either. Thus α and δ are ideal coordinates for describing the positions of the stars and other ‘fixed’ heavenly bodies.

Related to the right ascension is another ‘how far round’ coordinate called the **hour angle**, H (see Figure 18(b)). For the star Y it is defined as the angle subtended at O by the points R and D and is a measure of how far the star has travelled along the equator from the southern point R, that is a measure of the time since it crossed the meridian. H increases uniformly as the day proceeds; when H is zero, the star crosses the great circle NPZRS (Figure 18(a)). This circle is called the **meridian** and the star is said to **transit** or **culminate**. Its altitude (Section 17) is then maximum and its azimuth[†] is 180° (provided that its declination is less than the geographical latitude).

The declination is measured in degrees, positive north of the equator and negative south of it. The hour angle and the right ascension may also be measured in degrees, 0° to 360° . α is measured in the sense that it increases as you move *east* from Υ ; the point Υ itself is at 0° . (Note that this is in the *opposite* sense to that in which H is measured.) More commonly, however, these two coordinates H and α are measured in hours, minutes and seconds of time from 0 to 24 hours. One complete revolution, 360° , corresponds to 24 hours of sidereal time; thus 1 hour is equivalent to 15° . The two statements ‘the right ascension of X is 90° ’ and ‘the right ascension of X is 6 h’ are entirely equivalent. To convert from one to the other simply multiply or divide by 15.

A useful result of measuring the right ascension in time is that the star transits when the local sidereal time is equal to the right ascension.

[†] Some authors measure azimuth from the south point rather than the north point, in which case $A = 0^\circ$ at transit.

19 Ecliptic coordinates

The plane containing the Earth's orbit around the Sun is called the **ecliptic** and the other planets in our Solar System also move in orbits close to this plane. When making calculations on objects in the Solar System it is therefore often convenient to define positions with respect to the ecliptic, that is, to use the ecliptic coordinate system. This system, like the equatorial system described in Section 18, also uses the vernal equinox, Υ , as its reference direction. Figure 19, which is similar to Figure 18(b), shows how it goes.

The imaginary traces of the planes of the equator and the ecliptic are drawn on the sky, and their point of intersection is the vernal equinox, Υ . The two planes are inclined to each other at an angle of about 23.5 degrees, called the **obliquity** of the ecliptic and given the symbol ε . (See Section 27 for a formula for calculating ε .) This angle is the tilt of the Earth's NS axis from the perpendicular to the plane of the ecliptic. Also marked in Figure 19 is a planet, V. Part of the trace of the imaginary great circle from the pole of the ecliptic (i.e. the point where the line drawn through the Sun perpendicular to the ecliptic meets the celestial sphere) down through V is marked and this cuts the ecliptic at F. Then the **ecliptic longitude**, λ , of V is defined to be the angle subtended by the points Υ and F, and the **ecliptic latitude**, β , the angle subtended by the points F and V.

As with equatorial coordinates, β is positive if the planet is above (i.e. north of) the ecliptic and negative if it is below it. The sense of λ is such that λ increases as you move eastwards along the ecliptic. Both λ and β are usually measured in degrees.

During the course of the year the Sun moves eastwards along the trace of the ecliptic. By definition, its ecliptic latitude is always zero. On about 21 March, it is at the position Υ and its right ascension and declination are both zero. Its ecliptic longitude is also zero. Thereafter, its ecliptic longitude steadily increases until three months later it is 90° , midsummer in the northern hemisphere. After the course of 1 year, the Sun has returned to its starting position having traversed 360° of ecliptic longitude.

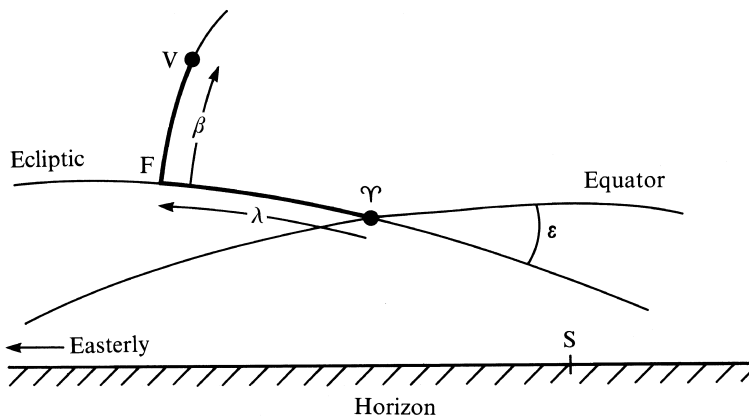


Figure 19. Ecliptic coordinates as seen from the ground (northern hemisphere).

20 Galactic coordinates

Astronomers occasionally need to describe the relationships between stars or other celestial objects within our own Galaxy and to do so it is convenient to use the galactic coordinate system. This time, the fundamental plane is the plane of the Galaxy and the fundamental direction is the line joining our Sun to the centre of the Galaxy. Figure 20 describes the situation. The point marked S represents the Sun, G is the centre of the Galaxy, and X a star which does not lie in the galactic plane. In equatorial coordinates, the position of G is $\alpha = 17^{\text{h}}42.4^{\text{m}}$ and $\delta = -28^{\circ}55'$. The lines SG and SX' both lie in the plane of the Galaxy; the point X' is the projection of the star's position onto the plane. The **galactic longitude** is defined to be the angle l measured in the plane, and the **galactic latitude** is defined to be the angle b measured perpendicular to it. The longitude increases from 0° to 360° in the same direction as increasing right ascension, and the latitude ranges from 0° to 90° north of the plane and from 0° to -90° south of it. These coordinates may be used, for example, to express the position of a star in the Milky Way.

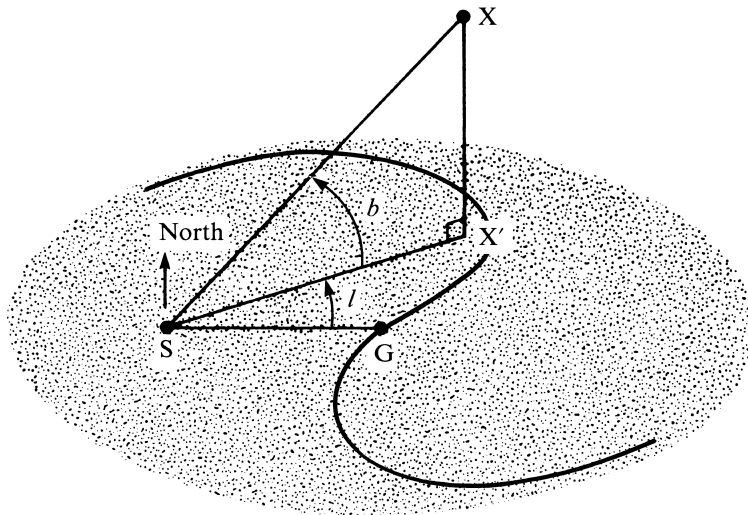


Figure 20. Galactic coordinates.

21 Converting between decimal degrees and degrees, minutes and seconds

Angles are often expressed as degrees, minutes and seconds; the minutes and seconds are called minutes and seconds of arc to distinguish them from time. Calculations are best done, however, with decimal degrees and the methods of conversion between these two forms are exactly the same as the methods for conversion between hours, minutes and seconds and decimal hours (Sections 7 and 8).

As an example, the angle $182^\circ 31' 27''$ is equal to 182.524 167 degrees, seen as follows:

<i>Method</i>	<i>Example</i>
1. Take the number of seconds and divide by 60.	$27/60 = 0.450\,000$
2. Add this to the number of minutes and divide by 60.	$31.45/60 = 0.524\,167$
3. Add the number of degrees.	$+182.0 = \mathbf{182.524\,167}$

Going the other way is equally straightforward. Continuing our example, express the angle 182.524 167 in degrees, minutes and seconds form.

<i>Method</i>	<i>Example</i>
1. Take the fractional part and multiply by 60. The integer part of the result is the number of minutes.	$0.524\,167 \times 60 = \mathbf{31.450\,020}$
2. Take the fractional part of the result and multiply by 60. This gives the number of seconds.	$0.450\,020 \times 60 = \mathbf{27.001\,200}$
Add the number of degrees.	$\mathbf{182^\circ 31' 27''}$

The spreadsheets shown in Figures 21 and 22, DMSDD, short for Degrees, Minutes and Seconds to Decimal Degrees conversion, and DDDMS, short for Decimal Degrees to Degrees, Minutes and Seconds conversion, will carry out these calculations, as will also the corresponding spreadsheet functions DMSDD(D,M,S), and DDDeg(D), DDDMin(D), and DDDSec(D). The first of these, DMSDD(D,M,S), takes three arguments D, M, S corresponding to the angle expressed in degrees, minutes and seconds, and returns the equivalent angle expressed in decimal degrees. Thus you could delete rows 7 to 10 of the spreadsheet shown in Figure 21 (save a copy) and insert the formula =DMSDD(C3,C4,C5) into cell H3.

The three functions DDDeg(D), DDDMin(D), and DDDSec(D) return the degrees part, the minutes part and the seconds part respectively of the angle given by the argument, D, expressed in decimal degrees. You can therefore delete rows 7 to 14 of the spreadsheet shown in Figure 22 (having saved a copy) and insert the following formulas into cells H3, H4 and H5 respectively:

=DDDeg(C3)

=DDMin(C3)

=DDSec(C3).

Why not try this for yourself?

	A	B	C	D	E	F	G	H	I
1	Converting degrees, minutes and seconds to decimal degrees								
2									
3	<i>Input</i>	degrees	182			<i>Output</i>	decimal degrees	182.5241667	=C10
4		minutes	31						
5		seconds	27						
6									
7	1	A	0.45	=ABS(C5)/60					
8	2	B	0.524166667	=(ABS(C4)+C7)/60					
9	3	C	182.5241667	=ABS(C3)+C8					
10	4	D	182.5241667	=IF((C3<0)+(C4<0)+(C5<0),-C9,C9)					

Figure 21. Converting angles expressed in degrees, minutes and seconds into decimal degrees.

	A	B	C	D	E	F	G	H	I
1	Converting decimal degrees to degrees, minutes and seconds								
2									
3	<i>Input</i>	decimal degrees	182.5241667			<i>Output</i>	degrees	182	=C14
4							minutes	31	=C12
5							seconds	27	=C10
6									
7	1	unsigned decimal	182.5241667	=ABS(C3)					
8	2	total seconds	657087.0001	=C7*3600					
9	3	seconds (2 dp)	27	=ROUND(MOD(C8,60),2)					
10	4	corrected seconds	27	=IF(C9=60,0,C9)					
11	5	corrected remainder	657087.0001	=IF(C9=60,C8+60,C8)					
12	6	minutes	31	=MOD(TRUNC(C11/60),60)					
13	7	unsigned degrees	182	=TRUNC(C11/3600)					
14	8	signed degrees	182	=IF(C3<0,-1*C13,C13)					

Figure 22. Converting angles expressed in decimal degrees into degrees, minutes and seconds.

22 Converting between angles expressed in degrees and angles expressed in hours

It is common astronomical practice to express the hour angle or right ascension of a star in hours, minutes and seconds of time rather than in degrees. We can transform one to the other by noting that 360° of Earth's rotation takes place in 1 day, or 24 hours. Thus 360° is equivalent to 24 hours or 15° to 1 hour. Table 4 illustrates this equivalence more completely. To convert between angles expressed in *decimal* hours and angles expressed in *decimal* degrees, simply multiply or divide by 15. For example, the right ascension 9h 36m 10.2s is equivalent to $144^\circ\ 02'\ 33''$.

Unit of time	Equivalent angle
1 day	360 degrees
1 hour	15 degrees
1 minute	15 arcmin
1 second	15 arcsec

Unit of angle	Equivalent time
1 radian	3.819 719 hours
1 degree	4 minutes
1 arcmin	4 seconds
1 arcsec	0.066 667 seconds

Table 4. Expressing angles in degrees or units of time.

We have not included spreadsheets for such simple calculations, but we have provided the spreadsheet functions DDDH(D), short for Decimal Degrees to Decimal Hours conversion, and DHDD(H), short for Decimal Hours to Decimal Degrees conversion. Each takes a single argument giving the angle expressed in decimal degrees or decimal hours respectively. The following spreadsheet formulas will carry out the example given above:

=DDDeg(DHDD(HMSDH(9,36,10.2))), which returns 144° ;
 =DDMin(DHDD(HMSDH(9,36,10.2))), which returns $02'$; and
 =DDSec(DHDD(HMSDH(9,36,10.2))), which returns $33''$.

Here we have nested several spreadsheet functions together. To work out how these work, always start from the middle and work outwards. Thus, in the first of the formulas above, HMSDH(9,36,10.2) returns the time 9h 36m 10.2s expressed in decimal hours. This value becomes the argument for the function DHDD that converts the decimal hours to decimal degrees. Its result is then the argument for the outer function DDDeg that returns the degrees part of the angle expressed as degrees, minutes and seconds. To convert the other way round you could write:

=DHHour(DDDH(DMSDD(144,2,33))), which returns 9h;
 =DHMin(DDDH(DMSDD(144,2,33))), which returns 36m; and
 =DHSec(DDDH(DMSDD(144,2,33))), which returns 10.2s.

23 Converting between one coordinate system and another

It is very often necessary to convert the coordinates of a heavenly body expressed in one coordinate system into the equivalent coordinates of another system. This is the case when, for example, you have found the position of a planet in ecliptic coordinates and you then wish to convert to horizon coordinates to see where to look in the sky. The formulas for conversion between the equatorial system and any of the other three systems, horizon, ecliptic, or galactic, are relatively straightforward. The conversion, therefore, is often best done via the equatorial system, as illustrated in Figure 23. The arrows indicate the conversions treated explicitly in this book in the section specified by the number. For example, to convert from galactic coordinates to horizon coordinates, first convert to equatorial coordinates (Section 30) and then to horizon coordinates (Section 25).

An alternative to using explicit formulas for each conversion, as in Sections 25 to 30, is to adopt the matrix method of generalised coordinate transformation. This is described in Section 31, and may be used to convert from any system to any other system directly, once you have worked out the appropriate matrix.

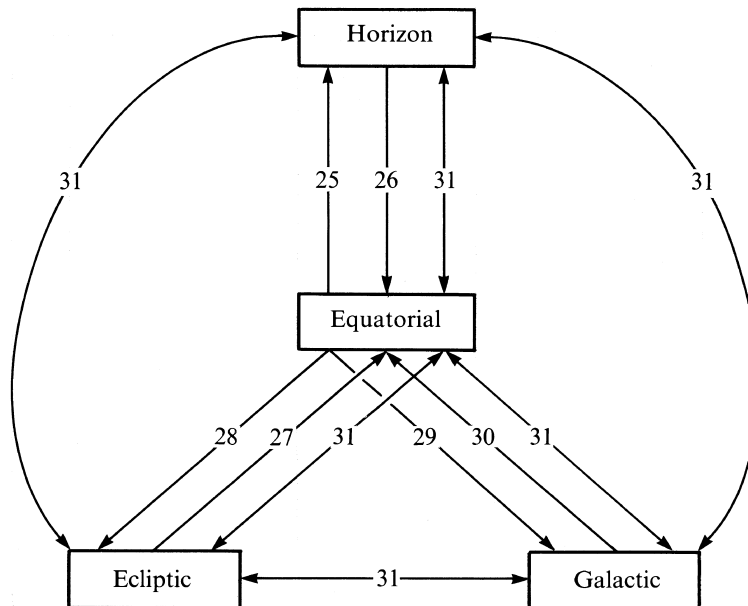


Figure 23. Converting between coordinate systems.

24 Converting between right ascension and hour angle

The hour angle, H , and the right ascension, α , are related by the simple formula

$$H = \text{LST} - \alpha,$$

where LST is the local sidereal time. All quantities must be expressed in the same units, i.e. as degrees or as hours. Let us take as an example the problem of finding the local hour angle of a star whose right ascension is $\alpha = 18\text{h } 32\text{m } 21\text{s}$, at a point whose longitude is 64° W , in time zone -4 h when daylight saving was not in operation, on local calendar date 22 April 1980 at local civil time 14h 36m 51.67s.

Method	Example
1. Find the UT and Greenwich calendar date corresponding to the local time and date (§9).	$\text{UT} = 18.614353$ $\text{GDay} = 22$ $\text{GMonth} = 4$ $\text{GYear} = 1980$
2. Find the corresponding Greenwich sidereal time (§12). ...	$\text{GST} = 8.679071$
3. ... hence find the local sidereal time (§14).	$\text{LST} = 4.412404$
4. Express the right ascension α in decimal hours (§7).	$\alpha = 18.539167 \text{ hours}$
5. Subtract this from the LST. If the answer is negative, add 24. This is the hour angle.	$H1 = -14.126763$ $H = 9.873237 \text{ hours}$
6. Convert to hours, minutes and seconds form (§8).	$H = \mathbf{9\text{h } 52\text{m } 23.66\text{s}}$

Figure 24 shows the spreadsheet for this calculation. We have made use of several spreadsheet functions, such as LCTUT, LCTGDay etc., to carry out much of the work. These functions are defined in their corresponding sections. The spreadsheet is called RAHA (Right Ascension to Hour Angle conversion). We have also provided a spreadsheet function of the same name with 12 arguments:

$\text{RAHA}(\text{RH}, \text{RM}, \text{RS}, \text{LH}, \text{LM}, \text{LS}, \text{DS}, \text{ZC}, \text{D}, \text{M}, \text{Y}, \text{GL})$.

RH, RM, RS are the right ascension in hours, minutes and seconds respectively, LH, LM, LS are the local civil time in hours, minutes, and seconds respectively, DS and ZC are the daylight saving and zone correction in hours, D, M, Y is the local calendar date as day, month, year, and GL is the geographical longitude in degrees, west negative. Having saved a copy, you can delete rows 16 to 24 of the spreadsheet and insert the following formulas in cells H3, H4, and H5 respectively:

$=\text{DHHour}(\text{RAHA}(\text{C3}, \text{C4}, \text{C5}, \text{C6}, \text{C7}, \text{C8}, \text{C9}, \text{C10}, \text{C11}, \text{C12}, \text{C13}, \text{C14}))$

$=\text{DHMin}(\text{RAHA}(\text{C3}, \text{C4}, \text{C5}, \text{C6}, \text{C7}, \text{C8}, \text{C9}, \text{C10}, \text{C11}, \text{C12}, \text{C13}, \text{C14}))$

$=\text{DHSec}(\text{RAHA}(\text{C3}, \text{C4}, \text{C5}, \text{C6}, \text{C7}, \text{C8}, \text{C9}, \text{C10}, \text{C11}, \text{C12}, \text{C13}, \text{C14}))$

to achieve the same result with a lot less typing!

	A	B	C	D	E	F	G	H	I	J
1	Conversion of right ascension to hour angle									
2										
3	Input	RA hours	18		Output	H hours	9	=DHHour(C24)		
4		RA mins	32			H mins	52	=DHMin(C24)		
5		RA secs	21			H secs	23.66	=DHSec(C24)		
6		LCT hours	14							
7		LCT mins	36							
8		LCT secs	51.67							
9		daylight saving	0							
10		zone correction	-4							
11		local day	22							
12		local month	4							
13		local year	1980							
14		geog long	-64							
15										
16	1	UT	18.61435278	=LCTUT(C6,C7,C8,C9,C10,C11,C12,C13)						
17	5	Gday	22	=LCTGDay(C6,C7,C8,C9,C10,C11,C12,C13)						
18	6	Gmonth	4	=LCTGMonth(C6,C7,C8,C9,C10,C11,C12,C13)						
19	7	Gyear	1980	=LCTGYear(C6,C7,C8,C9,C10,C11,C12,C13)						
20	8	GST	8.679070963	=UTGST(C16,0,0,C17,C18,C19)						
21	12	LST	4.412404296	=GSTLST(C20,0,0,C14)						
22	13	RA	18.53916667	=HMSDH(C3,C4,C5)						
23	14	H1	-14.12676237	=C21-C22						
24	15	H	9.873237629	=IF(C23<0,C24+C23,C23)						

Figure 24. Converting right ascension to local hour angle.

Converting an hour angle back to its equivalent right ascension is a very similar process. Continuing the example, what was the right ascension of the star whose local hour angle was 9h 52m 23.66s on local calendar date 22 April 1980 when observed in time zone -4 h from longitude 64° W at local time 14h 36m 51.67s, when daylight saving was not in operation?

Method	Example
1. Find the UT and Greenwich calendar date corresponding to the local time and date (§9).	UT = 18.614353 GDay = 22 GMonth = 4 GYear = 1980
2. Find the corresponding Greenwich sidereal time (§12)...	GST = 8.679071
3. ...hence find the local sidereal time (§14).	LST = 4.412404
4. Express the hour angle H in decimal hours (§7).	H = 9.873239 hours
5. Subtract this from the LST. If the answer is negative, add 24. This is the right ascension.	α_1 = -5.460835 α = 18.539165 hours
6. Convert to hours, minutes and seconds form (§8).	α = 18h 32m 21s

The spreadsheet for making this calculation is shown in Figure 25 and is called HARA (for Hour Angle to Right Ascension conversion). We have also provided a spreadsheet function with the same name which takes 12 arguments as follows:

HARA(HH, HM, HS, LH, LM, LS, DS, ZC, D, M, Y, GL).

The first three, HH, HM, HS take the hours, minutes and seconds of the hour angle, the next three, LH, LM, LS take the hours, minutes and seconds of the local civil time, then DS and ZC take the daylight saving and zone correction values in hours, D, M, Y take the day, month and year of the local calendar date, and finally GL takes the observer's geographical longitude in degrees, west negative. You can use this function to replace rows 16 to 24 of Figure 25 (save a copy first). Delete those rows and insert the following formulas into cells H3, H4, and H5 respectively:

```
=DHHour(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHMin(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHSec(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)).
```

As usual, the arguments are references to cells in the spreadsheet which contain the relevant values. Thus the first argument, which is the hours part of the hour angle expressed as hours, minutes, seconds, points to cell C3 which actually contains that number.

	A	B	C	D	E	F	G	H	I	J	
1	Conversion of hour angle to right ascension										
2											
3	Input	HA hours	9			Output	RA hours	18	=DHHour(C24)		
4		HA mins	52				RA mins	32	=DHMin(C24)		
5		HA secs	23.66				RA secs	21	=DHSec(C24)		
6		LCT hours	14								
7		LCT mins	36								
8		LCT secs	51.67								
9		daylight saving	0								
10		zone correction	-4								
11		local day	22								
12		local month	4								
13		local year	1980								
14		geog long	-64								
15											
16	1	UT	18.61435278	=LCTUT(C6,C7,C8,C9,C10,C11,C12,C13)							
17	5	Gday	22	=LCTGDay(C6,C7,C8,C9,C10,C11,C12,C13)							
18	6	Gmonth	4	=LCTGMonth(C6,C7,C8,C9,C10,C11,C12,C13)							
19	7	Gyear	1980	=LCTGYear(C6,C7,C8,C9,C10,C11,C12,C13)							
20	8	GST	8.679070963	=UTGST(C16,0,0,C17,C18,C19)							
21	12	LST	4.412404296	=GSTLST(C20,0,0,C14)							
22	13	HA	9.873238889	=HMSDH(C3,C4,C5)							
23	14	R1	-5.460834593	=C21-C22							
24	15	RA	18.53916541	=IF(C23<0,24+C23,C23)							

Figure 25. Converting local hour angle to right ascension.

25 Equatorial to horizon coordinate conversion

The formulas describing the relationships between hour angle, H , declination, δ , azimuth, A , and altitude, a , are:

$$\sin a = \sin \delta \sin \phi + \cos \delta \cos \phi \cos H,$$

$$\cos A = \frac{\sin \delta - \sin \phi \sin a}{\cos \phi \cos a},$$

where ϕ is the observer's geographical latitude. (The hour angle may be found from the right ascension by the method of Section 24.) These may be dealt with in the following way using the example 'what are the altitude and azimuth of a star whose hour angle is 5h 51m 44s and declination is $+23^\circ 13' 10''$?. The observer's latitude is 52° N.

Method	Example
1. Convert hour angle to decimal hours (§7).	$H = 5.862\,222$ hours
2. Multiply by 15 to convert H to degrees (§22).	$H = 87.933\,333$ degrees
3. Convert δ into decimal degrees (§21).	$\delta = 23.219\,444$ degrees
4. Find $\sin a = \sin \delta \sin \phi + \cos \delta \cos \phi \cos H$.	$\sin a = 0.331\,080$
5. Take inverse sin to find a .	$a = 19.334\,345$ degrees
6. Find $\cos A = \frac{\sin \delta - \sin \phi \sin a}{\cos \phi \cos a}$.	$\cos A = 0.229\,558$
7. Take inverse cos to find A' .	$A' = 76.728\,973$
8. Find $\sin H$.	$\sin H = 0.999\,350$
If negative, the true azimuth is $A = A'$.	(positive)
If positive, the true azimuth is $A = 360 - A'$.	$A = 283.271\,027$ degrees
9. Convert a and A to degrees, minutes and seconds (§21).	$a = 19^\circ 20' 3.64''$ $A = 283^\circ 16' 15.70''$

Step 8 is necessary because calculators can only return inverse trigonometrical functions correctly (inverse sin, inverse cos and inverse tan, which we will denote \sin^{-1} , \cos^{-1} and \tan^{-1} respectively) over half the range of 0° to 360° . For example, try $\cos 147^\circ$. The answer is -0.8387 which reverts to 147° when you take inverse cos. But now try $\cos 213^\circ$. The answer is again -0.8387 which, when you take inverse cos, gives 147° . Hence, whenever the inverse is taken an ambiguity arises that has to be cleared up by another means.

As an alternative to steps 6 to 8 you can calculate

$$\tan A' = \frac{y}{x} = \frac{-\cos \delta \cos \phi \sin H}{\sin \delta - \sin \phi \sin a}.$$

The azimuth is then found in the correct quadrant by using the rules (i) if x is negative, add 180° to A' , and (ii) if x is positive but y is negative, add 360° to A' .

Note that negative angles can be transformed back into the range 0° to 360° by simply adding 360. An example is -87.23 degrees which is the same as $360 - 87.23 = 272.77$ degrees.

The spreadsheet for converting from Equatorial coordinates (right ascension and declination) to Horizon coordinates is called EQHOR and is shown in Figure 26. Note that, in spreadsheet trigonometric formulas, the angles must be expressed in **radians** rather than degrees. We make extensive use of the functions RADIANS(D) and DEGREES(R) which convert, respectively, D given in degrees into radians, and R given

in radians into degrees. Examples can be found in rows 13 and 19. Another trick is to use the built-in function $\text{ATAN2}(x,y)$ to find the inverse tangent of y/x (row 22). In this case, the values of x and y are given explicitly in the two arguments, and the function returns the angle in its correct quadrant (in radians), thus overcoming the ambiguity on taking inverse tan. Some calculators also have a special key to do this. On ours, it is labelled $(y,x) \rightarrow (\theta,R)$. This is for converting the Cartesian coordinates of a point in two-dimensional space, (x,y) , into its equivalent polar coordinates (R,θ) . Don't worry about the details. Suffice it to say that if you use the key on the values of x and y you get the angle, θ , in its correct quadrant.

The spreadsheet functions are $\text{EqAz}(H,M,S,DD,DM,DS,GP)$ and $\text{EqAlt}(H,M,S,DD,DM,DS,GP)$. Both take the same seven arguments H, M, S , the hour angle expressed as hours, minutes and seconds, DD, DM, DS , the declination expressed as degrees, minutes, seconds, and GP , the observer's geographical latitude in decimal degrees (S negative). EqAz returns the azimuth, and EqAlt returns the altitude, both in decimal degrees. You can try these functions for yourself by first saving a copy of the spreadsheet EQHOR (in case

	A	B	C	D	E	F	G	H	I	J
1	Equatorial to Horizon coordinate conversion									
2										
3	Input	HA hours	5		Output	Az degs	283	=DDDeg(C24)		
4		HA mins	51			Az mins	16	=DDMin(C24)		
5		HA secs	44			Az secs	15.7	=DDSec(C24)		
6		dec degs	23			alt degs	19	=DDDeg(C19)		
7		dec mins	13			alt mins	20	=DDMin(C19)		
8		dec secs	10			alt secs	3.64	=DDSec(C19)		
9		geog lat	52							
10										
11	1	H	5.862222222	=HMSDH(C3,C4,C5)						
12	2	H degs	87.93333333	=C11*15						
13	3	H (rads)	1.534726189	=RADIANS(C12)						
14	4	dec (degs)	23.21944444	=DMSDD(C6,C7,C8)						
15	5	dec (rads)	0.405255756	=RADIANS(C14)						
16	6	lat (rads)	0.907571211	=RADIANS(C9)						
17	7	sin a	0.331080083	=SIN(C15)*SIN(C16)+COS(C15)*COS(C16)*COS(C13)						
18	8	a (rads)	0.337447983	=ASIN(C17)						
19	9	a (degs)	19.33434522	=DEGREES(C18)						
20	10	y	-0.565425853	=-COS(C15)*COS(C16)*SIN(C13)						
21	11	x	0.133359148	=SIN(C15)-SIN(C16)*C17						
22	12	A	-1.339173206	=ATAN2(C21,C20)						
23	13	B	-76.72897273	=DEGREES(C22)						
24	14	Az (degs)	283.2710273	=C23-(360*INT(C23/360))						

Figure 26. Converting equatorial to horizon coordinates.

you want it again), second deleting rows 11 to 24 inclusive, and third inserting the following formulas into cells H3 to H8:

```
=DDDeg(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDMin(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDSec(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDDeg(EqAlt(C3,C4,C5,C6,C7,C8,C9))
=DDMin(EqAlt(C3,C4,C5,C6,C7,C8,C9))
=DDSec(EqAlt(C3,C4,C5,C6,C7,C8,C9)).
```

The calculations of rows 11–24 are carried out by the BASIC programs behind the spreadsheet functions, and should return precisely the same values as the spreadsheet in Figure 26.

26 Horizon to equatorial coordinate conversion

This problem is the reverse of that of the preceding section, namely given a star's altitude, a , and azimuth, A , what are its declination, δ , and hour angle, H ? The appropriate formulas are:

$$\sin \delta = \sin a \sin \phi + \cos a \cos \phi \cos A,$$

$$\cos H = \frac{\sin a - \sin \phi \sin \delta}{\cos \phi \cos \delta},$$

where ϕ is the observer's geographical latitude. Notice that these formulas are exactly the same as those given in Section 25 except that δ and H have been substituted for a and A and vice-versa. This fact is useful when writing a program for a programmable calculator since exactly the same program can be used to convert δ , H to a , A or a , A to δ , H .

Let us take the following example: a star is observed by an observer at latitude 52° N to have an altitude of $19^\circ 20' 03.64''$ and an azimuth of $283^\circ 16' 15.7''$. What are its hour angle and declination? If the observer is on the Greenwich meridian and the GST is 0h 24m 05s, what is the right ascension?

Method	Example
1. Convert azimuth to decimal degrees (§21).	$A = 283.271\,028$ degrees
2. Convert altitude to decimal degrees (§21).	$a = 19.334\,344$ degrees
3. Find $\sin \delta = \sin a \sin \phi + \cos a \cos \phi \cos A$.	$\sin \delta = 0.394\,254$
4. Take inverse sin to find δ .	$\delta = 23.219\,444$ degrees
5. Find $\cos H = \frac{\sin a - \sin \phi \sin \delta}{\cos \phi \cos \delta}$.	$\cos H = 0.036\,062$
6. Take inverse cos to find H' .	$H' = 87.933\,334$ degrees
7. Find $\sin A$.	$\sin A = -0.973\,295$
If negative, the true hour angle is $H = H'$.	
If positive, the true hour angle is $H = 360 - H'$.	
8. Convert H into hours by dividing by 15 (§22).	$H = 5.862\,222$ hours
9. Convert H and δ into minutes and seconds form (§§8 and 21).	$H = 5\text{h } 51\text{m } 44\text{s}$
	$\delta = 23^\circ 13' 10''$

Again, step 7 is necessary to remove the ambiguity introduced by taking the inverse of cos. Alternatively,

calculate the hour angle from

$$\tan H' = \frac{y}{x} = \frac{-\cos a \cos \phi \sin A}{(\sin a - \sin \phi \sin \delta)}$$

H is then found in the correct quadrant by adding 180° to H' if x is negative, or 360° to H' if x is positive and y is negative.

The spreadsheet for converting Horizon to Equatorial coordinates is called HOREQ and is shown in Figure 27. Please see the notes in the previous section about converting between degrees and radians, and about using the ATAN2 function to remove the ambiguity on taking the inverse of a trigonometric quantity.

The spreadsheet functions

HorHa(AZD,AZM,AZS,ALD,ALM,ALS,GP) and

HorDec(AZD,AZM,AZS,ALD,ALM,ALS,GP)

will convert the azimuth expressed as degrees, minutes and seconds (AZD, AZM, AZS) and altitude also

	A	B	C	D	E	F	G	H	I	J
1	Horizon to Equatorial coordinate conversion									
2										
3	<i>Input</i>	Az degs	283		<i>Output</i>	HA hours	5	=DHHour(C24)		
4		Az mins	16			HA mins	51	=DHMin(C24)		
5		Az secs	15.7			HA secs	44	=DHSec(C24)		
6		alt degs	19			dec degs	23	=DDDeg(C18)		
7		alt mins	20			dec mins	13	=DDMin(C18)		
8		alt secs	3.64			dec secs	10	=DDSec(C18)		
9		geog lat	52							
10										
11	1	Az (degs)	283.2710278	=DMSDD(C3,C4,C5)						
12	2	alt (degs)	19.33434444	=DMSDD(C6,C7,C8)						
13	3	alt (rads)	0.337447969	=RADIANS(C12)						
14	4	lat (rads)	0.907571211	=RADIANS(C9)						
15	5	Az (rads)	4.94401211	=RADIANS(C11)						
16	6	sin dec	0.394253809	=SIN(C13)*SIN(C14)+COS(C13)*COS(C14)*COS(C15)						
17	7	dec (rads)	0.405255751	=ASIN(C16)						
18	8	dec (degs)	23.21944417	=DEGREES(C17)						
19	9	y	0.565425854	=-COS(C13)*COS(C14)*SIN(C15)						
20	10	x	0.020403829	=SIN(C13)-SIN(C14)*C16						
21	11	A	1.534726206	=ATAN2(C20,C19)						
22	12	B	87.93333428	=DEGREES(C21)						
23	13	HA (degs)	87.93333428	=C22-(360*INT(C22/360))						
24	14	HA (hours)	5.862222286	=DDD(H(C23))						

Figure 27. Converting horizon to equatorial coordinates.

expressed as degrees, minutes and seconds (ALD, ALM, ALS) into the corresponding hour angle in decimal hours and declination in decimal degrees respectively. The last argument, GP, is the observer's geographical latitude in decimal degrees (south negative). You can try these functions for yourself by first saving a copy of the spreadsheet HOREQ (in case you want it again), second deleting rows 11 to 24 inclusive, and third inserting the following formulas into cells H3–H8:

```
=DDDeg(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDMin(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDSec(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDDeg(HorDec(C3,C4,C5,C6,C7,C8,C9))
=DDMin(HorDec(C3,C4,C5,C6,C7,C8,C9))
=DDSec(HorDec(C3,C4,C5,C6,C7,C8,C9)).
```

The second part of the problem, converting the hour angle into the right ascension, was covered in Section 24. By applying the method shown there, you should obtain the result that the right ascension is 18h 32m 21s. Looking in our star atlas (see Bibliography on page 208), we find a sixth-magnitude star in the constellation of Hercules listed near this position.

27 Ecliptic to equatorial coordinate conversion

The ecliptic longitude, λ , and the ecliptic latitude, β , may be converted into right ascension, α , and declination, δ , using the formulas:

$$\alpha = \tan^{-1} \left\{ \frac{\sin \lambda \cos \varepsilon - \tan \beta \sin \varepsilon}{\cos \lambda} \right\},$$

$$\delta = \sin^{-1} \{ \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda \},$$

where ε is the **obliquity of the ecliptic**, the angle between the planes of the equator and the ecliptic. This angle changes slowly with time and for high accuracy the appropriate value should be used. If, for example, α and δ are referred to the standard epoch of 2000.0 (see Section 34), then ε should have its 2000.0 value. In the examples given here and in the following section, we make the calculation for a given Greenwich calendar date. Since the value of ε changes so slowly, it is often sufficient to find its value in the middle of a given year and use that in all calculations for that year. The method of calculating the mean obliquity of the ecliptic for any date is given by the equation

$$\varepsilon = 23^\circ 26' 21.45'' - 46.815''T - 0.0006''T^2 + 0.00181''T^3,$$

where T is the number of Julian centuries since epoch 2000 January 1.5 (Julian date 2 451 545.0). For example, what was the mean obliquity of the ecliptic on Greenwich calendar date 6 July 2009?

Method	Example
1. Calculate the Julian date (§4).	JD = 2 455 018.5
2. Subtract 2 451 545.0 (= JD for 2000 January 1.5).	MJD = 3 473.5 days
3. Divide by 36 525.0. The result is T .	T = 0.095 099 247 centuries
4. Calculate $DE = 46.815T + 0.0006T^2 - 0.00181T^3$.	DE = 4.452 075 122 arcsec
5. Divide by 3600 to convert to degrees.	DE = 0.001 236 688 degrees
6. Subtract DE from 23.439 292 to find ϵ .	ϵ = 23.438 055 31 degrees
7. If necessary, convert to degrees, minutes and seconds (§21).	ϵ = 23° 26' 17"

The spreadsheet *Obliq*, Figure 28, will make this calculation. We have also provided the spreadsheet function *Obliq*(D,M,Y), which returns the mean obliquity of the ecliptic calculated for the Greenwich calendar date, D days, M months and Y years. Whenever you need the obliquity in a spreadsheet, you could therefore insert the formula =*Obliq*(C3,C4,C5) where, for example, the day, month and year were in the cells C3, C4, C5 respectively. For very precise calculations, you also want to make allowance for **nutatation** (see Section 35) by adding the result of the function *NutatObl*(D,M,Y). The spreadsheet function *Obliq* already adds in this correction for you so you do not need to do so separately.

Having obtained the value of the obliquity of the ecliptic, we are now in a position to convert ecliptic coordinates into equatorial coordinates. Our example this time is: what were the right ascension and the declination of a planet whose ecliptic coordinates were longitude 139° 41' 10" and latitude 4° 52' 31" on 6 July 2009?

	A	B	C	D	E	F	G	H	I
1	The Mean Obliquity of the Ecliptic								
2									
3	Input	Gday	6			Output	Obliquity	23.43805531	=C12
4		Gmonth	7						
5		Gyear	2009						
6									
7	1	JD	2455018.5	=CDJD(C3,C4,C5)					
8	2	MJD	3473.5	=C7-2451545					
9	3	T	0.095099247	=C8/36525					
10	4	DE	4.452075122	=C9*(46.815+C9*(0.0006-(C9*0.00181)))					
11	5	DE	0.001236688	=C10/3600					
12	6	Obliquity	23.43805531	=23.439292-C11					

Figure 28. Calculating the mean obliquity of the ecliptic.

Method	Example
1. Convert λ and β into decimal degrees (§21).	$\lambda = 139.686\,111$ degrees $\beta = 4.875\,278$ degrees
2. Find $\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda$ (with $\varepsilon = 23.438\,055$ degrees ^a).	$\sin \delta = 0.334\,383$
3. Take inverse sin to find δ in decimal degrees.	$\delta = 19.535\,003$ degrees
4. Find $y = \sin \lambda \cos \varepsilon - \tan \beta \sin \varepsilon$.	$y = 0.559\,666$
5. Find $x = \cos \lambda$.	$x = -0.762\,512$
6. Find $\alpha' = \tan^{-1} \left(\frac{y}{x} \right)$.	$\alpha' = -36.277\,799$ degrees
7. We have to remove the ambiguity which arises from taking the inverse tan. The rule is that α should lie in the quadrant indicated by the signs of x and y in Figure 29. Add or subtract 180 or 360 to α' to bring it into the correct quadrant, unless it is already there, in which case $\alpha = \alpha'$.	x negative y positive $\therefore \alpha = \alpha' + 180.0$ $= 143.722\,173$ degrees
8. Convert α to hours by dividing by 15 (§22).	$\alpha = 9.581\,478$ hours
9. Convert α and δ to minutes and seconds form (§§21 and 8).	$\alpha = \mathbf{9^h\,34^m\,53.32s}$ $\delta = \mathbf{19^\circ\,32'\,6.01''}$

^aIncludes the nutation correction.

The spreadsheet for carrying out this calculation is shown in Figure 30 and is called ECEQ, short for Ecliptic coordinates to Equatorial coordinates conversion. The corresponding spreadsheet functions are EcRA and EcDec, returning the right ascension and declination in decimal degrees respectively. Each function takes the same nine arguments which are the ecliptic longitude as degrees, minutes, seconds, the ecliptic latitude as degrees, minutes, seconds, and the Greenwich calendar date as day, month, year. Thus you could replace the calculation part of the spreadsheet shown in Figure 30 using this function as follows. Save a copy of the spreadsheet, then delete rows 13 to 27 inclusive, and finally insert the following formulas into cells H3 to H8 respectively:

```
=DHHour(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DHMin(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DHSec(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DDDeg(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11)).
```

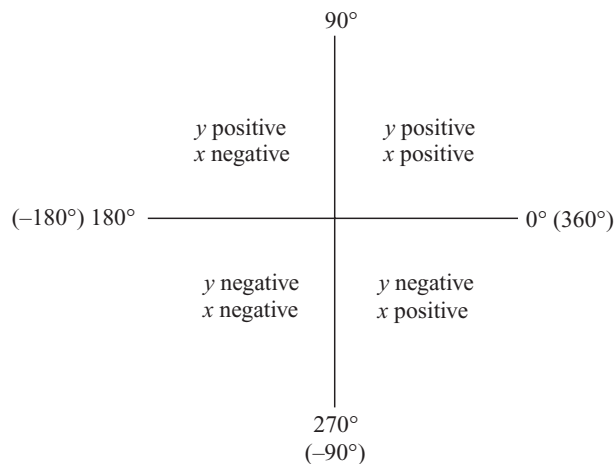


Figure 29. Removing the ambiguity on taking $\tan^{-1}(y/x)$.

	A	B	C	D	E	F	G	H	I	J
1	Ecliptic to Equatorial Coordinate Conversion									
2										
3	<i>Input</i>	ecl long deg	139		<i>Output</i>	RA hour	9	=DHHour(C27)		
4		ecl long min	41			RA min	34	=DHMin(C27)		
5		ecl long sec	10			RA sec	53.4	=DHSec(C27)		
6		ecl lat deg	4			dec deg	19	=DDDeg(C21)		
7		ecl lat min	52			dec min	32	=DDMin(C21)		
8		ecl lat sec	31			dec sec	8.52	=DDSec(C21)		
9		G day	6							
10		G month	7							
11		G year	2009							
12										
13	1	eclon (deg)	139.6861111	=DMSDD(C3,C4,C5)						
14	2	eclat (deg)	4.875277778	=DMSDD(C6,C7,C8)						
15	3	eclon (rad)	2.437982558	=RADIANS(C13)						
16	4	eclat (rad)	0.085089649	=RADIANS(C14)						
17	5	obliq (deg)	23.43923176	=Obliq(C9,C10,C11)						
18	6	obliq (rad)	0.409091768	=RADIANS(C17)						
19	7	sin dec	0.334394125	=SIN(C16)*COS(C18)+COS(C16)*SIN(C18)*SIN(C15)						
20	8	dec (rad)	0.340962273	=ASIN(C19)						
21	9	dec (deg)	19.53569924	=DEGREES(C20)						
22	10	y	0.559659329	=SIN(C15)*COS(C18)-TAN(C16)*SIN(C18)						
23	11	x	-0.762511521	=COS(C15)						
24	12	RA (rad)	2.508430994	=ATAN2(C23,C22)						
25	13	RA (deg)	143.7225092	=DEGREES(C24)						
26	14	RA (deg)	143.7225092	=C25-360*INT(C25/360)						
27	15	RA (hours)	9.58150061	=DDDHC(C26)						

Figure 30. Converting from ecliptic to equatorial coordinates.

28 Equatorial to ecliptic coordinate conversion

The reverse problem of the previous section is to find the celestial longitude and latitude, λ and β , given the right ascension and declination, α and δ . The formulas are:

$$\lambda = \tan^{-1} \left\{ \frac{\sin \alpha \cos \varepsilon + \tan \delta \sin \varepsilon}{\cos \alpha} \right\},$$

$$\beta = \sin^{-1} \{ \sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \sin \alpha \},$$

where ε is the obliquity of the ecliptic (see Section 27). These formulas are very nearly identical to those of the previous section with λ, β in place of α, δ and vice-versa; the symmetry is not quite complete, however, as the sign appearing in each formula is reversed.

Consider the example: what are the ecliptic coordinates of a planet whose right ascension and declination are given as $\alpha = 9^{\text{h}} 34^{\text{m}} 53.32^{\text{s}}$ and $\delta = 19^{\circ} 32' 6.01''$ when the Greenwich calendar date is 6 July 2009?

Method	Example
1. Convert α and δ into decimal degrees (§§21, 22 and 7).	$\delta = 19.535\,003$ degrees $\alpha = 143.722\,167$ degrees
2. Find $\sin \beta = \sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \sin \alpha$ (with, in 2009, $\varepsilon = 23.438\,055$ degrees ^a).	$\sin \beta = 0.084\,987$
3. Take inverse sin to find β in decimal degrees.	$\beta = 4.875\,276$ degrees
4. Find $y = \sin \alpha \cos \varepsilon + \tan \delta \sin \varepsilon$.	$y = 0.684\,007$
5. Find $x = \cos \alpha$.	$x = -0.806\,157$
6. Calculate $\lambda' = \tan^{-1} \left(\frac{y}{x} \right)$.	$\lambda' = -40.313\,894$ degrees
7. We have to remove the ambiguity which arises from taking the inverse tan. The rule is that λ should lie in the quadrant indicated by the signs of x and y in Figure 29. Add or subtract 180 or 360 to bring it into the correct quadrant, unless it is already there, in which case, $\lambda = \lambda'$.	x negative y positive $\therefore \lambda = \lambda' + 180$ $= 139.686\,106$ degrees
8. Convert λ and β to minutes and seconds form (§§21 and 8).	$\lambda = 139^{\circ} 41' 9.98''$ $\beta = 4^{\circ} 52' 30.99''$

^aIncludes the nutation correction.

Figure 31 shows the spreadsheet for this conversion, called EQEC (Equatorial to Ecliptic conversion). The corresponding spreadsheet functions are EqElong and EqElat returning, respectively, the ecliptic longitude and the ecliptic latitude, both in decimal degrees. They each take the same nine arguments, namely the right ascension in hour, minutes, seconds, the declination in degrees, minutes, seconds, and the Greenwich calendar date as day, month, year. You could therefore delete rows 13 to 26 inclusive of the spreadsheet shown in Figure 31 (having saved a copy), and insert into cells H3 to H8 respectively the following formulas:

```
=DDDeg(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDDeg(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11)).
```

	A	B	C	D	E	F	G	H	I	J
1	Equatorial to Ecliptic Coordinate Conversion									
2										
3	Input	RA (hour)	9		Output	ecl long (deg)	139	=DDDeg(C26)		
4		RA (min)	34			ecl long (min)	41	=DDMin(C26)		
5		RA (sec)	53.4			ecl long(sec)	9.97	=DDSec(C26)		
6		dec (deg)	19			ecl lat (deg)	4	=DDDeg(C21)		
7		dec (min)	32			ecl lat (min)	52	=DDMin(C21)		
8		dec (sec)	8.52			ecl lat (sec)	30.99	=DDSec(C21)		
9		Gday	6							
10		Gmonth	7							
11		Gyear	2009							
12										
13	1	RA (deg)	143.7225	=DHDD(HMSDH(C3,C4,C5))						
14	2	dec (deg)	19.5357	=DMSDD(C6,C7,C8)						
15	3	RA (rad)	2.508430834	=RADIANS(C13)						
16	4	dec (rad)	0.340962287	=RADIANS(C14)						
17	5	obliq (deg)	23.43923176	=Obliq(C9,C10,C11)						
18	6	obliq (rad)	0.409091768	=RADIANS(C17)						
19	7	sin ecl lat	0.084986972	=SIN(C16)*COS(C18)-COS(C16)*SIN(C18)*SIN(C15)						
20	8	ecl lat (rad)	0.085089613	=ASIN(C19)						
21	9	ecl lat (deg)	4.875275723	=DEGREES(C20)						
22	10	y	0.684010234	=SIN(C15)*COS(C18)+TAN(C16)*SIN(C18)						
23	11	x	-0.806160703	=COS(C15)						
24	12	ecl long (rad)	2.437982411	=ATAN2(C23,C22)						
25	13	ecl long (deg)	139.6861027	=DEGREES(C24)						
26	14	ecl long (deg)	139.6861027	=C25-360*INT(C25/360)						

Figure 31. Converting from equatorial to ecliptic coordinates.

29 Equatorial to galactic coordinate conversion

Occasionally we need to know the position of a star in relation to the rest of the stars in our Galaxy and to do this we can use the galactic coordinate system. The conversion formulas are:

$$b = \sin^{-1} \{ \cos \delta \cos (27.4^\circ) \cos (\alpha - 192.25^\circ) + \sin \delta \sin (27.4^\circ) \},$$

$$l = \tan^{-1} \left\{ \frac{\sin \delta - \sin b \sin (27.4^\circ)}{\cos \delta \sin (\alpha - 192.25^\circ) \cos (27.4^\circ)} \right\} + 33^\circ.$$

The numbers come from the following facts about our Galaxy: north galactic pole coordinates are $\alpha = 192^\circ 15'$, $\delta = +27^\circ 24'$; ascending node of the galactic plane on equator $l = 33^\circ$.

The example is: what are the galactic coordinates of a star whose right ascension and declination are $\alpha = 10\text{h } 21\text{m } 00\text{s}$ and $\delta = 10^\circ 03' 11''$?

Method	Example
1. Convert α , δ into decimal form (§§21 and 7).	$\delta = 10.053\,056$ degrees $\alpha = 10.350\,000$ hours
2. Convert α into degrees by multiplying by 15 (§22).	$\alpha = 155.250\,000$ degrees
3. Find $\sin b = \cos \delta \cos(27.4) \times \cos(\alpha - 192.25) + \sin \delta \sin(27.4)$.	$\sin b = 0.778\,487$
4. Take inverse sin to find b in degrees.	$b = 51.122\,268$ degrees
5. Find $y = \sin \delta - \sin b \sin(27.4)$ and note its sign.	$y = -0.183\,700$ (negative)
6. Find $x = \cos \delta \sin(\alpha - 192.25) \times \cos(27.4)$ and note its sign.	$x = -0.526\,097$ (negative)
7. Divide y by x .	$y/x = 0.349\,174$
8. Take inverse tan. Now we have to remove the ambiguity which arises from taking the inverse tan. To do so, look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant, unless it is already in the correct quadrant.	$\tan^{-1}\left(\frac{y}{x}\right) = 19.247\,881$ degrees From Fig. 29: + 180.0
9. Add 33 to get l .	+ 33.0 $l = 232.247\,881$ degrees
10. Convert l and b into minutes and seconds form (§21).	$l = 232^{\circ} 14' 52''$ $b = 51^{\circ} 07' 20''$

Figure 32 shows the spreadsheet, named EQGAL (Equatorial to Galactic conversion), which will carry out this conversion. We have also provided spreadsheet functions EqGlong and EqGlat which return the galactic longitude and latitude respectively, both in decimal degrees. Each function takes the same six arguments, namely the right ascension in hours, minutes, seconds, and the declination in degrees, minutes, seconds. The spreadsheet in Figure 32 could be modified to use these functions instead of the calculation part. To do this, save a copy of the spreadsheet, delete rows 10 to 20, then insert the following formulas into cells H3 to H8 inclusive:

```
=DDDeg(EqGlong(C3,C4,C5,C6,C7,C8))
=DDMin(EqGlong(C3,C4,C5,C6,C7,C8))
=DDSec(EqGlong(C3,C4,C5,C6,C7,C8))
=DDDeg(EqGlat(C3,C4,C5,C6,C7,C8))
=DDMin(EqGlat(C3,C4,C5,C6,C7,C8))
=DDSec(EqGlat(C3,C4,C5,C6,C7,C8)).
```

	A	B	C	D	E	F	G	H	I	J
1	Equatorial to Galactic Coordinate Conversion									
2										
3	Input	RA (hour)	10		Output	Gal long (deg)	232	=DDDeg(C20)		
4		RA (min)	21			Gal long (min)	14	=DDMin(C20)		
5		RA (sec)	0			Gal long (sec)	52.38	=DDSec(C20)		
6		dec (deg)	10			Gal lat (deg)	51	=DDDeg(C16)		
7		dec (min)	3			Gal lat (min)	7	=DDMin(C16)		
8		dec (sec)	11			Gal lat (sec)	20.16	=DDSec(C16)		
9										
10	1	RA (deg)	155.25	=DHDD(HMSDH(C3,C4,C5))						
11	2	dec (deg)	10.05305556	=DMSDD(C6,C7,C8)						
12	3	RA (rad)	2.709623664	=RADIANS(C10)						
13	4	dec (rad)	0.175458919	=RADIANS(C11)						
14	5	sin b	0.778487145	=COS(C13)*COS(RADIANS(27.4))*COS(C12-RADIANS(192.25))+SIN(C13)*SIN(RADIANS(27.4))						
15	6	b (radians)	0.892251894	=ASIN(C14)						
16	7	b (deg)	51.1222678	=DEGREES(C15)						
17	8	y	-0.183699587	=SIN(C13)-C14*SIN(RADIANS(27.4))						
18	9	x	-0.52609727	=COS(C13)*SIN(C12-RADIANS(192.25))*COS(RADIANS(27.4))						
19	10	long (deg)	-127.7521165	=DEGREES(ATAN2(C18,C17))+33						
20	11	long (deg)	232.2478835	=C19-360*INT(C19/360)						

Figure 32. Converting from equatorial to galactic coordinates.

30 Galactic to equatorial coordinate conversion

Given the galactic coordinates, l and b , of a star, what are the corresponding equatorial coordinates, α and δ ? To answer this question we need the conversion formulas:

$$\delta = \sin^{-1} \{ \cos b \cos(27.4^\circ) \sin(l - 33^\circ) + \sin b \sin(27.4^\circ) \},$$

$$\alpha = \tan^{-1} \left\{ \frac{\cos b \cos(l - 33^\circ)}{\sin b \cos(27.4^\circ) - \cos b \sin(27.4^\circ) \sin(l - 33^\circ)} \right\} + 192.25^\circ,$$

where both α and δ are expressed in degrees. As an example we shall find the right ascension and declination of the star whose galactic coordinates are $l = 232^\circ 14' 52''$ and $b = 51^\circ 07' 20''$.

Method	Example
1. Convert l and b into decimal form (§21).	$b = 51.122\,222$ degrees $l = 232.247\,778$ degrees
2. Find $\sin \delta = \cos b \cos (27.4) \times \sin (l - 33) + \sin b \sin (27.4)$.	$\sin \delta = 0.174\,560$
3. Take inverse sin to find δ in degrees.	$\delta = 10.053\,087$ degrees
4. Find $y = \cos b \cos (l - 33)$ and note its sign.	$y = -0.592\,576$ (negative)
5. Find $x = \sin b \cos (27.4) - \cos b \sin (27.4) \sin (l - 33)$ and note its sign.	$x = 0.786\,373$ (positive)
6. Divide y by x .	$y/x = -0.753\,556$
7. Take inverse tan. We have to remove the ambiguity which arises from taking the inverse tan. To do so, look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant, unless it is already there.	$\tan^{-1} \left(\frac{y}{x} \right) = -37.000\,075$ (already in correct quadrant)
8. Add 192.25 to find α in degrees.	$+ 192.25$ $\alpha = 155.249\,925$ degrees
9. Divide by 15 to find α in hours (§22).	$\alpha = 10.349\,995$ hours
10. Convert α and δ to minutes and seconds form (§§21 and 8).	$\alpha = \mathbf{10h\,21m\,00s}$ $\delta = \mathbf{10^\circ\,03'\,11''}$

The spreadsheet, GALEQ (Galactic to Equatorial conversion), for this calculation is shown in Figure 33. We have also provided the spreadsheet functions GalRA and GalDec which return the right ascension in decimal degrees and the declination in decimal degrees respectively. These functions each take the same six arguments, being the galactic longitude in degrees, minutes and seconds, and the galactic latitude in degrees, minutes and seconds. If you wish, you can delete rows 10 to 21 of the spreadsheet shown in Figure 33 and insert the following formulas in cells H3 to H8 respectively.

```
=DHHour(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DHMin(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DHSec(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DDDeg(GalDec(C3,C4,C5,C6,C7,C8))
=DDMin(GalDec(C3,C4,C5,C6,C7,C8))
=DDSec(GalDec(C3,C4,C5,C6,C7,C8))
```

Don't forget to save a copy of the original spreadsheet first.

	A	B	C	D	E	F	G	H	I	J
1	Galactic to Equatorial Coordinate Conversion									
2										
3	Input	Gal long (deg)	232		Output	RA (hour)	10	=DHHour(C21)		
4		Gal long (min)	14			RA (min)	21	=DHMin(C21)		
5		Gal long (sec)	52.38			RA (sec)	0	=DHSec(C21)		
6		Gal lat (deg)	51			dec (deg)	10	=DDDeg(C16)		
7		Gal lat (min)	7			dec (min)	3	=DDMin(C16)		
8		Gal lat (sec)	20.16			dec (sec)	11	=DDSec(C16)		
9										
10	1	Glong (deg)	232.2478833	=DMSDD(C3,C4,C5)						
11	2	Glat (deg)	51.12226667	=DMSDD(C6,C7,C8)						
12	3	Glong (rad)	4.053490245	=RADIANS(C10)						
13	4	Glat (rad)	0.892251874	=RADIANS(C11)						
14	5	sin dec	0.17456002	=COS(C13)*COS(RADIANS(27.4))*SIN(C12-RADIANS(33))+SIN(C13)*SIN(RADIANS(27.4))						
15	6	dec (radians)	0.17545891	=ASIN(C14)						
16	7	dec (deg)	10.05305504	=DEGREES(C15)						
17	8	y	-0.592575092	=COS(C13)*COS(C12-RADIANS(33))						
18	9	x	0.786373677	=SIN(C13)*COS(RADIANS(27.4))-COS(C13)*SIN(RADIANS(27.4))*SIN(C12-RADIANS(33))						
19	10	RA (deg)	155.249999	=DEGREES(ATAN2(C18,C17))+192.25						
20	11	RA (deg)	155.249999	=C19-360*INT(C19/360)						
21	12	RA (hours)	10.34999993	=DDDH(C20)						

Figure 33. Converting from galactic to equatorial coordinates.

31 Generalised coordinate transformations

The methods described in Sections 24 to 30 for converting between one coordinate system and another are quite satisfactory for normal use where you wish to make a single calculation. However, if you have a computer, or a programmable calculator that can handle matrices, you may be interested in a more general method of converting between one system and another. You can then write a single program that converts between any two systems, for example directly from galactic to horizon coordinates, or from horizon to ecliptic coordinates. The method makes use of **matrices**, ordered sets of numbers set out in rows and columns. The method of manipulation of these numbers by the computer is always the same. Conversion between different coordinate systems is achieved merely by changing the numbers in the matrices.

The matrices which we shall have to deal with are of size 3×3 , that is they consist of nine numbers set out in three rows of three like this:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix},$$

where each of the letters represents a number. We can specify the whole matrix by an upper-case letter in a bold sans-serif typeface. Let **A** represent the above matrix. For example, if b, c, d, f, g and h are all zero,

and a, e, i are all 1, then

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The coordinates of the point that we wish to convert are specified by means of a **column vector**. This is really just a simple case of a matrix which has three rows with a single number in each row. If x, y and z are the **elements** of the vector, we can write it as

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

where the bold-face, italic lower-case letter \mathbf{v} represents the vector. Our program simply has to put the correct numbers into \mathbf{A} and \mathbf{v} , and then multiply them together to form a new column vector \mathbf{w} . We can then extract the new coordinates from \mathbf{w} . In symbols we can write this as

$$\mathbf{w} = \mathbf{A} \cdot \mathbf{v}.$$

The multiplication of \mathbf{v} by \mathbf{A} follows a strict set of rules. Let the elements of \mathbf{w} be m, n and p . Then we have

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where $m = ax + by + cz$, $n = dx + ey + fz$ and $p = gx + hy + iz$. You can remember the rules as follows. To find the value of the element of the resultant column vector (e.g. n), point with a finger of your left hand to the first element of the corresponding row in the matrix (in this case d) and point with a finger of your right hand to the first element of the column vector (x). Multiply the elements together and add up the products as you move your left hand across the row, and your right hand down the column. All this sounds a bit complicated, but you'll soon get the hang of it, and remember that the computer or your calculator will be doing the manipulating in any case.

Matrices for conversion between (H, δ) and (A, a) , (α, δ) and (H, δ) , (α, δ) and (λ, β) and between (l, b) and (α, δ) are all given in Table 5. If you wish to make transformations between other combinations of coordinates, say (A, a) and (λ, β) , either you can do so by repeated operations using the given matrices, or you can form a new matrix first, and then use it. For example, let us convert from ecliptic coordinates (λ, β) , represented by \mathbf{v} , to horizon coordinates (A, a) , represented by \mathbf{w} . This can be written as

$$\mathbf{w} = \mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C}' \cdot \mathbf{v}.$$

You would carry out the operations as follows. First multiply \mathbf{C}' and \mathbf{v} to form a new column vector \mathbf{s} . Then multiply \mathbf{B} and \mathbf{s} to form a second column vector \mathbf{r} . Finally, multiply \mathbf{A} and \mathbf{r} to find \mathbf{w} . Note that the order in which you carry out the operations is important. $\mathbf{A} \cdot \mathbf{B}$ is not necessarily the same as $\mathbf{B} \cdot \mathbf{A}$.

The alternative method of proceeding is first to make a new matrix \mathbf{E} that is the product of $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C}'$, and then use it in a single stage of calculation:

$$\mathbf{w} = \mathbf{E} \cdot \mathbf{v}.$$

The rules for multiplying two matrices are similar to those for multiplying a matrix and a column vector. Let the elements of \mathbf{A} be $a, b, c, d, e, f, g, h, i$ as before, and of \mathbf{B} , j, k, l, m, n, o, p, q and r . Then we have

$$\mathbf{F} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix}.$$

Think of \mathbf{F} as being made up of three column vectors. Then the first column vector of \mathbf{F} is the product of \mathbf{A} and the first column of \mathbf{B} , and so on like this:

$$\mathbf{F} = \begin{pmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{pmatrix}.$$

To find \mathbf{E} you would need to make two matrix multiplications of this sort.

The other things which we need to know in order to perform all this magic are, first, how to convert a given pair of coordinates (μ, ν) into the column vector \mathbf{v} . Let (μ, ν) represent any pair, e.g. (A, a) or (l, b) , etc. Then, having done the matrix operations to find \mathbf{w} , how do we convert back to the new coordinates (θ, ψ) ? The procedure is quite simple and is as follows. If (μ, ν) represent the coordinates to be transformed, then the column vector \mathbf{v} is given by

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(\mu) \cos(\nu) \\ \sin(\mu) \cos(\nu) \\ \sin(\nu) \end{pmatrix}.$$

Having found \mathbf{w} , the new coordinates (θ, ψ) are extracted from its elements m, n and p using the formulas

$$\mathbf{w} = \begin{pmatrix} m \\ n \\ p \end{pmatrix}; \quad \theta = \tan^{-1} \left(\frac{n}{m} \right) \quad \text{and} \quad \psi = \sin^{-1}(p).$$

Let us clarify the methods by means of an example: what are the azimuth (A) and altitude (a) of a planet whose ecliptic coordinates are longitude $97^\circ 38' 17.228''$ and latitude $-17^\circ 51' 28.688''$ for an observer at geographical latitude $52^\circ 10' 31.0''$ at local sidereal time 5h 9m 21.103s? We shall assume that the obliquity of the ecliptic is $23^\circ 26' 46.45''$. We do this first by successive matrix multiplications, and then repeat the calculation using one matrix formed from several others.

$$\mathbf{A} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{H,\delta} = \begin{pmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ 0 & -1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{A,a}$$

$$\mathbf{A} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{A,a} = \begin{pmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ 0 & -1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta}$$

$$\mathbf{B} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{H,\delta} = \begin{pmatrix} \cos(\text{ST}) & \sin(\text{ST}) & 0 \\ \sin(\text{ST}) & -\cos(\text{ST}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{B} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} \cos(\text{ST}) & \sin(\text{ST}) & 0 \\ \sin(\text{ST}) & -\cos(\text{ST}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta}$$

$$\mathbf{C} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\lambda,\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & \sin(\varepsilon) \\ 0 & -\sin(\varepsilon) & \cos(\varepsilon) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{C}' : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & -\sin(\varepsilon) \\ 0 & \sin(\varepsilon) & \cos(\varepsilon) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\lambda,\beta}$$

where ϕ is the geographical latitude, ST is the local sidereal time (expressed in radians or degrees) and ε is the obliquity of the ecliptic. The matrices for converting between galactic and equatorial coordinates are constant and best expressed by decimal numbers as follows:

$$\mathbf{D} : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{l,b} = \begin{pmatrix} -0.0669887 & -0.8727558 & -0.4835389 \\ 0.4927285 & -0.4503470 & 0.7445846 \\ -0.8676008 & -0.1883746 & 0.4601998 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{D}' : \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} -0.0669887 & 0.8727558 & -0.4835389 \\ -0.4927285 & -0.4503470 & -0.7445846 \\ -0.8676008 & 0.1883746 & 0.4601998 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{l,b}$$

The column vector x, y, z is formed from the coordinates (μ, ν) as follows:

$$\mathbf{v} : \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mu,\nu} = \begin{pmatrix} \cos(\mu)\cos(\nu) \\ \sin(\mu)\cos(\nu) \\ \sin(\nu) \end{pmatrix}.$$

The new coordinates (θ, ψ) are extracted from the column vector m, n, p by means of the equations:

$$\theta = \tan^{-1}\left(\frac{n}{m}\right); \quad \psi = \sin^{-1}(p).$$

Table 5. Matrix conversions.

First method	Example
1. Convert λ and β into decimal form (§§7 and 21).	$\lambda = 97.638\,119$ degrees $\beta = -17.857\,969$ degrees
2. Form the column vector \mathbf{v} using the equations given in Table 5.	$\mathbf{v} = \begin{pmatrix} -0.126\,512 \\ 0.943\,374 \\ -0.306\,658 \end{pmatrix}$
3. Convert ε to degrees and construct the matrix \mathbf{C}' (see Table 5).	$\varepsilon = 23.446\,236$ degrees $\mathbf{C}' = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.917\,434 & -0.397\,888 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
4. Multiply \mathbf{C}' and \mathbf{v} to form column vector \mathbf{s} .	$\mathbf{s} = \begin{pmatrix} -0.126\,512 \\ 0.987\,499 \\ 0.094\,019 \end{pmatrix}$
5. Convert the local sidereal time into hours, and then into degrees by multiplying by 15. Construct the matrix \mathbf{B} (see Table 5).	$\text{ST} = 5.155\,862$ hours $\text{ST} = 77.337\,930$ degrees $\mathbf{B} = \begin{pmatrix} 0.219\,200 & 0.975\,680 & 0.0 \\ 0.975\,680 & -0.219\,200 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$
6. Multiply \mathbf{B} and \mathbf{s} to form column vector \mathbf{r} .	$\mathbf{r} = \begin{pmatrix} 0.935\,752 \\ -0.339\,895 \\ 0.094\,019 \end{pmatrix}$
7. Convert the geographical latitude to degrees and construct the matrix \mathbf{A} (see Table 5).	$\phi = 52.175\,278$ degrees $\mathbf{A} = \begin{pmatrix} -0.789\,890 & 0.0 & 0.613\,248 \\ 0.0 & -1.0 & 0.0 \\ 0.613\,248 & 0.0 & 0.789\,890 \end{pmatrix}$
8. Multiply \mathbf{A} and \mathbf{r} to find column vector \mathbf{w} .	$\mathbf{w} = \begin{pmatrix} -0.681\,485 \\ 0.339\,895 \\ 0.648\,113 \end{pmatrix}$
9. Find the azimuth, A , and altitude, a , from the elements of \mathbf{w} (see Table 5). Remove the ambiguity on taking inverse tan: look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant.	$\tan^{-1}\left(\frac{n}{m}\right) = -26.508\,056$ degrees $+ 180.0$ (from Fig. 29) $\therefore A = 153.491\,944$ degrees $a = \sin^{-1}(p)$ $= 40.399\,444$ degrees
10. Convert A and a into degrees, minutes and seconds form (§21).	$A = 153^\circ 29' 31''$ $a = 40^\circ 23' 58''$

Second method	Example
1. Convert λ and β into decimal form (§§7 and 21).	$\lambda = 97.638\,119$ degrees $\beta = -17.857\,969$ degrees
2. Form the column vector \mathbf{v} using the equations given in Table 5.	$\mathbf{v} = \begin{pmatrix} -0.126\,512 \\ 0.943\,374 \\ -0.306\,658 \end{pmatrix}$
3. Convert ε to degrees and construct the matrix \mathbf{C}' (see Table 5).	$\varepsilon = 23.446\,236$ degrees $\mathbf{C}' = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.917\,434 & -0.397\,888 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
4. Convert the local sidereal time into hours, and then into degrees by multiplying by 15. Construct the matrix \mathbf{B} (see Table 5).	$\text{ST} = 5.155\,862$ hours $\text{ST} = 77.337\,930$ degrees $\mathbf{B} = \begin{pmatrix} 0.219\,200 & 0.975\,680 & 0.0 \\ 0.975\,680 & -0.219\,200 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$
5. Multiply \mathbf{B} and \mathbf{C}' to form a new matrix \mathbf{F} .	$\mathbf{F} = \mathbf{B} \cdot \mathbf{C}' = \begin{pmatrix} 0.219\,200 & 0.895\,122 & -0.388\,212 \\ 0.975\,680 & -0.201\,102 & 0.087\,217 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
6. Convert the geographical latitude to degrees and construct the matrix \mathbf{A} (see Table 5).	$\phi = 52.175\,278$ degrees $\mathbf{A} = \begin{pmatrix} -0.789\,890 & 0.0 & 0.613\,248 \\ 0.0 & -1.0 & 0.0 \\ 0.613\,248 & 0.0 & 0.789\,890 \end{pmatrix}$
7. Multiply \mathbf{A} and \mathbf{F} to form a new matrix \mathbf{E} .	$\mathbf{E} = \mathbf{A} \cdot \mathbf{F} = \begin{pmatrix} -0.173\,144 & -0.463\,044 & 0.869\,259 \\ -0.975\,680 & 0.201\,102 & -0.087\,217 \\ 0.134\,424 & 0.863\,220 & 0.486\,602 \end{pmatrix}$
8. Multiply \mathbf{E} and \mathbf{v} to find column vector \mathbf{w} .	$\mathbf{w} = \begin{pmatrix} -0.681\,485 \\ 0.339\,895 \\ -0.648\,113 \end{pmatrix}$
9. Find the azimuth, A , and altitude, a , from the elements of \mathbf{w} (see Table 5). Remove the ambiguity on taking inverse tan: look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant.	$\tan^{-1}\left(\frac{n}{m}\right) = -26.508\,056$ degrees $+ 180.0$ (from Fig. 29) $\therefore A = 153.491\,944$ degrees $a = \sin^{-1}(p)$ $= 40.399\,444$ degrees
10. Convert A and a into degrees, minutes and seconds form (§21).	$A = 153^\circ 29' 31''$ $a = 40^\circ 23' 58''$

We have not included a spreadsheet or spreadsheet functions to carry out generalised coordinate transformations.

32 The angle between two celestial objects

Sometimes it is of interest to know what is the angle between two objects in the sky, and this can be calculated very easily provided their equatorial coordinates (α, δ) or ecliptic coordinates (λ, β) are known. The formula is:

$$\cos d = \sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2 \cos (\alpha_1 - \alpha_2)$$

or

$$\cos d = \sin \beta_1 \sin \beta_2 + \cos \beta_1 \cos \beta_2 \cos (\lambda_1 - \lambda_2),$$

where d is the angle between the objects whose coordinates are α_1, δ_1 (or λ_1, β_1) and α_2, δ_2 (or λ_2, β_2). These formulas are exact and mathematically correct for any values of α, δ or λ, β . However, when d becomes either very small, or close to 180° , your calculator may not have enough precision to return the correct answer, in which case a better expression is

$$d = \sqrt{(\cos \delta \times \Delta \alpha)^2 + \Delta \delta^2}$$

or

$$d = \sqrt{(\cos \beta \times \Delta \lambda)^2 + \Delta \beta^2},$$

where $\Delta \alpha, \Delta \delta$ (or $\Delta \lambda, \Delta \beta$) are the differences in the two coordinates (i.e. $\Delta \alpha = \alpha_1 - \alpha_2$, etc.). These expressions may be used for values of d within about 10 arcmin of 0° or 180° . Both $\Delta \alpha$ ($\Delta \lambda$) and $\Delta \delta$ ($\Delta \beta$) must be expressed in the same units (e.g. arcseconds) and d will then be returned in those units.

For example, what is the angular distance between the stars β Orionis ($\alpha = 5\text{h } 13\text{m } 31.7\text{s}$; $\delta = -8^\circ 13' 30''$) and α Canis Majoris ($\alpha = 6\text{h } 44\text{m } 13.4\text{s}$; $\delta = -16^\circ 41' 11''$)?

Method	Example
1. Convert both sets of coordinates to decimal form (§§7 and 21).	$\alpha_1 = 5.225\,472$ hours $\delta_1 = -8.225\,000$ degrees $\alpha_2 = 6.737\,056$ hours $\delta_2 = -16.686\,389$ degrees
2. Find $\alpha_1 - \alpha_2$, and convert to degrees by multiplying by 15 (§22).	$\alpha_1 - \alpha_2 = -1.511\,583$ hours $= -22.673\,750$ degrees
3. Calculate $\cos d = \sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2 \cos (\alpha_1 - \alpha_2)$.	$\cos d = 0.915\,846$
4. Take inverse cos to find d . Convert to minutes and seconds form if required (§21).	$d = 23.673\,850$ degrees $= \mathbf{23^\circ 40' 26''}$

The spreadsheet, labelled Angle, is shown in Figure 34. It can use coordinates expressed either in equatorial or ecliptic form, specified via a switch in cell C15. Set this to H (as here) if the coordinates are α, δ (i.e. α is in Hours, minutes and seconds) or D if the coordinates are λ, β (i.e. λ is in Degrees, minutes and seconds). The corresponding spreadsheet function is also called Angle, and it takes the same 13 arguments as entered in the spreadsheet in cells C3 to C15, i.e. the right ascension/longitude of the first object expressed in hours/degrees, minutes, seconds, the declination/latitude of the first object expressed in degrees, minutes and seconds, the same again for the second object, and finally the character H or D specifying the coordinate format.

	A	B	C	D	E	F	G	H	I	J
1	The angle between two objects									
2										
3	<i>Input</i>	RA/long 1 (hour/deg)	5		<i>Output</i>	angle (deg)	23	=DDDeg(C29)		
4		RA/long 1 (min)	13			angle(min)	40	=DDMin(C29)		
5		RA/long 1 (sec)	31.7			angle(sec)	25.86	=DDSec(C29)		
6		dec/lat 1 (deg)	-8							
7		dec/lat 1 (min)	13							
8		dec/lat 1 (sec)	30							
9		RA/long 2 (hour/deg)	6							
10		RA/long 2 (min)	44							
11		RA/long 2 (sec)	13.4							
12		dec/lat 2 (deg)	-16							
13		dec/lat 2 (min)	41							
14		dec/lat 2 (sec)	11							
15		Hour/degree [H or D]	H							
16										
17	1	RA/long 1 (decimal)	5.225472222	=IF(C15="H",HMSDH(C3,C4,C5),DMSDD(C3,C4,C5))						
18	2	RA/long 1 (deg)	78.38208333	=IF(C15="H",DHDD(C17),C17)						
19	3	RA/long 1 (rad)	1.368025429	=RADIANS(C18)						
20	4	dec/lat 1 (deg)	-8.225	=DMSDD(C6,C7,C8)						
21	5	dec/lat 1 (rad)	-0.143553331	=RADIANS(C20)						
22	6	RA/long 2 (decimal)	6.737055556	=IF(C15="H",HMSDH(C9,C10,C11),DMSDD(C9,C10,C11))						
23	7	RA/long 2 (deg)	101.0558333	=IF(C15="H",DHDD(C22),C22)						
24	8	RA/long 2 (rad)	1.76375702	=RADIANS(C23)						
25	9	dec/lat 2 (deg)	-16.68638889	=DMSDD(C12,C13,C14)						
26	10	dec/lat 2 (rad)	-0.291232426	=RADIANS(C25)						
27	11	cos(d)	0.915845952	=SIN(C21)*SIN(C26)+COS(C21)*COS(C26)*COS(C18-C24)						
28	12	d (rad)	0.413186619	=ACOS(C27)						
29	13	d (deg)	23.67384942	=DEGREES(C28)						

Figure 34. Finding the angle between two celestial objects.

Thus you could delete rows 17 to 29 of the spreadsheet shown in Figure 34 (having saved a copy), and insert into cells H3, H4 and H5 the following formulas:

=DDDeg(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15))

=DDMin(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15))

=DDSec(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15)).

33 Rising and setting

During the course of a sidereal day, the stars and other ‘fixed’ celestial objects appear to move in circles about the rotation axis of the Earth, making one complete revolution in 24 hours. At the moment, there is a star called Polaris very close to the north pole of the Earth’s axis so that stars in the northern sky appear to revolve about Polaris. There is nothing special about this star, however, and no corresponding object exists for the south pole. In any case, the poles are gradually changing their positions in the sky

because of precession (see the next section) so that Polaris will no longer be the pole star in a few thousand years.

The apparent radius of a star's rotation depends, of course, on the angular separation, or **polar distance**, between it and the pole; those stars with a small enough polar distance never dip below the horizon during the course of their rotation. Such stars are called **circumpolar**. As the polar distance increases, however, a point comes when the star just touches the horizon at some time during the day. Stars with polar distances greater than this spend part of their time below the horizon, out of sight to the observer. When the star crosses the horizon on the way down it is said to **set** and as it reappears it is said to **rise**.

There are several effects, including atmospheric refraction (Section 37) and parallax (for bodies relatively close to the Earth: Sections 38 and 39), that shift an object's apparent position and this may alter the apparent times of rising or setting by several minutes. The situation at rising or setting is shown in Figure 35. The celestial body appears to cross the horizon at B, although its 'true' position, as calculated from its uncorrected coordinates, is at A. Provided we know the vertical shift[†], v , we can include its effects on the circumstances of rising and setting.

The local sidereal times of rising and setting, and the azimuths at which they occur, can be calculated using the formulas

$$\cos H = -\frac{(\sin v + \sin \phi \sin \delta)}{\cos \phi \cos \delta},$$

$$\text{LST}_r = \alpha - H,$$

$$\text{LST}_s = \alpha + H,$$

$$\cos A_r = \frac{\sin \delta + \sin v \sin \phi}{\cos v \cos \phi},$$

$$A_s = 360^\circ - A_r,$$

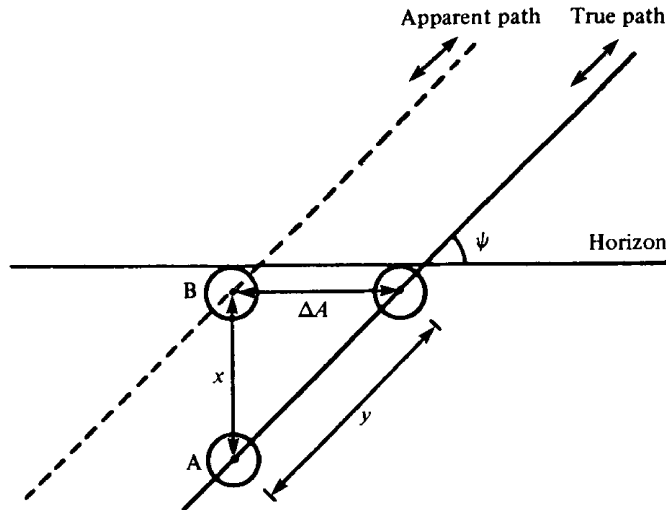


Figure 35. The true and apparent positions of a celestial object at rising or setting.

[†] v is positive if the star stays longer above the horizon.

where the subscripts r and s correspond to rising and setting respectively, A is the azimuth, LST is the local sidereal time in hours, α is the right ascension, δ is the declination, ϕ is the observer's geographical latitude and H is the hour angle. The value of $\cos H$ can be used as an indicator of whether the star never rises, or is circumpolar. If $\cos H$ is greater than 1, the star is permanently below the horizon and never rises. If $\cos H$ is more negative than -1 , the star is permanently above the horizon and never sets (i.e. is circumpolar).

The LST can be converted to UT and hence to the local civil time by the methods given in Sections 15, 13 and 10. Hence, all the circumstances of a star's rising and setting can be calculated. However, there is a difficulty that you may need to overcome if you live far away from the Greenwich meridian. In order to convert the Greenwich sidereal time into the UT, you need to know the calendar date at Greenwich on which the rising or setting occurs. But in order to find this from your local calendar date you need to know the UT at which the rising or setting occurs. This difficulty is easily overcome however. If you take the date at Greenwich to be the same as your local calendar date, the times of rising and setting will usually not be more than a few minutes out. You can then use those times to recalculate the calendar date(s) at Greenwich and iterate until there are no further changes.

As an example, let us calculate the UTs of rising and setting over a sea horizon of a star whose equatorial coordinates are $\alpha = 23^{\text{h}} 39^{\text{m}} 20^{\text{s}}$ and $\delta = 21^{\circ} 42' 00''$ on 24 August 2010, and find the corresponding azimuths. The geographical latitude is 30° N, and the longitude is 64° E, and the value of v due to atmospheric refraction is 34 arcmin.

Method	Example
1. Convert α and δ into decimal form (§§7 and 21).	$\alpha = 23.655558$ hours $\delta = 21.700000$ degrees
2. Find $\cos H = -\frac{(\sin v + \sin \phi \sin \delta)}{\cos \phi \cos \delta}$.	$\cos H = -0.242047$
3. If $\cos H$ is between -1 and $+1$, take the inverse cos to find ^a H .	$H = 6.933827$ hours
4. Find $\text{LST}_r = \alpha - H$. Restore to the range 0 to 24 by adding or subtracting 24.	$\text{LST}_r = 16.721728$ hours
5. Find $\text{LST}_s = \alpha + H$. Restore to the range 0 to 24 by adding or subtracting 24.	$\text{LST}_s = 6.589383$ hours
6. Find $A_r = \cos^{-1} \left\{ \frac{\sin \delta + \sin v \sin \phi}{\cos v \cos \phi} \right\}$. Restore to the range 0 to 360 by adding or subtracting 360.	$A_r = 64.362348$ degrees
7. Find $A_s = 360 - A_r$.	$A_s = 296.637652$ degrees
8. Convert the LST values to GST values, then to universal times (§§15 and 13).	$\text{UT}_r = 14.271670$ hours $\text{UT}_s = 4.166990$ hours
9. Finally, express the times as hours, minutes and seconds (§8).	$\text{UT}_r = 14^{\text{h}} 16^{\text{m}}$ $\text{UT}_s = 4^{\text{h}} 10^{\text{m}}$

^aIf the star's declination is such that it never rises above the horizon, or if it is circumpolar, then you will find that you will be trying to take inverse cos of a number greater than 1 or less than -1 . This is impossible and your calculator should respond with 'error'.

Note that the UTs you calculate are appropriate for the date you have applied. As here, the setting time on a given date may be earlier than the rising time.

Figure 36 shows the spreadsheet for making this lengthy calculation. We have used several techniques that are worthy of note. First, in rows 29 and 30, we have used the trick of adding 30 s ($= 0.008333$ hours) to the UTs so that, when displayed as hours and minutes, the time will be rounded correctly to the nearest

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Rising and Setting													
2														
3	Input	RA (hour)	23	Output	Rise/set status	OK =IF(C20>1,"** never rises",IF(C20<=-1,"** circumpolar","OK"))								
4		RA (min)	39		UT rise:	14:16 =IF(H3="OK",CONCATENATE(DHHour(C29),":",DHMin(C29)),"")								
5		RA (sec)	20		UT set:	4:10 =IF(H3="OK",CONCATENATE(DHHour(C30),":",DHMin(C30)),"")								
6		dec (deg)	21		Azimuth rise:	64.36 =IF(H3="OK",ROUND(C25,2))								
7		dec (min)	42		Azimuth set:	295.64 =IF(H3="OK",ROUND(C26,2))								
8		dec (sec)	0											
9		Greenwich date (day)	24			=IF(H3="OK","UT rise:","")								
10		Greenwich date (month)	8			=IF(H3="OK","UT set:","")								
11		Greenwich date (year)	2010			=IF(H3="OK","Azimuth rise:","")								
12		geog longitude (deg)	64			=IF(H3="OK","Azimuth set:","")								
13		geog lat (deg)	30											
14		vertical shift (deg)	0.5667											
15														
16	1	RA (hours)	23.6555556	=HMSDH(C3,C4,C5)										
17	2	dec (rad)	0.37873645	=RADIANS(DMSDD(C6,C7,C8))										
18	3	vertical displ (radians)	0.00989078	=RADIANS(C14)										
19	4	geo lat (radians)	0.52359878	=RADIANS(C13)										
20	5	cos (H)	-0.2420474	=-(SIN(C18)+SIN(C19)*SIN(C17))/(COS(C19)*COS(C17))										
21	6	H (hours)	6.93382728	=DDDH(DEGREES(ACOS(C20)))										
22	7	LST rise (hours)	16.7217283	=(C16-C21)-24*INT((C16-C21)/24)										
23	8	LST set (hours)	6.58938284	=(C16+C21)-24*INT((C16+C21)/24)										
24	9	A (deg)	64.362348	=DEGREES(ACOS((SIN(C17)+SIN(C18)*SIN(C19))/(COS(C18)*COS(C19))))										
25	10	Az rise (deg)	64.362348	=C24-360*INT(C24/360)										
26	11	Az set (deg)	295.637652	=(360-C24)-360*INT((360-C24)/360)										
27	12	UT rise (hours)	14.2716699	=GSTUT(LSTGST(C22,0,0,C12),0,0,C9,C10,C11)										
28	13	UT set (hours)	4.16699015	=GSTUT(LSTGST(C23,0,0,C12),0,0,C9,C10,C11)										
29	14	UT rise adjusted (hours)	14.2800029	=C27+0.008333										
30	15	UT set adjusted (hours)	4.17532315	=C28+0.008333										

Figure 36. Finding the circumstances of rising and setting.

minute. If the seconds part is less than 30, then adding 30 s will not take the result over the minute boundary and the minutes part will be unaffected. If the seconds part is 30 or more, then adding 30 s will cause the minutes part to increment by 1, as required when rounding to the nearest minute.

Second, we have used $\cos H$ as an indicator of whether the star is circumpolar, or never rises. In cell H3, we use an IF formula to return a status word. This is OK if the star rises and sets, ** never rises if it is permanently below the horizon, and ** circumpolar if it never sets. We then test the status word in cells G4 to G7, and H4 to H7, not displaying anything in these cells unless the status word is OK. This avoids rather ugly error messages appearing in the output cells when there is no rising and setting. Note that the formulas in cells G4 to G7 are displayed in cells H9 to H12. Finally, we have used the spreadsheet formula =CONCATENATE(a,b,c,...) in cells H4 and H5 to display the time formatted as hh:mm. The formula interprets each of its arguments a, b, c,... as text, strings them all together without any spaces in between, and displays the result. Thus =CONCATENATE(DHHour(C29),":",DHMin(C29)) displays the hour part of the time contained in cell C29, then a colon, then the minute part.

There are five spreadsheet functions provided for this calculation. Each takes eight arguments corresponding to the right ascension in hours, minutes, and seconds, the declination in degrees, minutes and seconds, the vertical shift in decimal degrees, and the geographical latitude in decimal degrees. The functions are RSLSTR, RSLSTS, RSAZR, RSAZS and eRS, returning respectively the local sidereal times of rising and setting in hours, the azimuths of rising and setting in degrees, and a status word of OK, ** never

risers, or ** circumpolar as appropriate. Parts of the spreadsheet of Figure 36 could therefore be replaced with these functions as follows (save a copy first). Delete rows 16 to 21 inclusive, and row 24. Insert the following spreadsheet formulas into the cells which were C22, C23, C25 and C26:

```
=RSLSTR(C3,C4,C5,C6,C7,C8,C14,C13)
=RSLSTS(C3,C4,C5,C6,C7,C8,C14,C13)
=RSAZR(C3,C4,C5,C6,C7,C8,C14,C13)
=RSAZS(C3,C4,C5,C6,C7,C8,C14,C13).
```

Finally, you need to insert =eRS(C3,C4,C5,C6,C7,C8,C14,C13) into cell H3. Try this for yourself.

34 Precession

In Section 18 we found that equatorial coordinates were ideal for fixing the positions of the stars because they were independent of the Earth's motion and therefore constant. This is true to quite a high accuracy, but we find that the coordinates do in fact change slowly with time. This is because of a gyrating motion of the Earth's axis. Rather as the rotation axis of a quickly-spinning top revolves slowly about a vertical line, so the rotation axis of the Earth rotates slowly about a fixed direction in space. The motion is called **luni-solar precession**, or **precession of the equinoxes**, and it is caused by the gravitational effects of the Moon and Sun on the Earth. We need not be worried by the details. It is sufficient to say that the effect is small over periods of a few years, the north pole of the Earth making one complete circuit in 25 800 years, but for high precision we must be able to allow for it. In this section you will find two methods of doing so. The first, which is suitable for most purposes (i.e. precession over periods of a few tens of years), is an approximate method that is quite easy to apply. The second is a rigorous method, correct for long or short periods, which uses matrices (see Section 31 for a description of the use of matrices).

Low-precision method

The coordinates α and δ of the stars and galaxies are given in catalogues correct at some particular time or **epoch**. The ones you are quite likely to see at present will be correct at the epoch 1950.0 (strictly 1950 January 0.923) or 2000.0 (2000 January 1.5). For example, you can convert coordinates from their 1950.0 values to the values they will have at some other date using the formulas:

$$\alpha_1 = \alpha_0 + (3^s.073\,27 + 1^s.336\,17 \sin \alpha_0 \tan \delta_0) \times N,$$

$$\delta_1 = \delta_0 + (20''.042\,6 \cos \alpha_0) \times N,$$

where N is the number of years since 1950.0, α_0 and δ_0 are the coordinates at 1950.0, and α_1 and δ_1 are the new coordinates. These formulas may not work well for regions around the north and south poles where the magnitude of $\tan \delta$ tends towards infinity. You must use the rigorous method (see below) in such cases.

To convert from coordinates given at an epoch other than 1950.0, use the following formulas with the values of m , n and n' given in Table 6:

$$\alpha_1 = \alpha_0 + (m + n \sin \alpha_0 \tan \delta_0) \times N,$$

$$\delta_1 = \delta_0 + (n' \cos \alpha_0) \times N.$$

Epoch	m (seconds)	n (seconds)	n' (arcsec)
1900.0	3.072 34	1.336 45	20.046 8
1950.0	3.073 27	1.336 17	20.042 6
2000.0	3.074 20	1.335 89	20.038 3
2050.0	3.075 13	1.335 60	20.034 0

Table 6. Precessional constants.

For our example we shall work out the 1979.5 coordinates of a star whose 1950.0 coordinates were $\alpha_0 = 9^{\text{h}} 10^{\text{m}} 43^{\text{s}}$ and $\delta_0 = 14^\circ 23' 25''$.

Method	Example
1. Convert α_0, δ_0 into decimal form (§§21 and 7).	$\alpha_0^{\text{h}} = 9.178\,611$ hours $\delta_0 = 14.390\,278$ degrees
2. Convert α_0 to degrees by multiplying by 15 (§22).	$\alpha_0^{\text{d}} = 137.679\,167$ degrees
3. Find $S_1 = (3.073\,27 + 1.336\,17 \sin \alpha_0^{\text{d}} \tan \delta_0) \times N$ (where $N = 1979.5 - 1950.0 = 29.5$).	$S_1 = 97.470\,656$ seconds
4. Divide by 3600 to convert to hours.	$S_1^{\text{h}} = 0.027\,075$ hours
5. Add S_1^{h} to α_0^{h} to get α_1^{h} .	$\alpha_1^{\text{h}} = 9.205\,686$ hours
6. Convert to hours, minutes and seconds (§8).	$\alpha_1 = \mathbf{9^{\text{h}} 12^{\text{m}} 20^{\text{s}}}$
7. Find $S_2 = (20.042\,6 \cos \alpha_0^{\text{d}}) \times N$.	$S_2 = -437.167\,123$ arcsec
8. Divide by 3600 to convert to degrees.	$S_2^{\text{d}} = -0.121\,435$ degrees
9. Add S_2^{d} to δ_0 to get δ_1 .	$\delta_1 = 14.268\,842$ degrees
10. Convert to degrees, minutes and seconds (§21).	$\delta_1 = \mathbf{14^\circ 16' 08''}$

Figure 37 shows the spreadsheet for carrying out the low-precision method, called *Precess1*. It differs slightly from the method given in the table in that it calculates the precession constants explicitly in rows 19 and 20 rather than looking them up in the table. The small differences in the answers are also in part caused by rounding errors. The spreadsheet functions provided for this section use the rigorous method (see below).

Rigorous method

The rigorous reduction of coordinates from one epoch to another makes use of matrices. These were described in Section 31, and if you are not familiar with them, you should read and understand that section first.

The method proceeds in two parts. First, we convert the given coordinates α_1, δ_1 , which are appropriate to date (or epoch) number 1, into the corresponding coordinates of 2000.0, i.e. 2000 January 1.5, epoch 0. Second, we convert from epoch 0 to the required date, epoch 2.

	A	B	C	D	E	F	G	H	I	J
1	Low-precision Precession									
2										
3	<i>Input</i>	RA (hour)	9		<i>Output</i>	corrected RA (hour)	9	=DHHour(C23)		
4		RA (min)	10			corrected RA (min)	12	=DHMin(C23)		
5		RA (sec)	43			corrected RA (sec)	20.18	=DHSec(C23)		
6		dec (deg)	14			corrected Dec (deg)	14	=DDDeg(C25)		
7		dec (min)	23			corrected Dec (min)	16	=DDMin(C25)		
8		dec (sec)	25			corrected Dec (sec)	9.12	=DDSec(C25)		
9		epoch1 (day)	0.923							
10		epoch1 (month)	1							
11		epoch1 (year)	1950							
12		epoch2 (day)	1							
13		epoch2 (month)	6							
14		epoch2 (year)	1979							
15										
16	1	RA 1 (rad)	2.40295477	=RADIANS(DHDD(HMSDH(C3,C4,C5)))						
17	2	dec 1 (rad)	0.251157727	=RADIANS(DMSDD(C6,C7,C8))						
18	3	T (centuries)	0.499997892	=(CDJD(C9,C10,C11)-2415020)/36525						
19	4	m (sec)	3.073269996	=3.07234+(0.00186*C18)						
20	5	n' (arcsec)	20.04255002	=20.0468-(0.0085*C18)						
21	6	N (years)	29.41294182	=(CDJD(C12,C13,C14)-CDJD(C9,C10,C11))/365.25						
22	7	S1 (hours)	0.02699528	=((C19+(C20*SIN(C16)*TAN(C17)/15))*C21)/3600						
23	8	RA 2 (hours)	9.205606391	=HMSDH(C3,C4,C5)+C22						
24	9	S2 (deg)	-0.121076639	=(C20*COS(C16)*C21)/3600						
25	10	dec 2 (deg)	14.26920114	=DMSDD(C6,C7,C8)+C24						

Figure 37. Low-precision precession.

We begin by calculating the precessional variables ζ_A , z_A and θ_A , for the date at which the coordinates are specified (epoch 1). These are given in degrees by the following formulas:

$$\begin{aligned}\zeta_A &= 0.6406161T + 0.0000839T^2 + 0.0000050T^3, \\ z_A &= 0.6406161T + 0.0003041T^2 + 0.0000051T^3, \\ \theta_A &= 0.5567530T - 0.0001185T^2 - 0.0000116T^3,\end{aligned}$$

where T is the number of Julian centuries of 36 525 days at epoch 1 since the epoch J2000.0. The value of T may be calculated from

$$T = \frac{(JD1 - 2451545)}{36525},$$

where JD1 is the Julian date of epoch 1. We then construct the matrix \mathbf{P}' which allows us to convert the coordinates from epoch 1 to epoch 0. The matrix \mathbf{P}' is given by

$$\mathbf{P}' = \begin{pmatrix} CX \cdot CT \cdot CZ - SX \cdot SZ & CX \cdot CT \cdot SZ + SX \cdot CZ & CX \cdot ST \\ -SX \cdot CT \cdot CZ - CX \cdot SZ & -SX \cdot CT \cdot SZ + CX \cdot CZ & -SX \cdot ST \\ -ST \cdot CZ & -ST \cdot SZ & CT \end{pmatrix}$$

where $CX = \cos \zeta_A$, $SX = \sin \zeta_A$, $CZ = \cos z_A$, $SZ = \sin z_A$, $CT = \cos \theta_A$, and $ST = \sin \theta_A$. We next calculate

the column vector, \mathbf{v} , corresponding to the coordinates at epoch 1, α_1 and δ_1 , from

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha_1, \delta_1} = \begin{pmatrix} \cos \alpha_1 \cos \delta_1 \\ \sin \alpha_1 \cos \delta_1 \\ \sin \delta_1 \end{pmatrix}.$$

Now we multiply \mathbf{P}' and \mathbf{v} to form the column vector \mathbf{s} corresponding to the epoch-0 coordinates,

$$\mathbf{s} = \mathbf{P}' \cdot \mathbf{v}.$$

The second part of the process is to convert from epoch 0 to the epoch at which the coordinates are actually required, epoch 2. We need to repeat the above procedure, calculating values for ζ_A , z_A and θ_A using T appropriate to epoch 2 (i.e. use JD2 instead of JD1). Then we construct the matrix \mathbf{P} , which is just the **transpose** of \mathbf{P}' , from

$$\mathbf{P} = \begin{pmatrix} CX \cdot CT \cdot CZ - SX \cdot SZ & -SX \cdot CT \cdot CZ - CX \cdot SZ & -ST \cdot CZ \\ CX \cdot CT \cdot SZ + SX \cdot CZ & -SX \cdot CT \cdot SZ + CX \cdot CZ & -ST \cdot SZ \\ CX \cdot ST & -SX \cdot ST & CT \end{pmatrix}.$$

The transpose of a 3×3 matrix is found by interchanging the rows and columns, ‘flipping’ them about the diagonal. For example, the transpose of

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \text{ is } \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}.$$

Now, the column vector, \mathbf{w} , corresponding to the coordinates at epoch 2, can be found by multiplying \mathbf{P} and \mathbf{s} :

$$\mathbf{w} = \mathbf{P} \cdot \mathbf{s}.$$

Finally, we extract the new coordinates, α_2 , δ_2 , from the elements of \mathbf{w} using

$$\alpha_2 = \tan^{-1} \left(\frac{n}{m} \right) \text{ and } \delta_2 = \sin^{-1}(p).$$

Let us clarify this procedure by repeating the example used for the low-precision method.

Method	Example	
1. Find the Julian date corresponding to epoch 1 (1,1,1950; §4).	JD1	= 2 433 282.5
2. Find $T = (\text{JD1} - 2451\,545)/36\,525$.	T	= -0.500 000
3. Calculate ζ_A , z_A and θ_A for JD1 using the formulas given above.	ζ_A	= -0.320 288 degrees
	z_A	= -0.320 233 degrees
	θ_A	= -0.278 405 degrees
4. Construct the matrix \mathbf{P}' .	\mathbf{P}'	= $\begin{pmatrix} 0.999926 & -0.011\,179 & -0.004\,859 \\ 0.011\,179 & 0.999938 & -0.000027 \\ 0.004\,859 & -0.000027 & 0.999988 \end{pmatrix}$
5. Convert α_1 and δ_1 into decimal degrees (§§21 and 22).	α_1	= 137.679 167 degrees
	δ_1	= 14.390 278 degrees
6. Construct the column vector \mathbf{v} .	x	= -0.716 188
	y	= 0.652 157
	z	= 0.248 526
7. Multiply \mathbf{P}' and \mathbf{v} to find \mathbf{s} (§31).	\mathbf{s}	= $\begin{pmatrix} -0.724633 \\ 0.644\,104 \\ 0.245025 \end{pmatrix}$
8. Find the Julian date corresponding to epoch 2 (1,6,1979; §4).	JD2	= 2 444 025.5
9. Calculate $T = (\text{JD2} - 2451\,545)/36\,525$.	T	= -0.205 873
10. Now find new values of ζ_A , z_A and θ_A , appropriate to JD2.	ζ_A	= -0.131 882 degrees
	z_A	= -0.131 873 degrees
	θ_A	= -0.114 625 degrees
11. Construct the matrix \mathbf{P} .	\mathbf{P}	= $\begin{pmatrix} 0.999987 & 0.004\,603 & 0.002\,001 \\ -0.004\,603 & 0.999989 & -0.000005 \\ -0.002\,001 & -0.000005 & 0.999998 \end{pmatrix}$
12. Multiply \mathbf{P} and \mathbf{s} to find \mathbf{w} , corresponding to the coordinates at epoch 2 (§31).	m	= -0.721 169
	n	= 0.647 432
	p	= 0.246 471
13. Find the new coordinates using $\alpha_2 = \tan^{-1}(\frac{n}{m})$ and $\delta_2 = \sin^{-1}(p)$. Remove the ambiguity on taking \tan^{-1} by adding or subtracting 180 or 360 according to Figure 29.	α_2	= -41.916 009 degrees
	$+$	= 180.0
	α_2	= 138.083 991 degrees
	δ_2	= 14.268 792 degrees
14. Convert α_2 and δ_2 to minutes and seconds form (§§21 and 22).	α_2	= 9h 12m 20.16s
	δ_2	= 14° 16' 7.65''

You can see that the low-precision method gave quite a good result in this case.

We have not provided a spreadsheet for this calculation. Manipulation of matrices in spreadsheets is possible, but is a complication that is instead carried out more easily in the background by the spreadsheet functions `Precess2RA` and `Precess2Dec`, which return the right ascension in decimal hours and the declination in decimal degrees respectively, each corrected for precession using the rigorous method. Both functions take the same 12 arguments, being the right ascension in hours, minutes and seconds, the declination in degrees, minutes and seconds, the calendar date at which those coordinates were specified (epoch 1)

as day, month, year, and finally the calendar date to which the coordinates must be precessed (epoch 2) as day, month, and year.

We can now simplify the spreadsheet *Precess1* and, at the same time, make it more accurate by replacing the calculation part by these functions. Save a copy of the spreadsheet shown in Figure 37 and delete rows 16–22 and 24. In the cells which were C23 and C25 (now C16 and C17 after the deletion) insert the following spreadsheet formulas respectively:

```
=Precess2RA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)
=Precess2Dec(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14).
```

You should also change the title of the spreadsheet (cell A1) to *Rigorous precession* and rename the spreadsheet *Precess2*.

35 Nutation

The combined gravitational fields of the Sun and the Moon acting on the non-spherical Earth cause the direction of the Earth's rotation axis to gyrate slowly with a period of about 25 800 years. We saw how to allow for this effect, called **precession**, in Section 34. Superimposed on the regular motion there are also small additional periodic terms caused by the varying distances and relative directions of the Moon and Sun, which continuously alter the strength and direction of the gravitational field. This slight wobbling motion is called **nutation**, and can be taken into account by calculating its effects on ecliptic longitude, $\Delta\psi$, and on the mean obliquity of the ecliptic, $\Delta\epsilon$.

The method described here uses the pre-1984 theory of nutation. We consider only the most important terms to achieve an accuracy of about half an arcsecond. We first calculate the number of Julian centuries, T , on the date in question since 1900 January 0.5 using

$$T = \frac{\text{JD} - 2415020.0}{36525},$$

where JD is the Julian date. Note that this definition of T is different from that in other parts of the book where the fundamental epoch 2000 January 1.5 is used. Next, we find the values of the Sun's mean longitude, L , and the longitude of the Moon's ascending node, Ω , from the formulas

$$L = 279.6967 + 360.0 \times (A - \text{INT}(A)) \text{ degrees},$$

where

$$A = 100.002136 \times T,$$

and

$$\Omega = 259.1833 - 360.0 \times (B - \text{INT}(B)) \text{ degrees},$$

where

$$B = 5.372617 \times T.$$

(See Section 4 about the meaning of INT.) The effects of nutation on the ecliptic longitude and obliquity of

the ecliptic are then given by

$$\begin{aligned}\Delta\psi &= -17.2 \sin(\delta\odot) - 1.3 \sin(2L) \text{ arcsec}, \\ \Delta\epsilon &= 9.2 \cos(\delta\odot) + 0.5 \cos(2L) \text{ arcsec}.\end{aligned}$$

As an example we calculate the nutation on 1 September 1988.

Method	Example
1. Find the Julian day number for this calendar date.	JD = 2 447 405.5
2. Calculate $T = (\text{JD} - 2\,415\,020.0)/36\,525.0$.	$T = 0.886\,667$
3. Find $A = 100.002\,136T$.	$A = 88.668\,560$
4. Calculate the Sun's mean longitude $L = 279.696\,7 + 360.0(A - \text{INT}(A))$. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	$L = 160.378\,686 \text{ degrees}$
5. Find $B = 5.372\,617T$.	$B = 4.763\,720$
6. Calculate the Moon's node $\delta\odot = 259.183\,3 - 360.0(B - \text{INT}(B))$. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	$\delta\odot = 344.243\,954 \text{ degrees}$
7. Calculate nutation in longitude $\Delta\psi = -17.2 \sin(\delta\odot) - 1.3 \sin(2L)$.	$\Delta\psi = 5.5''$
8. Calculate nutation in obliquity $\Delta\epsilon = 9.2 \cos(\delta\odot) + 0.5 \cos(2L)$.	$\Delta\epsilon = 9.2''$

The *Astronomical Almanac* gives the values $\Delta\psi = 5.1''$ and $\Delta\epsilon = 9.2''$ for this date, so we are well within the accuracy claimed for this method of half an arcsecond. $\Delta\psi$ must be added to the ecliptic longitude, and $\Delta\epsilon$ to the mean obliquity, to allow for nutation.

The spreadsheet for making this calculation is shown in Figure 38. It gives the small amounts to be added to the ecliptic longitude and the mean obliquity of the ecliptic in degrees in cells H3 and H4. The corresponding quantities in arcseconds are in cells C18 and C19. We have also provided the spreadsheet functions NutatLong(D,M,Y) and NutatObl(D,M,Y) giving, respectively, the nutation amount to be added in ecliptic longitude and in mean obliquity, both in degrees. Both functions take the calendar date as arguments expressed as day, month and year (strictly, the calendar date at Greenwich, although your local calendar date will usually do). Note that when you use the function Obliq (Section 27), the nutation term is already included in the result so you will hardly ever need to use NutatObl explicitly.

You can use these two functions to simplify the spreadsheet shown in Figure 38. Save a copy in case you want it later (or you make a mistake and need to start again), then delete rows 7 to 19 (the calculation part), and insert the following formulas into cells H3 and H4:

```
=NutatLong(C3,C4,C5)
=NutatObl(C3,C4,C5).
```

	A	B	C	D	E	F	G	H	I
1	Nutation in ecliptic longitude and obliquity								
2									
3	<i>Input</i>	Gday	1		<i>Output</i>	nut in long (deg)	0.001525808	=C18/3600	
4		Gmonth	9			nut in obl (deg)	0.0025671	=C19/3600	
5		Gyear	1988						
6									
7	1	JD (days)	2447405.5	=CDJD(C3,C4,C5)					
8	2	T (centuries)	0.886666667	=(C7-2415020)/36525					
9	3	A (deg)	88.66856041	=100.0021358*C8					
10	4	L 1 (deg)	279.6969382	=279.6967+(0.000303*C8*C8)					
11	5	L (deg)	520.3786856	=C10+360*(C9-INT(C9))					
12	6	L (deg)	160.3786856	=C11-360*INT(C11/360)					
13	7	L (rad)	2.799136113	=RADIANS(C12)					
14	8	B (deg)	4.763720407	=5.372617*C8					
15	9	N (deg)	-15.7560464	=259.1833-360*(C14-INT(C14))					
16	10	N (deg)	344.2439536	=C15-360*(INT(C15/360))					
17	11	N (rad)	6.00819042	=RADIANS(C16)					
18	12	nut in long (arcsec)	5.492910079	=-17.2*SIN(C17)-1.3*SIN(2*C13)					
19	13	nut in obl (arcsec)	9.24156161	=9.2*COS(C17)+0.5*COS(2*C13)					

Figure 38. Calculating nutation in ecliptic longitude and obliquity.

36 Aberration

There are several small effects which must be taken into account to improve the accuracy of our calculations. One of them is called **aberration**, and is caused by the speed of the Earth in its orbit around the Sun. Since light does not travel at infinite speed, the motion of the Earth causes the apparent direction of a celestial body to be shifted slightly from its true direction, just as rain falling vertically downwards appears to come at an angle to a cyclist moving through it. The correction is small, amounting to a maximum shift of 20.5 arcsec (0.000 569 degrees). We can find its effect on the ecliptic longitude, $\Delta\lambda$, and on the ecliptic latitude, $\Delta\beta$, using the formulas

$$\Delta\lambda = -20.5 \cos(\lambda_{\odot} - \lambda) / \cos \beta \text{ arcsec}$$

and

$$\Delta\beta = -20.5 \sin(\lambda_{\odot} - \lambda) \sin \beta \text{ arcsec},$$

where λ and β are the true ecliptic longitude and latitude, and λ_{\odot} is the ecliptic longitude of the Sun (see Section 46). The apparent longitude and latitude are then given by

$$\lambda' = \lambda + \Delta\lambda \text{ and } \beta' = \beta + \Delta\beta.$$

Corrections to the true right ascension, $\Delta\alpha$, and to the declination, $\Delta\delta$, of a body can be calculated using the formulas

$$\Delta\alpha = -20.5 \frac{\cos\alpha \cos\lambda_{\odot} \cos\varepsilon + \sin\alpha \sin\lambda_{\odot}}{\cos\delta} \text{ arcsec},$$

$$\Delta\delta = -20.5 \left[\cos\lambda_{\odot} \cos\varepsilon (\tan\varepsilon \cos\delta - \sin\alpha \sin\delta) + \cos\alpha \sin\delta \sin\lambda_{\odot} \right] \text{ arcsec},$$

where α and δ are the true coordinates and ε is the obliquity of the ecliptic (see Section 27). These corrections must be added to the true coordinates to find the apparent coordinates.

Let's take as an example the effect of aberration on the position of Mars on 8 September 1988. Its true ecliptic coordinates were $\lambda = 352^{\circ} 37' 10.1''$ and $\beta = -1^{\circ} 32' 56.4''$. The longitude of the Sun on that day (at 0h TT) was $165^{\circ} 33' 44.1''$.

Method	Example
1. Convert λ , β and λ_{\odot} to decimal degrees (§21).	$\lambda = 352.619\,472$ degrees $\beta = -1.549\,000$ degrees $\lambda_{\odot} = 165.562\,250$ degrees
2. Calculate $\Delta\lambda = -20.5 \cos(\lambda_{\odot} - \lambda) / \cos\beta$.	$\Delta\lambda = 20.352\,128$ arcsec
3. Calculate $\Delta\beta = -20.5 \sin(\lambda_{\odot} - \lambda) \sin\beta$.	$\Delta\beta = 0.068\,084$ arcsec
4. Convert $\Delta\lambda$ and $\Delta\beta$ to degrees by dividing by 3600.	$\Delta\lambda = 0.005\,653$ degrees $\Delta\beta = 0.000\,019$ degrees
5. Add the corrections to λ and β to find the apparent coordinates.	$\lambda' = 352.625\,126$ degrees $\beta' = -1.548\,981$ degrees
6. Convert back to minutes and seconds (§21).	$\lambda' = 352^{\circ} 37' 30.5''$ $\beta' = -1^{\circ} 32' 56.3''$

Notice that $\Delta\beta$ is negligible (much less than 1 arcsecond) for ecliptic latitudes close to zero, as in this case.

Our spreadsheet for this calculation is shown in Figure 39 and is called **Aberration**. Of special note is that we have anticipated the method of Section 46 for calculating the ecliptic longitude of the Sun and have used the spreadsheet function **SunLong** in row 18. Here, we have also supplied the spreadsheet functions **AbLong** and **AbLat** which return, respectively, the ecliptic longitude in degrees corrected for the effect of aberration (**Aberration in Longitude**) and the ecliptic latitude in degrees corrected for the effect of aberration (**Aberration in Latitude**). They both take 12 arguments, these being the universal time in hours, minutes and seconds, the Greenwich calendar date as day, month, year, the true ecliptic longitude in degrees, minutes and seconds, and the true ecliptic latitude in degrees, minutes and seconds. If you wish, you can therefore simplify the spreadsheet of Figure 39 (save a copy first) by deleting rows 16 to 22 inclusive, and inserting the following spreadsheet formulas into cells H3 to H8 inclusive:

```
=DDDeg(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDMin(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDSec(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDDeg(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDMin(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDSec(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)).
```

	A	B	C	D	E	F	G	H	I	J
1	Correcting for Aberration									
2										
3	Input	UT (hour)	0		Output	apparent ecl long (deg)	352	=DDDdeg(C21)		
4		UT (min)	0			apparent ecl long (min)	37	=DDMin(C21)		
5		UT (sec)	0			apparent ecl long (sec)	30.45	=DDSec(C21)		
6		G date (day)	8			apparent ecl lat (deg)	-1	=DDDdeg(C22)		
7		G date (month)	9			apparent ecl lat (min)	32	=DDMin(C22)		
8		G date (year)	1988			apparent ecl lat (sec)	56.33	=DDSec(C22)		
9		true ecl long (deg)	352							
10		true ecl long (min)	37							
11		true ecl long (sec)	10.1							
12		true ecl lat (deg)	-1							
13		true ecl lat (min)	32							
14		true ecl lat (sec)	56.4							
15										
16	1	true long (deg)	352.6194722	=DMSDD(C9,C10,C11)						
17	2	true lat (deg)	-1.549	=DMSDD(C12,C13,C14)						
18	3	Sun true long (deg)	165.5633044	=Sunlong(C3,C4,C5,0,0,C6,C7,C8)						
19	4	dlong (arcsec)	20.35217443	=-20.5*COS(RADIANS(C18-C16))/COS(RADIANS(C17))						
20	5	dlat (arcsec)	0.068073433	=-20.5*SIN(RADIANS(C18-C16))*SIN(RADIANS(C17))						
21	6	apparent long (deg)	352.6251256	=C16+(C19/3600)						
22	7	apparent lat (deg)	-1.548981091	=C17+(C20/3600)						

Figure 39. Correcting ecliptic coordinates for the effects of aberration.

37 Refraction

In all our calculations so far, we have assumed that the light from distant objects reaches us by the most direct route, a straight line. This is not actually the case (except for observations made at the zenith) as the Earth's atmosphere bends the light a little, making the rays reach the ground at a slightly different angle from that which they would have had if the atmosphere had not been there (see Figure 40). This is called **atmospheric refraction** and its effect is to make the star appear to be closer to the zenith than it really is. The amount of refraction depends on the **zenith angle** or **zenith distance** ($90^\circ - \text{altitude}$) and on the atmospheric conditions, particularly the temperature and pressure. If we observe a star with zenith angle ζ from the surface of the Earth, its true zenith angle, z , is given by $z = \zeta + R$, where R is the refraction angle. An approximate expression for R that is suitable for altitudes above 15° is

$$R = 0.00452P \tan z / (273 + T) \text{ degrees,}$$

where T is the temperature in degrees centigrade and P is the barometric pressure in millibars, both measured at the observation point. This formula is usually accurate to about 6 arcsec for altitudes greater than 15° . At lower altitudes, better results can be obtained using the approximate formula

$$R = \frac{P(0.1594 + 0.0196a + 0.00002a^2)}{(273 + T)(1 + 0.505a + 0.0845a^2)} \text{ degrees,}$$

where a is the altitude in degrees.[†]

[†]Strictly, a is the apparent altitude as measured through the atmosphere, rather than the true altitude as measured with no atmosphere.

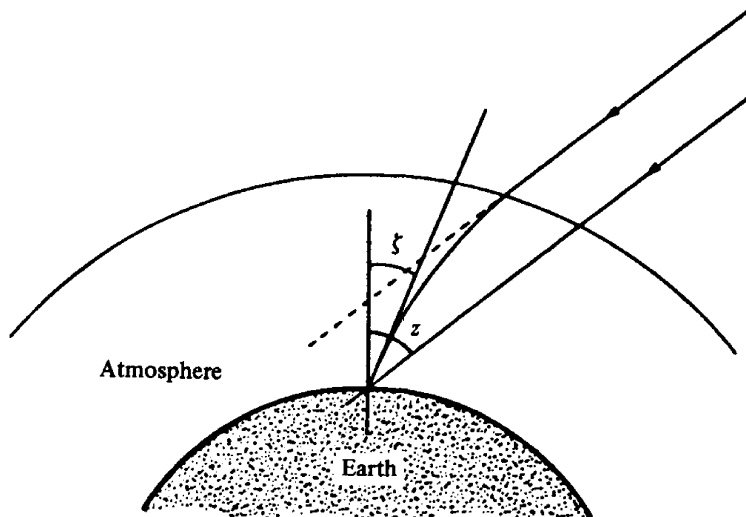


Figure 40. Atmospheric refraction.

The effect of refraction on true equatorial, ecliptic and galactic coordinates is best computed by first converting to horizon coordinates, increasing the altitude by adding R , and then converting back to the original coordinate system to find the apparent position. We will now illustrate this by calculating the refraction for a star whose true hour angle is 5h 51m 44s and true declination $+23^{\circ} 13' 10''$ as observed at a geographical latitude of 52° N. The temperature is 13°C and the pressure is 1008 mbar.

Method	Example
1. Calculate the true altitude and azimuth of the star (§25).	$a = 19.334\,345$ degrees $A = 283.271\,027$ degrees
2. Find the refraction angle R from the formula appropriate to the altitude: $a > 15^{\circ}$, $R = 0.004\,52P \tan z / (273 + T)$.	$z = 70.665\,655$ degrees $R = 0.045\,403$ degrees
3. Add R to the altitude to find the apparent altitude a' .	$a' = 19.379\,748$ degrees
4. Convert A and a' back into equatorial coordinates (§26).	$H' = \mathbf{5h\,51m\,36s}$ $\delta' = \mathbf{23^{\circ}\,15'\,14''}$

The magnitude of R right at the horizon is usually assumed to be 34 arcmin. (Its actual value may be different depending on atmospheric conditions.) Since its effect is to increase the apparent altitude, the times of rising and setting will be earlier and later, respectively, than they would have been without the atmosphere. The effective length of the day, therefore, is increased by atmospheric refraction. We can calculate its effects on the azimuths and times of rising and setting by the method given in Section 33. Alternatively, we can calculate the effect on the hour angle, H , at rising or setting by

$$\Delta H = \frac{34}{15 \cos \phi \cos \delta \sin H} \text{ minutes of time,}$$

where ΔH is the amount by which the true hour angle is reduced.

The spreadsheet for calculating the effects of refraction is labelled Refract and is shown in Figure 41. It goes some way beyond the method given in the method table above in that it handles conversion from apparent to true coordinates as well as from true to apparent, it converts right ascension and declination, and it makes use of the spreadsheet function Refract to do the refraction calculation. This function, used in row 26, takes four arguments, the altitude to be corrected for the effect of refraction, a switch which is set to TRUE or APPARENT depending on the altitude type, the atmospheric pressure in millibars, and the atmospheric temperature in degrees centigrade. In the example shown in the spreadsheet, the true right ascension and declination are converted to the apparent right ascension and declination, this direction being specified by the text in cell C9. You can check that the conversion in the reverse direction is working properly by writing down the corrected coordinates on a piece of paper, then inserting them into cells C3 to C8 in place of the true coordinates, and changing the text in cell C9 to APPARENT (actually, anything which does not begin with T or t will do as the function only looks at the first character). You should see that the true coordinates appear in the output, cells H3 to H8, correct to the second decimal place in the seconds part.

	A	B	C	D	E	F	G	H	I	J
1	Atmospheric refraction									
2										
3	Input	true RA (hour)	23		Output	corrected RA (hour)	23	=DHHour(C28)		
4		true RA (min)	14			corrected RA (min)	13	=DHMin(C28)		
5		true RA (sec)	0			corrected RA (sec)	44.74	=DHSec(C28)		
6		true dec (deg)	40			corrected dec (deg)	40	=DDDeg(C29)		
7		true dec (min)	10			corrected dec (min)	19	=DDMin(C29)		
8		true dec(sec)	0			corrected dec (sec)	45.76	=DDSec(C29)		
9	.. these coordinates are of type		TRUE							
10		geog longitude (deg)	0.17							
11		geog latitude (deg)	51.203611							
12		daylight saving (hours)	0							
13		time zone (hours)	0							
14		local calendar date (day)	23							
15		local calendar date (month)	3							
16		local calendar date (year)	1987							
17		local civil time (hour)	1							
18		local civil time (min)	1							
19		local civil time (sec)	24							
20		atmospheric pressure (mBar)	1012							
21		atmospheric temperature (C)	21.7							
22										
23		HA (hour)	13.8009048	=RAHA(C3,C4,C5,C17,C18,C19,C12,C13,C14,C15,C16,C10)						
24		azimuth (deg)	20.3710605	=EqAz(C23,0,0,C6,C7,C8,C11)						
25		altitude (deg)	4.36733747	=EqAlt(C23,0,0,C6,C7,C8,C11)						
26		corrected altitude (deg)	4.53712816	=Refract(C25,C9,C20,C21)						
27		corrected HA (hour)	13.8051426	=HorHa(C24,0,0,C26,0,0,C11)						
28		corrected RA (hour)	23.2290955	=HARA(C27,0,0,C17,C18,C19,C12,C13,C14,C15,C16,C10)						
29		corrected dec (deg)	40.3293776	=HORDec(C24,0,0,C26,0,0,C11)						

Figure 41. Correcting equatorial coordinates for the effects of refraction.

38 Geocentric parallax and the figure of the Earth

In later sections of this book we calculate the coordinates of the Sun, the Moon and other members of our Solar System. These coordinates are the ones which would be observed from the centre of the Earth, called **geocentric coordinates**, and if the celestial body is at a very great distance from the Earth, they are also the coordinates which would be measured by anyone on the Earth's surface. However, objects relatively close at hand like the Sun, and especially the Moon, appear to be at slightly different positions depending upon the exact viewpoint of the observer. This is illustrated in Figure 42 where two observers, O_1 and O_2 , are viewing the Moon, M , from the surface of the Earth, E . Each measures the angle between the Moon and a very distant star in the direction S . Since this star is so far away the lines of sight to the star, O_1S and O_2S , are parallel so that both observers see it in the same place in the sky relative to other stars. However, they do not measure the same angles, a_1 and a_2 , and hence do not agree about the Moon's apparent position. If a_0 represented, say, the right ascension of the Moon as calculated from the Earth's centre, then each observer would have to add a different correction to a_0 to get a_1 or a_2 , the apparent right ascension at each place. This apparent shift of position is known as **geocentric parallax** and we often need to be able to correct for it as, for example, when we wish to calculate the circumstances of an eclipse.

The problem is complicated slightly by the fact that the Earth is not quite spherical, but is, instead, more like a **spheroid of revolution**, being flattened along the line joining the north and south poles. A cross-section through the Earth along any line of longitude would be approximately elliptical, while a cross-section along any line of latitude would be circular. We have to take account of the **figure of the Earth**, its deviation from a perfect sphere, if we are to make precise corrections for parallax. The situation is shown much exaggerated in Figure 43 where the Earth, E , is drawn with its north and south poles, N and S . An observer at O locates his zenith by means of a plumb line to be along the dashed line OZ ; the angle this makes with the equator defines his **geographical or astronomical latitude**, ϕ . Since the Earth is not

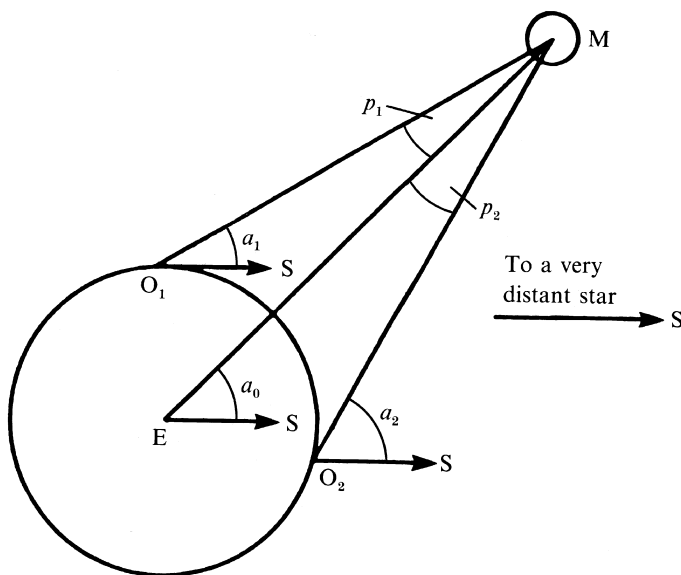


Figure 42. Geocentric parallax.

quite spherical, his geocentric vertical, EZ' , is slightly different and so too is his **geocentric latitude**, ϕ' . In calculations of the effect of parallax, we need to know the quantities $\rho \sin \phi'$ and $\rho \cos \phi'$, where ρ is the distance of the observer from the centre of the Earth in units of the Earth's equatorial radius. For a place whose height above sea-level is h metres, we have

$$\rho \sin \phi' = 0.996\,647 \sin u + \frac{h}{6\,378\,140} \sin \phi,$$

$$\rho \cos \phi' = \cos u + \frac{h}{6\,378\,140} \cos \phi,$$

where

$$u = \tan^{-1} \{0.996\,647 \tan \phi\}.$$

ϕ must be reckoned as positive in the northern hemisphere and negative in the southern hemisphere. For example, let us calculate the values of $\rho \sin \phi'$ and $\rho \cos \phi'$ for an observer whose height above sea-level is 60 metres at longitude 100° W and latitude 50° N.

Method	Example
1. Calculate $u = \tan^{-1} \{0.996\,647 \tan \phi\}$.	$\phi = +50.0$ degrees $u = 49.905\,217$ degrees
2. Calculate $h' = \frac{h}{6\,378\,140}$.	$h = 60.0$ metres $h' = 0.000\,009$
3. Calculate $\rho \sin \phi' = 0.996\,647 \sin u + h' \sin \phi$.	$\rho \sin \phi' = \mathbf{0.762\,422}$
4. Calculate $\rho \cos \phi' = \cos u + h' \cos \phi$.	$\rho \cos \phi' = \mathbf{0.644\,060}$

In Figure 43, it is the angle p which is formally called the geocentric parallax. This is the angle between the observer and the Earth's centre as seen by the celestial body in question. If the observer views the body right at his horizon (i.e. the zenith angle = 90°), then p is called the **horizontal parallax**. Further, if the observer is also on the equator, this angle becomes the **equatorial horizontal parallax**, and is given the symbol P . (We will meet parallax again in Section 69.) The spreadsheet for parallax calculations is given in the next section (Figure 44).

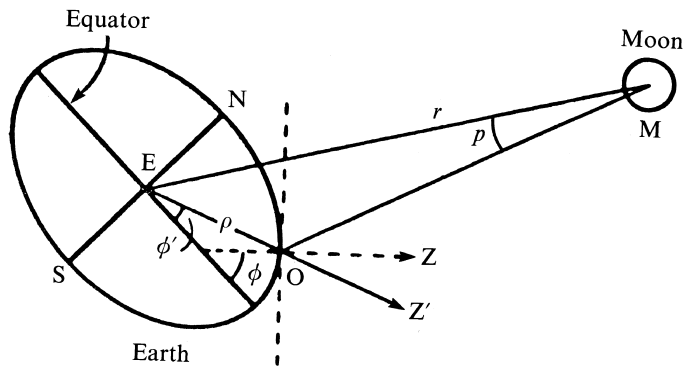


Figure 43. Allowing for the figure of the Earth.

39 Calculating corrections for parallax

If a body has geocentric hour angle, H , and geocentric right ascension, α , then its apparent hour angle, H' , and right ascension, α' (taking account of parallax), are given by

$$H' = H + \Delta,$$

$$\alpha' = \alpha - \Delta,$$

with

$$\Delta = \tan^{-1} \left\{ \frac{\rho \cos \phi' \sin H}{r \cos \delta - \rho \cos \phi' \cos H} \right\},$$

where $\rho \cos \phi'$ is the quantity calculated in Section 38 and r is the distance of the body from the centre of the Earth measured in units of (equatorial) Earth radii, 6378.14 km. If r' is this distance in kilometres, then

$$r = \frac{r'}{6378.14}.$$

r can also be found from the equatorial horizontal parallax of the body, P . Thus

$$r = \frac{1}{\sin P}.$$

The formula for finding the apparent declination, δ' , from the geocentric declination, δ , is

$$\delta' = \tan^{-1} \left\{ \cos H' \frac{r \sin \delta - \rho \sin \phi'}{r \cos \delta \cos H - \rho \cos \phi'} \right\}.$$

Again, $\rho \sin \phi'$ and $\rho \cos \phi'$ can be found by the method described in Section 38.

As an example, let us calculate the apparent right ascension and declination of the Moon on 26 February 1979 at 16h 45m UT when observed from a location 60 metres above sea-level on longitude 100° W and latitude 50° N. The geocentric coordinates were $\alpha = 22^{\text{h}} 35^{\text{m}} 19^{\text{s}}$ and $\delta = -7^\circ 41' 13''$, and the Moon's equatorial horizontal parallax was $1^\circ 01' 09''$.

Method	Example
1. Convert UT to GST and hence to LST by the methods of §§12 and 14.	UT = 16h 45m GST = 3.145 778 hours LST = 20.479 111 hours
2. Convert α and δ to decimal form (§§7 and 21).	α = 22.588 611 hours δ = -7.686 944 degrees
3. Find the hour angle, H (§24), and convert to degrees (§22).	H = -2.109 500 hours = -31.642 500 degrees
4. Find $\rho \cos \phi'$ and $\rho \sin \phi'$ (§38).	$\rho \cos \phi'$ = 0.644 060 $\rho \sin \phi'$ = 0.762 422
5. Find $r = (1/\sin P)$ (remember to convert P to decimal degrees first (§21)).	P = 1.019 167 degrees r = 56.221 228 Earth-radii
6. Calculate $\Delta = \tan^{-1} \left\{ \frac{\rho \cos \phi' \sin H}{r \cos \delta - \rho \cos \phi' \cos H} \right\}$.	Δ = -0.350 915 degrees
7. Find $H' = H + \Delta$.	H' = -31.994 414 degrees
8. Convert Δ to hours by dividing by 15 (§22).	Δ = -0.023 394 hours
9. Subtract Δ from α to find α' .	α' = 22.612 005 hours
10. Calculate $\delta' = \tan^{-1} \left\{ \cos H' \frac{r \sin \delta - \rho \sin \phi'}{r \cos \delta \cos H - \rho \cos \phi'} \right\}$.	δ' = -8.538 165 degrees
11. Convert α' and δ' to minutes and second form (§§8 and 21).	α' = 22h 36m 43s δ' = -8° 32' 17"

Such lengthy calculations are strictly only necessary for the Moon which has a very large parallax. The Sun, planets and comets usually have much smaller values, enabling us to simplify the formulas slightly without serious loss of accuracy. Let r again denote the distance of the body from the centre of the Earth, but this time measured in astronomical units (AU). Then

$$\pi = \frac{8.794}{r} \text{ arcsec}$$

and

$$\alpha' = \alpha - \frac{\pi \sin H \times \rho \cos \phi'}{\cos \delta},$$

$$\delta' = \delta - \pi (\rho \sin \phi' \cos \delta - \rho \cos \phi' \cos H \sin \delta).$$

π is the symbol often used for parallax. Take care to distinguish its use for parallax from its use to represent the circular constant 3.141 592 654.

Let us now calculate the apparent position of the Sun when observed by the same observer at the same time as in the previous example. The geocentric right ascension of the Sun was 22h 36m 44s and its declination was $-8^\circ 44' 24''$. Its distance was 0.9901 AU.

Method	Example
1. Calculate $\pi = \frac{8.794}{r}$.	$\pi = 8.881\,931$ arcsec
2. Convert to degrees by dividing by 3600.	$\pi = 0.002\,467$ degrees
3. Convert to hours by dividing by 15 (§22).	$\pi = 0.000\,164$ hours
4. Convert UT to GST and hence to LST by the methods of §§12 and 14.	GST = 3.145 778 hours LST = 20.479 111 hours
5. Convert α and δ to decimal form (§§7 and 21).	$\alpha = 22.612\,222$ hours $\delta = -8.740\,000$ degrees
6. Find $\rho \cos \phi'$ and $\rho \sin \phi'$ (§38).	$\rho \cos \phi' = 0.644\,060$ $\rho \sin \phi' = 0.762\,422$
7. Find the hour angle, H , and convert to degrees (§§24 and 22).	$H = -2.133\,111$ hours $H = -31.996\,666$ degrees
8. Calculate $\Delta_1 = \frac{\pi \sin H \rho \cos \phi'}{\cos \delta}$ (π expressed in hours).	$\Delta_1 = -0.000\,051$ hours
9. Subtract from α to get α' .	$\alpha' = 22.612\,279$ hours
10. Find $\Delta_2 = \pi (\rho \sin \phi' \cos \delta - \rho \cos \phi' \cos H \sin \delta)$ (π expressed in degrees).	$\Delta_2 = 0.002\,064$ degrees
11. Subtract from δ to get δ' .	$\delta' = -8.742\,064$ degrees
12. Convert α' and δ' to minutes and seconds form (§§8 and 21).	$\alpha' = 22^{\text{h}}\,36^{\text{m}}\,44^{\text{s}}$ $\delta' = -8^{\circ}\,44'\,31''$

Note that the correction for parallax has had hardly any effect in this case. Except for the Moon, geocentric parallax can often be ignored. Note also that the Sun and Moon have almost the same apparent positions in this example; we have chosen the moment of a total solar eclipse (see Section 74).

The calculations of Sections 38 and 39 have been swept up into just one spreadsheet, called *Parallax*, shown in Figure 44. Rather than carrying out all the calculations of the last three method tables explicitly, we have put them into two spreadsheet functions called *ParallaxHA* and *ParallaxDec*, returning respectively the corrected hour angle in decimal hours and the corrected declination in decimal degrees. Each takes the same ten arguments, these being the hour angle as hours, minutes, seconds, the declination as degrees, minutes, seconds, a text word set to TRUE or APPARENT specifying whether the hour angle and declination are true coordinates or are apparent coordinates, the geographical latitude in degrees, the height above sea-level in metres, and the horizontal parallax in degrees. These functions are used in rows 24 and 26 of the spreadsheet. The example given in Figure 44 is the same as that of the first method table of this section, except that the local civil time has been specified for time zone -6 h; the result is the same.

Although the usual calculation is to find the apparent coordinates given the true coordinates (setting the switch in cell C9 to TRUE), the spreadsheet also allows the calculation to be done the other way around. Try this for yourself. Write down the apparent coordinates and then enter them into the spreadsheet in cells C3 to C8, setting also cell C9 to APPARENT (or anything which does not begin with T or t). You should find the true (geocentric) coordinates returned in cells H3 to H8 correct to within the second decimal place in the seconds part.

	A	B	C	D	E	F	G	H	I	J
1	Corrections for geocentric parallax									
2										
3	<i>Input</i>	RA (hour)	22		<i>Output</i>	corrected RA (hour)	22	=DHour(C25)		
4		RA (min)	35			corrected RA (min)	36	=DHMin(C25)		
5		RA (sec)	19			corrected RA (sec)	43.22	=DHSec(C25)		
6		dec (deg)	-7			corrected dec (deg)	-8	=DDDeg(C26)		
7		dec (min)	41			corrected dec (min)	32	=DDMin(C26)		
8		dec(sec)	13			corrected dec (sec)	17.4	=DDSec(C26)		
9		.. these are true or apparent?	TRUE							
10		equatorial hor parallax (deg)	1.019167							
11		geographic longitude (deg)	-100							
12		geographic latitude (deg)	50							
13		height (m)	60							
14		daylight Saving	0							
15		time zone	-6							
16		local calendar date (day)	26							
17		local calendar date (month)	2							
18		local calendar date (year)	1979							
19		local civil time (hour)	10							
20		local civil time (min)	45							
21		local civil time (sec)	0							
22										
23	1	HA (hours)	21.89050003	=RAHA(C3,C4,C5,C19,C20,C21,C14,C15,C16,C17,C18,C11)						
24	2	corrected HA (hours)	21.86710571	=ParallaxHA(C23,0,0,C6,C7,C8,C9,C12,C13,C10)						
25	3	corrected RA (hours)	22.61200543	=HARA(C24,0,0,C19,C20,C21,C14,C15,C16,C17,C18,C11)						
26	4	corrected dec (deg)	-8.538165508	=ParallaxDec(C23,0,0,C6,C7,C8,C9,C12,C13,C10)						

Figure 44. Correcting equatorial coordinates for the effects of parallax.

40 Heliographic coordinates

Heliographic coordinates enable us to define the position of any point (such as a sunspot) on the surface of the Sun. As with any other set of astronomical coordinates, latitudes are referred to a fundamental plane and longitudes to a fixed point in that plane. In this case the fundamental plane is taken to be the solar equator, inclined at an angle $I = 7^\circ 15'$ to the ecliptic, and the fixed point is the present position of the point occupied by the ascending node of the solar equator on the ecliptic at noon on 1 January 1854 (JD 2 398 220.0). There are no permanent features on the Sun's disc by which we can locate this point, so we have to work out its position assuming a rotation period of 25.38 days.

The situation is illustrated in Figure 45. The sphere represents the surface of the Sun and the great circle ONJ the solar equator. $P_N C$ is the rotation axis of the Sun and any point on the equator rotates in the direction from O to N. The plane of the ecliptic intersects the Sun's surface along the great circle ΥEN ; the point N is therefore the ascending node of the solar equator on the plane of the ecliptic. Imaginary lines drawn from the centre of the Sun, C, towards the Earth and towards the vernal equinox cut the Sun's surface at E and Υ respectively. O is the point which, at midday on 1 January 1854, was at N. A sunspot at X has heliographic latitude B (positive north of the equator, negative south of it) and heliographic longitude L , reckoned in the same sense as the solar rotation and measured along the equator from O.

When we observe the Sun (which, to avoid permanent injury to the eye, must only be by projection onto a screen, or using proper solar filters) we see a flat disc, the centre of which is the point E. This is shown

in Figure 46, together with the point P_N , the north pole of the Sun's rotation axis, and X , the position of the sunspot. The dashed line $N'S'$ represents the projection of the Earth's axis of rotation, NS , onto the disc. We define the position of X by the coordinates ρ_1 and θ . The trick is to turn ρ_1 and θ into B and L .

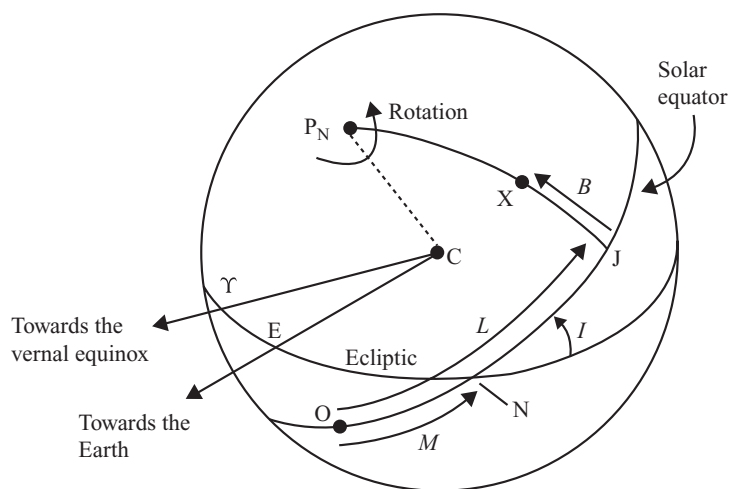


Figure 45. Defining heliographic coordinates.

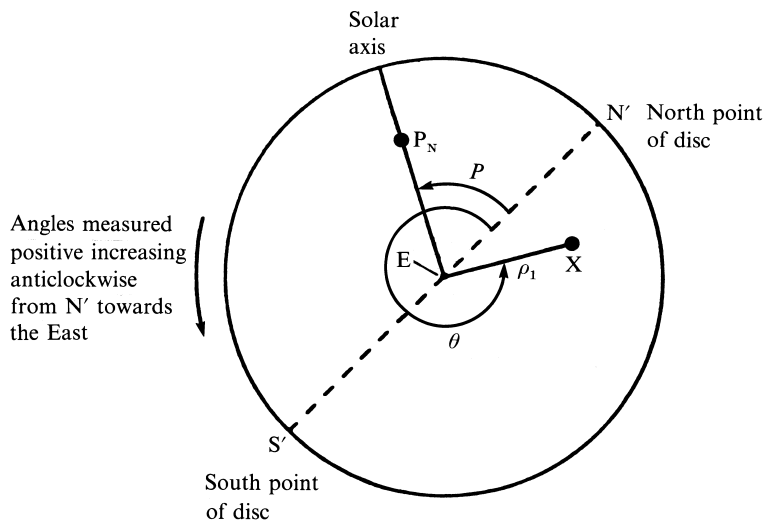


Figure 46. The Sun's disc.

To do this we need first to calculate the heliographic coordinates, B_0 and L_0 , of the centre of the disc, E . The equations are:

$$L_0 = \tan^{-1} \left\{ \frac{\sin(\delta_\odot - \lambda_\odot) \cos I}{-\cos(\delta_\odot - \lambda_\odot)} \right\} + M,$$

$$B_0 = \sin^{-1} \left\{ \sin(\lambda_\odot - \delta_\odot) \sin I \right\},$$

where λ_\odot is the geocentric ecliptic longitude of the Sun, I is the inclination of the solar equator to the plane of the ecliptic ($= 7^\circ 15'$), δ_\odot is the longitude of the ascending node (the angle ΥN in Figure 45), and M is the angle between O and N (Figure 45; here and elsewhere the angles on the celestial sphere are subtended at the centre C , and measured along the corresponding great-circle arcs.). δ_\odot is given by

$$\delta_\odot = 74^\circ 22' + 84'T,$$

where T is the number of Julian centuries since the epoch 1900 January 0.5[†]. M is given by

$$M = 360 - M',$$

with

$$M' = \frac{360}{25.38} (\text{JD} - 2398220.0),$$

where JD is the Julian date. M' must be reduced to the range 0 to 360 by subtracting integral multiples of 360.

For example, let us calculate the heliographic coordinates of the centre of the solar disc on 1 May 1988. The geocentric longitude of the Sun, λ_\odot , can be found by the method described in Section 46; its value at 0h UT on this day was $40^\circ 50' 37''$.

[†]Note that this definition of T is different from that used elsewhere in the book.

Method	Example
1. Find the Julian date (§4).	JD = 2 447 282.5
2. Subtract 2 415 020 and divide by 36 525 to find T in centuries since 1900 January 0.5.	T = 0.883 299 centuries
3. Calculate $\Delta = 84T$. Divide by 60 to convert to degrees.	Δ = 74.197 125 arcmin = 1.236 619 degrees
4. Convert $74^\circ 22'$ to decimal degrees (§21) and add to Δ to find δ_\odot .	δ_\odot = 75.603 285 degrees
5. Convert λ_\odot to decimal degrees (§21). (You can calculate λ_\odot by the method of §46.)	λ_\odot = 40.843 611 degrees
6. Find $y = \sin(\delta_\odot - \lambda_\odot) \cos I$ ($I = 7.25$ degrees).	$\delta_\odot - \lambda_\odot$ = 34.759 674 degrees y = 0.565 577
7. Find $x = -\cos(\delta_\odot - \lambda_\odot)$.	x = -0.821 551
8. Find $A' = \tan^{-1}\left(\frac{y}{x}\right)$. We have to remove the ambiguity introduced by taking inverse tan. To do so, look up Figure 29 and add or subtract 180 or 360 until A' is in the correct quadrant. If it is already in the correct quadrant, $A = A'$.	A' = -34.544 552 y positive x negative + 180.0 A = 145.455 448
9. Calculate $M' = \frac{360}{25.38}(\text{JD} - 2\,398\,220)$. Subtract multiples of 360 to bring it back into the range 0 to 360.	M' = 695 921.985 816 - 360×1933 = 41.985 816 degrees
10. Find $M = 360 - M'$.	M = 318.014 184 degrees
11. Add M to A to find L_0 . Subtract 360 if more than 360.	L_0 = 103.47 degrees
12. Calculate $B_0 = \sin^{-1}\left\{\sin(\lambda_\odot - \delta_\odot) \sin I\right\}$.	B_0 = -4.13 degrees

The *Astronomical Almanac* lists these values as $L_0 = 103.47$ degrees and $B_0 = -4.12$ degrees.

In addition to B_0 and L_0 we also need the position-angle of the Sun's rotation axis, the angle P in Figure 46. This is given by

$$P = \theta_1 + \theta_2,$$

with

$$\theta_1 = \tan^{-1}\left\{-\cos \lambda_\odot \tan \varepsilon\right\}$$

and

$$\theta_2 = \tan^{-1}\left\{-\cos(\delta_\odot - \lambda_\odot) \tan I\right\},$$

where ε is the obliquity of the ecliptic (see Section 27).

For example, what was the value of P on 1 May 1988?

Method	Example
Referring to the previous example for values of λ_\odot , δ_\odot and I :	λ_\odot = 40.843 611 degrees δ_\odot = 75.603 285 degrees I = 7.25 degrees
1. Calculate $\theta_1 = \tan^{-1}\left\{-\cos \lambda_\odot \tan \varepsilon\right\}$ (with $\varepsilon = 23.442$ degrees).	θ_1 = -18.160 747 degrees
2. Calculate $\theta_2 = \tan^{-1}\left\{-\cos(\delta_\odot - \lambda_\odot) \tan I\right\}$.	θ_2 = -5.966 575 degrees
3. Find $P = \theta_1 + \theta_2$.	P = -24.127 321 degrees

The *Astronomical Almanac* gives $P = -24.11$ degrees.

We are now in a position to calculate the heliographic coordinates of the sunspot X, given its position-angle, θ (see Figure 46), and ρ_1 , the angle subtended at the Earth by X and E. The formulas are as follows:

$$B = \sin^{-1} \{ \sin B_0 \cos \rho + \cos B_0 \sin \rho \cos (P - \theta) \},$$

$$L = \sin^{-1} \left\{ \frac{\sin \rho \sin (P - \theta)}{\cos B} \right\} + L_0,$$

with

$$\rho = \sin^{-1} \left\{ \frac{\rho_1}{S} \right\} - \rho_1,$$

where S is the angular radius of the Sun.

Continuing our example, what were the heliographic coordinates of a sunspot measured at position-angle $\theta = 220^\circ$ and displacement $\rho_1 = 10.5$ arcminutes on 1 May 1988? The angular radius of the Sun was $15' 52''$.

Method	Example
1. Calculate L_0 , B_0 and P (see previous examples).	$L_0 = 103.47$ degrees $B_0 = -4.13$ degrees $P = -24.11$ degrees
2. Find $\sin^{-1} \left(\frac{\rho_1}{S} \right)$ (remember to convert S to decimal arcminutes first).	$S = 15.867$ arcmin $\sin^{-1} \left(\frac{\rho_1}{S} \right) = 41.435$ degrees
3. Convert ρ_1 to decimal degrees (§21) and subtract to find ρ .	$\rho_1 = 0.175$ degrees $\rho = 41.260$ degrees
4. Calculate $B = \sin^{-1} \{ \sin B_0 \cos \rho + \cos B_0 \sin \rho \cos (P - \theta) \}$.	$(P - \theta) = -244.127$ degrees $B = -19.945$ degrees
5. Calculate $A = \sin^{-1} \left\{ \frac{\sin \rho \sin (P - \theta)}{\cos B} \right\}$.	$A = 39.141$ degrees
6. Add L_0 to find L ; subtract 360 if L is greater than 360.	$L = 142.611$ degrees

All of the calculations of this section are made in one spreadsheet called Heliographic (see Figure 47). It makes a forward reference to finding the position of the Sun, and uses the spreadsheet function SunLong referred to in Figure 47 (row 12). The small differences between the answers calculated by the spreadsheet and those calculated in the method table above are caused by rounding errors, and by the difference between the solar longitudes calculated by SunLong (higher precision) and the method of Section 46 (lower precision). We have also provided spreadsheet functions HeliogLong and HeliogLat to carry out the calculation. These return, respectively, the Heliographic Longitude and the Heliographic Latitude, both in degrees. Each function takes the same five arguments. They are the heliographic position angle in degrees, the heliographic displacement from the centre of the Sun's disc in arcminutes, and the Greenwich calendar date as day, month and year. You can simplify the spreadsheet by using these functions instead of the calculation part. Save a copy, and then delete rows 9 to 29 inclusive. Insert the following spreadsheet formulas into cells H3 and H4:

=ROUND(HeliogLong(C3,C4,C5,C6,C7),2)

=ROUND(HeliogLat(C3,C4,C5,C6,C7),2).

Type a question-for help									
A	B	C	D	E	F	G	H	I	J
1	Heliographic coordinates								
2									
3	Input helio position angle (deg)	220		Output	helio long (deg)	142.59	=ROUND(C29,2)		
4	helio displacement (arcmin)	10.5			helio lat (deg)	-19.94	=ROUND(C27,2)		
5	Greenwich date day	1							
6	Greenwich date month	5							
7	Greenwich date year	1988							
8									
9	1 Julian date (days)	2447282.5	=CDJD(C5,C6,C7)						
10	2 T (centuries)	0.88329911	=(C9-2415020)/36525						
11	3 Long asc node (deg)	75.60328542	=DMSDD(74,22,0)+(84*C10/60)						
12	4 Sun long (deg)	40.84263343	=SunLong(0,0,0,0,C5,C6,C7)						
13	5 y	0.565591133	=SIN(RADIANS(C11-C12))*COS(RADIANS(DMSdd(7,15,0)))						
14	6 x	-0.821540954	=-COS(RADIANS(C11-C12))						
15	7 A (deg)	145.4544725	=DEGREES(ATAN2(C14,C13))						
16	8 M (deg)	-695561.9858	=360-(360*(C9-2398220)/25.38)						
17	9 M (deg)	318.0141844	=C16-360*INT(C16/360)						
18	10 L0 (deg)	463.4686569	=C17+C15						
19	11 L0 (deg)	103.4686569	=C18-360*INT(C18/360)						
20	12 B0 (rad)	-0.07201451	=ASIN(SIN(RADIANS(C12-C11))*SIN(RADIANS(DMSdd(7,15,0))))						
21	13 theta1 (rad)	-0.316988267	=ATAN(-COS(RADIANS(C12))*TAN(RADIANS(oblq(C5,C6,C7))))						
22	14 theta2 (rad)	-0.104135152	=ATAN(-COS(RADIANS(C11-C12))*TAN(RADIANS(DMSdd(7,15,0))))						
23	15 P (deg)	-24.12859453	=DEGREES(C21+C22)						
24	16 rho1 (deg)	0.175	=C4/60						
25	17 rho (rad)	0.71975524	=ASIN(2*C24/SunDia(0,0,0,0,C5,C6,C7))-RADIANS(C24)						
26	18 B (rad)	-0.347984957	=ASIN(SIN(C20)*COS(C25)+COS(C20)*SIN(C25)*COS(RADIANS(C23-C3)))						
27	19 B (deg)	-19.93806936	=DEGREES(C26)						
28	20 L (deg)	502.5889605	=DEGREES(ASIN(SIN(C25)*SIN(RADIANS(C23-C3))/COS(C26)))+C18						
29	21 L (deg)	142.5889605	=C28-360*INT(C28/360)						

Figure 47. The calculation of heliographic coordinates.

41 Carrington rotation numbers

Solar rotations are numbered by the Carrington rotation number, CRN, the first of which began on 9 November 1853. One rotation is the period during which the value of L_0 (Section 40) decreases by 360° , and its mean length is 27.2753 days. We can calculate CRN quite accurately by noting from the *Astronomical Ephemeris* that rotation number 1690 began on 1979 December 27.84. Thus

$$\text{CRN} = 1690 + \left\lceil \frac{\text{JD} - 2444235.34}{27.2753} \right\rceil,$$

where JD is the Julian date. Round the result to the nearest integer. You may be in error by ± 1 just at the point where the rotation number changes.

For example, what was the CRN on 27 January 1975?

Method	Example
1. Calculate the Julian date (§4).	JD = 2 442 439.50
2. Find $\text{CRN} = 1690 + \left\lceil \frac{\text{JD} - 2444235.34}{27.2753} \right\rceil$, rounding the result to the nearest integer.	CRN = 1624

The spreadsheet for this straightforward calculation is shown in Figure 48. We have also provided the spreadsheet function CRN which returns the Carrington Rotation Number for the given Greenwich calendar date specified in the three arguments as day, month, and year. Thus you could, if you wished, get the same result by deleting rows 7 and 8 of the spreadsheet and inserting the following formula in cell H3 (save a copy of the spreadsheet first!):

=CRN(C3,C4,C5).

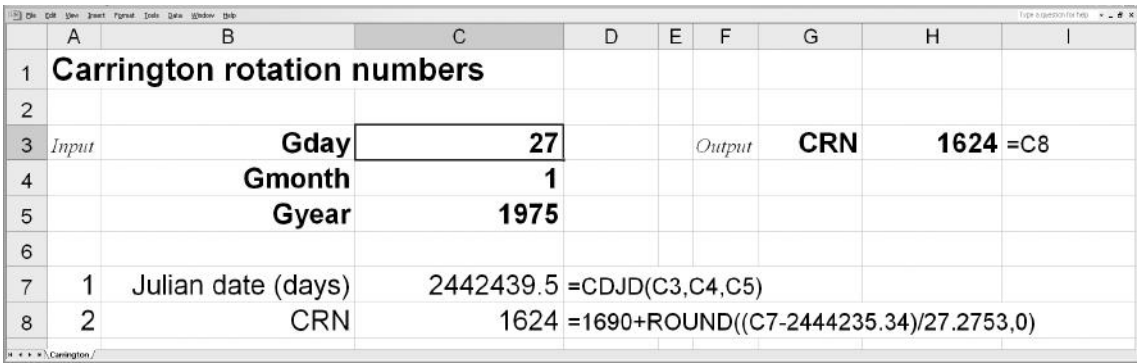


Figure 48. Carrington rotation number calculation.

42 Selenographic coordinates

The position of any point on the surface of the Moon can be described by means of a pair of **selenographic coordinates**. As with all other astronomical systems, latitudes are referred to a fundamental plane, and longitudes to a fixed point in that plane. In this case, the plane is taken to be the Moon's equator, inclined at an angle of $I = 1^\circ 32' 32.7''$ to the plane of the ecliptic. However, there is a problem with defining a fixed point of zero longitude because the Moon wobbles about so much. In particular, the rotation axis of the Moon oscillates about its mean position, an effect known as **physical libration**, and the mean position itself does not have a fixed sidereal direction, but is subject to a regular precession with a period of 18.6 years. Nevertheless, we can identify a point on the lunar surface which has an average position exactly in the centre of the apparent disc as it would be seen from the centre of the Earth. This is the reference point for measuring longitudes. The actual centre of the apparent lunar disc at any time may be as much as 8° in longitude and 6° in latitude away from the average position.

The selenocentric celestial sphere (i.e. the celestial sphere centred on the Moon) is shown in Figure 49. The traces of the planes of the ecliptic and the lunar equator are drawn, and they intersect with one another at angle I . The point P_0 represents the pole of the equator, and M is the mean centre of the Moon's apparent disc as observed from the centre of the Earth. The great circle through P_0 and M is the prime meridian, longitude 0° . Point R is a crater on the surface of the Moon, and the great circle through P_0 and R cuts the equator at X . Then the selenographic longitude of R is the angle l measured from M along the equator to X , and the selenographic latitude is the angle b subtended at the centre of the Moon by X and R . Longitudes increase to the west (towards Mare Crisium) and latitudes to the north, as seen on the apparent disc.

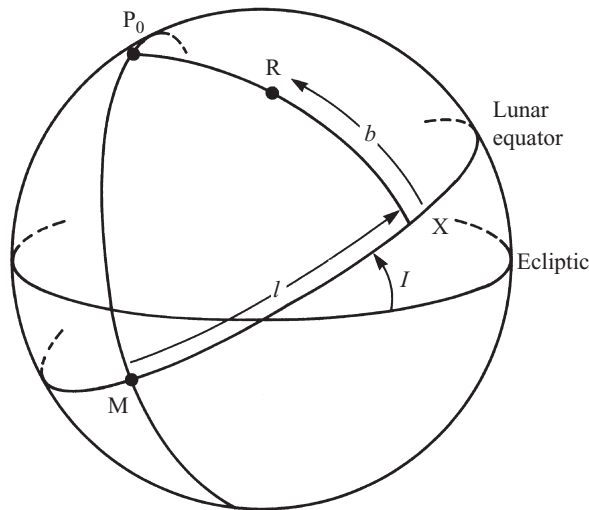


Figure 49. Selenographic coordinates.

The point which, at any moment, is in the centre of the apparent lunar disc is called the **sub-Earth point**. Its selenographic coordinates, l_e , and b_e , may be calculated from the equations

$$b_e = \sin^{-1} [-\cos I \sin \beta + \sin I \cos \beta \sin (\delta\Omega - \lambda_m)],$$

$$l_e = \tan^{-1} \left\{ \frac{-\sin \beta \sin I - \cos \beta \cos I \sin (\delta\Omega - \lambda_m)}{\cos \beta \cos (\delta\Omega - \lambda_m)} \right\} - F,$$

where λ_m and β are the apparent geocentric coordinates of the Moon, $\delta\Omega$ is the longitude of the ascending node of the Moon's mean orbit on the ecliptic, and $F = L - \delta\Omega$, where L is the mean longitude of the Moon. $\delta\Omega$ and F can be found from the approximate relations

$$\delta\Omega = 125.044522 - 1934.136261T \text{ degrees},$$

$$F = 93.271910 + 483202.0175T \text{ degrees},$$

where T is the number of Julian centuries of 36 525 days since the epoch 2000 January 1.5, i.e.

$$T = (\text{JD} - 2451545.0) / 36525.$$

The position-angle, C , of the pole of the lunar equator (P_0) is measured in the same way as position-angles on the surface of the Sun, i.e. anticlockwise from the north point of the apparent lunar disc. We can calculate it from the following set of equations:

$$C = C_1 + C_2,$$

$$C_1 = \tan^{-1} \left\{ \frac{\cos (\delta\Omega - \lambda_m) \sin I}{\cos \beta \cos I + \sin \beta \sin I \sin (\delta\Omega - \lambda_m)} \right\},$$

$$C_2 = \tan^{-1} \left\{ \frac{\sin \varepsilon \cos \lambda_m}{\sin \varepsilon \sin \beta \sin \lambda_m - \cos \varepsilon \cos \beta} \right\},$$

where ε is the obliquity of the ecliptic (Section 27).

We shall find as an example of these calculations the values of l_e , b_e and C on 1 May 1988. The Moon's geocentric longitude was 209.12° , and latitude -3.08° . The obliquity was 23.4433° .

Method	Example
1. Convert the date to the Julian day number (§4).	JD = 2447 282.5
2. Calculate $T = (\text{JD} - 2451\,545.0) / 36\,525$.	$T = -0.116\,701$
3. Find the value of $\delta\mathcal{L}$ using the equation given in this section. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	$\delta\mathcal{L} = 350.759\,945$ degrees
4. Find the value of F using the equation given in this section. Reduce to the range 0 to 360.	$F = 223.166\,514$ degrees
5. Find $\sin b_e = (-\cos I \sin \beta + \sin I \cos \beta \sin(\delta\mathcal{L} - \lambda_m))$.	$\sin b_e = 0.070\,391$
6. Take the inverse sine to find b_e .	$b_e = \mathbf{4.04}$ degrees
7. Find, and note the sign of, $y = (-\sin \beta \sin I - \cos \beta \cos I \sin(\delta\mathcal{L} - \lambda_m))$.	$y = -0.618\,034$ (negative)
8. Find, and note the sign of, $x = \cos \beta \cos(\delta\mathcal{L} - \lambda_m)$.	$x = -0.782\,994$ (negative)
9. Take $A = \tan^{-1}(\frac{y}{x})$. Consult Figure 29 and add or subtract 180 or 360 to bring the result into the quadrant specified by the signs of x and y .	$A = 38.284\,805$ degrees + 180.0
10. Subtract F from A and reduce the result to the range -180 to $+180$ by adding or subtracting 360. This is l_e .	$A = 218.284\,805$ degrees $l_e = \mathbf{-4.88}$ degrees
11. Calculate the values of C_1 and C_2 using the equations given in this section. There is no need to worry about ambiguities of 180° when taking inverse tan since, in this case, the results will always be in the correct quadrants.	$C_1 = -1.212\,401$ degrees $C_2 = 20.993\,347$ degrees
12. Add C_1 and C_2 to find the position angle C .	$C = \mathbf{19.78}$ degrees

The *Astronomical Almanac* gives values of $l_e = -4.918$, $b_e = 4.051$ and $C = 19.761$, having taken account also of physical libration.

The spreadsheet for finding the coordinates of the sub-Earth point and the position-angle of the equator is shown in Figure 50, labelled *Selenographic1*. It steals spreadsheet functions from Section 65 to find the Moon's longitude and latitude (rows 12 and 13), but otherwise follows the method given in the method table. We have not provided a corresponding spreadsheet function.

The selenographic coordinates of the Sun (strictly, the **sub-solar** point) can also be found quite easily. We can use the same equations as for calculating l_e and b_e , but with λ_m and β replaced by the heliocentric ecliptic coordinates of the Moon, λ'_m , and β' . Their values may be found by the approximations

$$\lambda'_m = \lambda_\odot + 180 + \left\{ \frac{26.4 \cos \beta \sin(\lambda_\odot - \lambda_m)}{\pi R} \right\} \text{ degrees,}$$

$$\beta' = \frac{0.14666\beta}{\pi R} \text{ degrees,}$$

where π is the equatorial horizontal parallax of the Moon (see Section 69) expressed in arcminutes, R is the Sun–Earth distance in astronomical units (Section 48), and λ_\odot is the true geocentric longitude of the Sun (Section 46). Note that the Sun's selenographic longitude is often expressed as the (selenographic) **colongitude**, which is just $90^\circ - \text{longitude}$ (reduced to the range 0 to 360 by adding or subtracting 360). If we know the colongitude of the Sun, then the selenographic longitude of the morning **terminator** (the division between night and morning on the Moon) is approximately $360^\circ - \text{colongitude of the Sun}$, and the selenographic longitude of the evening terminator is approximately $180^\circ - \text{colongitude of the Sun}$.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Selenographic coordinates 1											
2												
3	Input	Greenwich date (day)	1		Output	sub-Earth longitude	-4.88 =ROUND(C22,2)					
4		Greenwich date (month)	5			sub-Earth latitude	4.04 =ROUND(C17,2)					
5		Greenwich date (year)	1988			position angle of pole	19.78 =ROUND(C26,2)					
6												
7	1	Julian date (days)	2447282.5 =CDJD(C3,C4,C5)									
8	2	T (centuries)	-0.11670089 =(C7-2451545)/36525									
9	3	long asc node (deg)	350.7599447 =125.044522-1934.136261*C8									
10	4	F	-56296.83349 =93.27191+483202.0175*C8									
11	5	F	223.1665139 =C10-360*INT(C10/360)									
12	6	Geocentric Moon long (deg)	209.1175282 =MoonLong(0,0,0,0,C3,C4,C5)									
13	7	Geocentric Moon lat (rad)	-0.053838535 =RADIANS(MoonLat(0,0,0,0,C3,C4,C5))									
14	8	inclination (rad)	0.026920249 =RADIANS(DMSDD(1,32,32.7))									
15	9	node-long (rad)	2.472126528 =RADIANS(C9-C12)									
16	10	sin(be)	0.070472641 =-COS(C14)*SIN(C13)+SIN(C14)*COS(C13)*SIN(C15)									
17	11	sub-Earth lat (deg)	4.041134615 =DEGREES(ASIN(C16))									
18	12	A (rad)	-2.473441682 =ATAN2(COS(C13)*COS(C15),-SIN(C13)*SIN(C14)-COS(C13)*COS(C14)*SIN(C15))									
19	13	A (deg)	-141.7177693 =DEGREES(C18)									
20	14	sub-Earth long (deg)	-364.8842831 =C19-C11									
21	15	sub-Earth long (deg)	355.1157169 =C20-360*INT(C20/360)									
22	16	sub-Earth long (deg)	-4.884283136 =IF(C21>180,C21-360,C21)									
23	17	C 1 (rad)	-0.021161235 =ATAN(COS(C15)*SIN(C14)/(COS(C13)*COS(C14)+SIN(C13)*SIN(C14)*SIN(C15)))									
24	18	obliquity (rad)	0.409163585 =RADIANS(OBLIQ(C3,C4,C5))									
25	19	C 2 (rad)	0.366418907 =ATAN(SIN(C24)*COS(RADIANS(C12)))/(SIN(C24)*SIN(C13)*SIN(RADIANS(C12))-COS(C24)*COS(C13))									
26	20	C (deg)	19.78180743 =DEGREES(C23+C25)									

Figure 50. Calculating the selenographic coordinates of the sub-Earth point and the position angle of the lunar equator.

Continuing our previous example: what were the selenographic coordinates of the Sun on 1 May 1988 at 0h UT? The equatorial horizontal parallax of the Moon was 55.952 arcminutes, the Sun–Earth distance was 1.0076 AU, and the true geocentric longitude of the Sun was 40.8437°.

Method	Example
1. Find the values of λ'_m and β' using the expressions given above.	λ'_m = 220.749 degrees β' = -0.008 degrees
2. Repeat the calculations in steps 1–10 in the previous example, using λ'_m and β' instead of λ_m and β .	b_s = 1.19 degrees y = -0.766 x = -0.643 A = 229.978 degrees l_s = 6.811 degrees
3. Subtract l_s from 90 and reduce to the range 0 to 360 by adding or subtracting 360. This is the colongitude.	colong. = 83.19 degrees

The *Astronomical Almanac* gives the coordinates as colongitude = 83.16° and latitude = 1.2°.

The spreadsheet for calculating the selenographic coordinates of the sub-solar point is shown in Figure 51, called Selenographic2. It follows the method given in the method table above except that it makes use of five spreadsheet functions defined in later sections to obtain the Sun's longitude (row 12; Section 47), the Moon's equatorial horizontal parallax (row 13; Section 65), the distance between the Sun and the Earth (row 14; Section 48), and the Moon's latitude and longitude (rows 15 and 16; Section 65). We have not provided a corresponding spreadsheet function for this calculation.

	A	B	C	D	E	F	G	H	I	J
1	Selenographic coordinates 2									
2										
3	Input	Greenwich date (day)	1		Output	sub-solar longitude	6.81	=ROUND(C27,2)		
4		Greenwich date (month)	5			sub-solar colongitude	83.19	=ROUND(C28,2)		
5		Greenwich date (year)	1988			sub-solar latitude	1.19	=ROUND(C22,2)		
6										
7	1	Julian date (days)	2447282.5	=CDJD(C3,C4,C5)						
8	2	T (centuries)	-0.11670089	=(C7-2451545)/36525						
9	3	long asc node (deg)	350.7599447	=125.044522-1934.136261*C8						
10	4	F	-56296.83349	=93.27191+483202.0175*C8						
11	5	F	223.1665139	=C10-360*INT(C10/360)						
12	6	Sun geocentric long (deg)	40.84263343	=SunLong(0,0,0,0,0,C3,C4,C5)						
13	7	Moon equ hor parallax (arc min)	55.95238522	=MoonHP(0,0,0,0,0,C3,C4,C5)*60						
14	8	Sun-Earth dist (AU)	1.00760326	=SunDist(0,0,0,0,0,C3,C4,C5)						
15	9	geocentric Moon lat (rad)	-0.053838535	=RADIANS(MoonLat(0,0,0,0,0,C3,C4,C5))						
16	10	geocentric Moon long (deg)	209.1175282	=MoonLong(0,0,0,0,0,C3,C4,C5)						
17	11	adjusted Moon long (deg)	220.7476113	=C12+180+(26.4*COS(C15)*SIN(RADIANS(C12-C16)))/(C13*C14))						
18	12	adjusted Moon lat (rad)	-0.000140054	=0.14666*C15/(C13*C14)						
19	13	inclination (rad)	0.026920249	=RADIANS(DMSdd(1,32,32.7))						
20	14	node-long (rad)	2.269143285	=RADIANS(C9-C17)						
21	15	sin(bs)	0.020755895	=-COS(C19)*SIN(C18)+SIN(C19)*COS(C18)*SIN(C20)						
22	16	sub-solar lat (deg)	1.189310602	=DEGREES(ASIN(C21))						
23	17	A (rad)	-2.269324173	=ATAN2(COS(C18)*COS(C20),-SIN(C18)*SIN(C19)-COS(C18)*COS(C19)*SIN(C20))						
24	18	A (deg)	-130.0226975	=DEGREES(C23)						
25	19	sub-solar long (deg)	-353.1892113	=C24-C11						
26	20	sub-solar long (deg)	6.810788676	=C25-360*INT(C25/360)						
27	21	sub-solar long (deg)	6.810788676	=IF(C26>180,C26-360,C26)						
28	22	sub-solar colong (deg)	83.18921132	=90-C27						

Figure 51. Calculating the selenographic coordinates of the Sun.

43 Atmospheric extinction

The light that reaches us on the surface of the Earth from heavenly bodies first has to pass through the atmosphere where some of it is scattered by dust, electrons, oxygen and nitrogen molecules, and other sundry particles. The amount of this **Rayleigh scattering** depends on the physical conditions in the atmosphere (it will be enhanced, for example, by extra dust from a volcanic eruption) and on the wavelength of the light. In general, the shorter wavelengths (blue) are scattered much more than the longer wavelengths (red); for this reason, the sky looks blue (we see the scattered light) and the apparent colour of a star observed from the Earth's surface is reddened. If we take the visual wavelengths as a whole, we can make a rough estimate of the amount of absorption to expect when the atmosphere is clear, from

$$\Delta m = \frac{0.2}{\cos z} \text{ magnitudes,}$$

where Δm is the quantity to be added to the **magnitude**, and z is the zenith angle ($z = 90^\circ - \text{altitude}$). For example, a planet whose altitude is 15° may appear dimmer by about 0.8 magnitudes in good conditions when the atmosphere is clear; in general this will be an underestimate since there are additional causes of absorption. The formula breaks down for zenith angles greater than about 85° .

The Sun

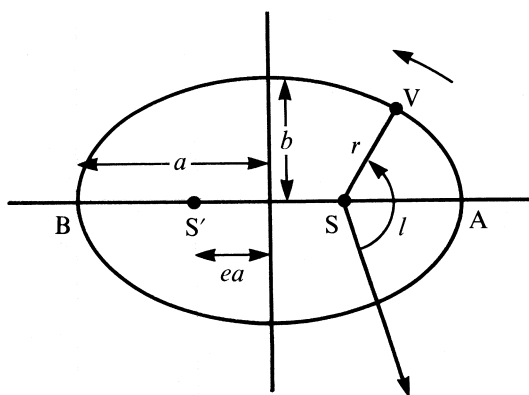
The nearest star to the Earth is the Sun, being some 91 million miles distant at its closest approach. The sunlight reaching us is already 8 minutes old when we see it, having taken this long to travel the radius of the Earth's orbit. Yet despite this distance the Sun is so huge that it appears as one of the largest celestial objects in the sky, equalled only by the Moon which, by coincidence, has more or less the same angular size. It is certainly the brightest. It dominates the Solar System, controlling the motions of the planets and supplying the energy needed for life on Earth. Although we always know by experience approximately where the Sun is in the sky, we often need to know its position more accurately as, for example, when we wish to calculate an eclipse or the orientation of a sundial. The next few sections deal with methods for calculating the Sun's orbit, distance from the Earth, apparent angular size, the times of sunrise and sunset, the solar elongations of other celestial bodies, the equation of time which you will need if you wish to set your watch by your sundial, and the duration of twilight.

44 Orbits

The motions of the planets around the Sun, and of the satellites about their planets, are all controlled by the action of gravity, that is by the mutual force of attraction between the masses. One of the consequences of the way this force varies with distance is that the planetary orbits trace out the forms of ellipses (Figure 52), geometrical shapes with well-known mathematical properties which enable us to calculate a planet's course precisely. You can imagine an ellipse as a squashed circle; in fact, a circle is a special case of an ellipse where the two **foci** or **foci**, S and S' , have moved together into the middle. The amount of squashing is measured by the **eccentricity**, e ; for a circle, $e = 0$, and the most flattened ellipses have values of e approaching 1. Most planetary orbits have eccentricities less than 0.1, so that their deviations from circular orbits are small. This is fortunate as it enables us to calculate planetary positions quite accurately by relatively simple methods.

All of the planetary orbits have the Sun at one focus, S . In Figure 52 the planet V moves in the direction of the arrow around the ellipse, its distance from the Sun varying from a minimum at A to a maximum at B . These points are called **perihelion** and **aphelion** respectively. The line joining the planet to the Sun, r , is called the **radius vector**, and the angle, l , it makes with a fixed direction in space defines the position of the planet in its orbit at any time. The size of the ellipse is completely defined by the **semi-major axis**, a , and the eccentricity. The length of the **semi-minor axis**, b , is obtained from these two quantities by the equation

$$b^2 = a^2(1 - e^2).$$



$$b^2 = a^2(1 - e^2)$$

Figure 52. An orbital ellipse.

45 The apparent orbit of the Sun

During the course of a year, the Earth moves in its own elliptical orbit around the Sun, making one complete revolution in about $365\frac{1}{4}$ days. Viewed from the Earth, it seems to us that the Sun is moving in orbit around the Earth and for the purposes of calculating the Sun's position it is convenient to regard this as the case. Hence we now assume that it is we who are at the focus and the Sun describes an ellipse about us. When the Sun is closest to the Earth, we say it is at **perigee** and when it is farthest away it is at **apogee**.

Since the plane which contains the Sun–Earth orbit defines the plane of the ecliptic, it is particularly easy to calculate the Sun's apparent motion as we do not have to worry about deviations from the ecliptic. Once we have calculated the ecliptic longitude, we have defined the Sun's position as the ecliptic latitude is zero.

46 Calculating the position of the Sun

The first thing to do is to define the epoch on which we shall base our calculations; we choose 2010 January 0.0 (JD = 2455 196.5). The Sun's mean ecliptic longitude at the epoch is $\epsilon_g = 279.557\,208$ degrees; this is the position it would have had if it had been moving in a circular orbit rather than an ellipse. The value of ϵ_g represents our starting point. We simply have to add on the correct number of degrees moved by the Sun since then (which may be negative if we are making the calculation for a date before the epoch), and to make due allowance for its elliptical motion, to find where it is at any other time. To do so we need two other constants: $\varpi_g = 283.112\,438$ degrees, the longitude of the Sun at perigee, and $e = 0.016\,705$, the eccentricity of the Sun–Earth orbit. If you wish to find these values for any other epoch, you can do so by using the equations

$$\begin{aligned}\epsilon_g &= 279.696\,6778 + 36\,000.768\,92T + 0.000\,302\,5T^2 \text{ degrees,} \\ \varpi_g &= 281.220\,8444 + 1.719\,175T + 0.000\,452\,778T^2 \text{ degrees,} \\ e &= 0.016\,751\,04 - 0.000\,041\,8T - 0.000\,000\,126T^2,\end{aligned}$$

where $T = (\text{JD} - 2415\,020.0)/36\,525$ and is the number of Julian centuries since 1900 January 0.5. Note that this is a different definition of T from that used in some other parts of the book.

We now imagine that the Sun moves in a circle around the Earth at a constant speed, rather than along the ellipse that it actually traces. We can easily calculate the angle, M_\odot , called the **mean anomaly**, through which this fictitious **mean Sun** has moved since it passed through perigee by

$$M_\odot = \frac{360}{365.242\,191}d \text{ degrees,}$$

where d is the number of days since perigee, because during the course of one tropical year of 365.242 191 days the Sun completes a circle of 360° . But rather than basing our calculations on the moment of perigee, we have decided for convenience to use the epoch 2010.0. Then if D is the number of days since the epoch, the mean anomaly is given by (Figure 53):

$$M_\odot = \frac{360}{365.242\,191}D + \epsilon_g - \varpi_g \text{ degrees,}$$

where ϵ_g and ϖ_g are the mean longitudes of the Sun at the epoch and perigee respectively (values given in Table 7).

ε_g	(ecliptic longitude at epoch 2010.0)	=	279.557 208 degrees
ϖ_g	(ecliptic longitude of perigee at epoch 2010.0)	=	283.112 438 degrees
e	(eccentricity of orbit at epoch 2010.0)	=	0.016 705
r_0	(semi-major axis)	=	$1.495\,985 \times 10^8$ km
θ_0	(angular diameter at $r = r_0$)	=	0.533 128 degrees

Table 7. Details of the Sun's apparent orbit at epoch 2010.0.

M_\odot refers to the motion of a mean Sun moving in a circle. We actually need the **true anomaly**, ν , which applies for the true motion of the Sun in an ellipse. This can be found from the **equation of the centre**, which is (to a sufficient accuracy for our purposes; see Figure 68):

$$\nu = M_\odot + \frac{360}{\pi} e \sin M_\odot,$$

where ν and M_\odot are expressed in degrees and $\pi = 3.141\,592\,7$. Having found ν , we simply add ϖ_g (see Figure 53) to get the longitude of the Sun, λ_\odot . Hence

$$\lambda_\odot = \nu + \varpi_g,$$

or

$$\lambda_\odot = \frac{360}{365.242\,191} D + \frac{360}{\pi} e \sin \left\{ \frac{360}{365.242\,191} D + \varepsilon_g - \varpi_g \right\} + \varepsilon_g.$$

Having calculated λ_\odot the method given in Section 27 may be used to find the right ascension and declination (remembering that the ecliptic latitude is zero because the Sun is in the ecliptic).

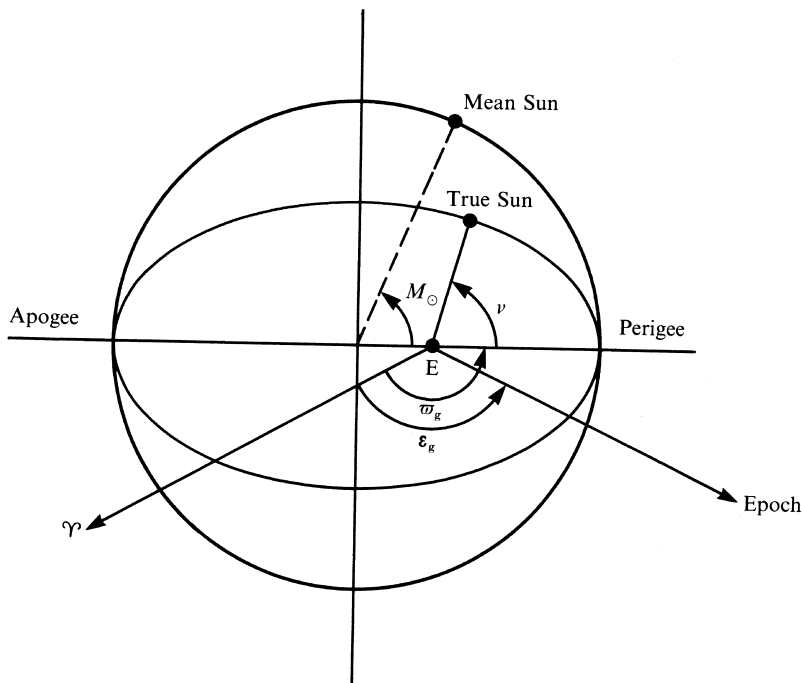


Figure 53. Defining the apparent orbit of the Sun.

Let us clarify all this by working out an example: what were the right ascension and declination of the Sun at 0 h UT on Greenwich date 27 July 2003?

Method	Example
1. Find the number of days since January 0.0 at the beginning of the year (§3).	27th July = 181 + 27 = 208
2. Add 365 days for every year since 2010 plus 1 extra day for every leap year (see Table 2). The result is D . (Note that we subtract the total in this case as 2003 is before 2010.)	D = 2 557 = -2 349 days
3. Calculate $N = \frac{360}{365.242191} D$; subtract (or add) multiples of 360 until N lies in the range 0 to 360.	N = 204.714 360 degrees
4. Find $M_{\odot} = N + \varepsilon_g - \varpi_g$ (Table 7). If the result is negative, add 360.	M_{\odot} = 201.159 131 degrees
5. Find $E_c = \frac{360}{\pi} e \sin M_{\odot}$ ($\pi = 3.141 592 7$ and e from Table 7).	E_c = -0.690 967 degrees
6. Find $\lambda_{\odot} = N + E_c + \varepsilon_g$. If the result is more than 360, subtract 360. This is the Sun's geocentric ecliptic longitude.	λ_{\odot} = 123.580 601 degrees
7. Now convert to right ascension and declination (§27). Remember that $\beta_{\odot} = 0$.	α_{\odot} = 8h 23m 34s δ_{\odot} = 19° 21' 10''

The *Astronomical Almanac* gives $\alpha = 8\text{h } 23\text{m } 33\text{s}$ and $\delta = 19^{\circ} 21' 16''$ so our result is really quite accurate. In general, we should find that we can calculate α to within about 10 s and δ to within a few arcminutes by this method. The inaccuracies arise because we have only used the first term in the **equation of the centre**, and we have not taken account of all sorts of tiny perturbations due to the influences of the other planets in the Solar System.

Figure 54 shows the spreadsheet for this calculation, called SunPos1. It follows the method outlined above, but we have cheated slightly in that we have defined the spreadsheet functions SunElong, SunPeri, and SunEcc to calculate the values of ε_g , ϖ_g , and e respectively using the formulas given above for the Greenwich calendar date supplied in the three arguments as day, month, year. In particular, we have used the values of ϖ_g and e for 27 July 2003 (see rows 20 and 22) and not the values given in Table 7, so the result is a bit more accurate. In the next section, we will see how to get even better accuracy.

File Edit View Insert Format Tools Data Window Help									
	A	B	C	D	E	F	G	H	I
1	Finding the position of the Sun (approximate method)								
2									
3	<i>Input</i>	local civil time (hour)	0		<i>Output</i>	Sun RA (hour)	8	=DHHour(C26)	
4		local civil time (min)	0			Sun RA (min)	23	=DHMin(C26)	
5		local civil time (sec)	0			Sun RA (sec)	33.73	=DHSec(C26)	
6		local date (day)	27			Sun dec (deg)	19	=DDDdeg(C27)	
7		local date (month)	7			Sun dec (min)	21	=DDMin(C27)	
8		local date (year)	2003			Sun dec (sec)	14.33	=DDSec(C27)	
9		daylight saving	0						
10		zone correction	0						
11									
12	1	Greenwich date (day)	27	=LctGDay(C3,C4,C5,C9,C10,C6,C7,C8)					
13	2	Greenwich date (month)	7	=LctGMonth(C3,C4,C5,C9,C10,C6,C7,C8)					
14	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C9,C10,C6,C7,C8)					
15	4	UT (hours)	0	=LctUT(C3,C4,C5,C9,C10,C6,C7,C8)					
16	5	UT (days)	0	=C15/24					
17	6	JD (days)	2452847.5	=CDJD(C12,C13,C14)+C16					
18	7	D (days)	-2349	=C17-CDJD(0,1,2010)					
19	8	N (deg)	-2315.28564	=360*C18/365.242191					
20	10	M (deg)	-2318.840869	=C19+SunElong(0,1,2010)-SunPeri(C12,C13,C14)					
21	11	M (deg)	201.1591307	=C20-360*INT(C20/360)					
22	12	E c (deg)	-0.690963565	=360*SunEcc(0,1,2010)*SIN(RADIANS(C21))/PI()					
23	13	L s (deg)	-2036.419395	=C19+C22+SunElong(0,1,2010)					
24	14	L s (deg)	123.5806048	=C23-360*INT(C23/360)					
25	15	RA (deg)	125.8905256	=ECRA(C24,0,0,0,0,0,C12,C13,C14)					
26	16	RA (hours)	8.392701704	=DDDh(C25)					
27	17	dec (deg)	19.35398081	=ECDec(C24,0,0,0,0,0,C12,C13,C14)					

Figure 54. Finding the position of the Sun by an approximate method.

47 Calculating orbits more precisely

In this section we discover how to find the true anomaly, v , by a slightly more accurate method.[†] For most purposes the accuracy of the simpler method given in Section 46 will suffice, but if you have a good programmable calculator, or access to a computer, you may find this section useful.

As before, we find the mean anomaly, but this time we use the equations for ε_g and ϖ_g given in the previous section. This requires us to find the number of Julian centuries since 1900 January 0.5, T , and to use that value in the formulas. We calculate the mean anomaly from

$$M_{\odot} = \varepsilon_g - \varpi_g,$$

where M_{\odot} , ε_g and ϖ_g are all expressed in degrees. Then we find the **eccentric anomaly**, E , which is defined in Figure 55. E is given by **Kepler's equation**, named after the famous German astronomer, Johannes Kepler (1571–1630), who made a detailed study of the planets:

$$E - e \sin E = M_{\odot} \text{ radians},$$

where this time E and M_{\odot} are expressed in circular measure, or **radians**. Unfortunately, this equation is not easily solved, but with the aid of an iterative routine a numerical solution may be reached. Such a routine is given in Routine R2 and is appropriate for values of e less than about 0.1 (but see Section 61 for comets, which have larger values of e). You can recast the equation to use degrees rather than radians by noting that 1 radian is $180/\pi$ degrees, so if E and M_{\odot} are expressed in degrees, we have

$$E - \frac{180}{\pi} e \sin E = M_{\odot} \text{ degrees}.$$

The value of e for the Sun's orbit is given in Table 7.

Having found a solution to Kepler's equation, we can find the true anomaly, v , from

$$\tan\left(\frac{v}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2},$$

and then, as in the previous section, the ecliptic longitude is given by

$$\lambda_{\odot} = v + \varpi_g \text{ degrees}.$$

[†]Figure 68 (Section 56) shows the error introduced by using only the first term in the equation of the centre.

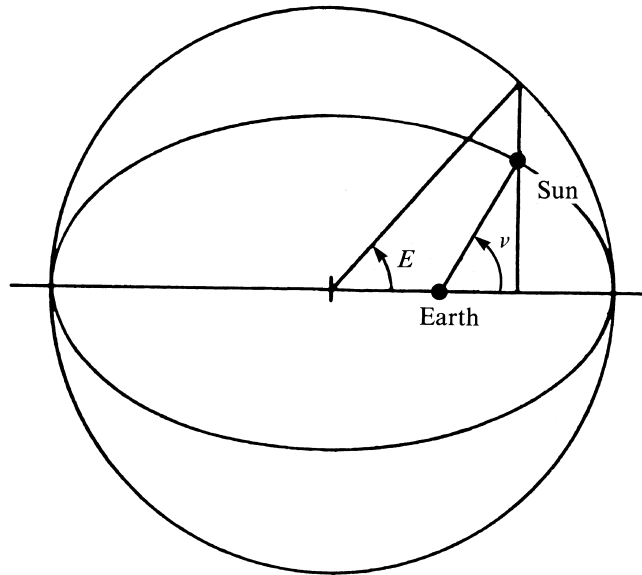


Figure 55. True and eccentric anomalies.

Routine R2: To find a solution to Kepler's equation $E - e \sin E = M$ for small values of e . All angles are expressed in radians.

1. First guess, put $E = E_0 = M$.
2. Find the value of $\delta = E - e \sin E - M$.
3. If $^\dagger |\delta| \leq \varepsilon$ go to step 6.
If $|\delta| > \varepsilon$ proceed with step 4.
 ε is the required accuracy ($= 10^{-6}$ radians).
4. Find $\Delta E = \delta / (1 - e \cos E)$.
5. Take new value $E_1 = E - \Delta E$. Go to step 2.
6. The present value of E is the solution, correct to within ε of the true value.

$^\dagger |\delta|$ is the absolute value of δ .

Let us use this method to solve another example: what were the right ascension and declination of the Sun on Greenwich date 27 July 1988 at 0 h UT?

Method	Example
1. Find the Julian date (§4).	JD = 2 447 369.5
2. Subtract 2 415 020.0 and divide by 36 525. The result is T .	T = 0.885 681 centuries
3. Find ϵ_g , ϖ_g and e using the formulas given in the previous section. Reduce to the range 0 to 360 by adding or subtracting multiples of 360.	ϵ_g = 124.895 390 degrees ϖ_g = 282.743 840 degrees e = 0.016 714
4. Find $M_{\odot} = \epsilon_g - \varpi_g$. Add or subtract 360 to bring into the range 0 to 360. Multiply by $\frac{\pi}{180}$ to convert to radians.	M_{\odot} = 202.151 550 degrees M_{\odot} = 3.528 210 radians
5. Use Routine R2 to find the eccentric anomaly. Divide by $\frac{\pi}{180}$ to convert to degrees.	E = 3.522 004 radians E = 201.795 968 degrees
6. Find $\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$.	$\tan \frac{v}{2}$ = -5.281 459
7. Take the inverse tan and multiply by 2 to find v . Add 360 if negative.	v = 201.443 110 degrees
8. Find $\lambda_{\odot} = v + \varpi_g$. If the result is more than 360, subtract 360. This is the Sun's geocentric ecliptic longitude.	λ_{\odot} = 124.186 950 degrees
9. Now convert to right ascension and declination (§27). Remember that $\beta_{\odot} = 0$.	α_{\odot} = 8h 26m 4s δ_{\odot} = 19° 12' 43''

The *Astronomical Almanac* gives $\alpha = 8^{\text{h}} 26^{\text{m}} 3^{\text{s}}$ and $\delta = 19^{\circ} 12' 52''$.

The spreadsheet for a more precise calculation of the position of the Sun is shown in Figure 56. It incorporates the method given above, together with many correction terms for slight perturbations to the Earth's orbit, all of which are calculated by a single spreadsheet function SunLong (row 15). This returns the Sun's ecliptic longitude as seen at a given local calendar date and local civil time. It takes eight arguments, namely the local civil time in hours, minutes and seconds, the daylight saving correction and time zone offset, both in hours, and the local calendar date as day, month and year.

	A	B	C	D	E	F	G	H	I	J
1	Finding the Position of the Sun (more precise method)									
2										
3	Input	local civil time (hour)	0		Output	Sun RA (hour)	8	=DHHour(C17)		
4		local civil time (min)	0			Sun RA (min)	26	=DHMin(C17)		
5		local civil time (sec)	0			Sun RA (sec)	3.83	=DHSec(C17)		
6		local date (day)	27			Sun dec (deg)	19	=DDDeg(C18)		
7		local date (month)	7			Sun dec (min)	12	=DDMin(C18)		
8		local date (year)	1988			Sun dec (sec)	49.72	=DDSec(C18)		
9		daylight saving	0							
10		zone correction	0							
11										
12	1	Gyear	27	=LctGDay(C3,C4,C5,C9,C10,C6,C7,C8)						
13	2	Gmonth	7	=LctGMonth(C3,C4,C5,C9,C10,C6,C7,C8)						
14	3	Gyear	1988	=LctGYear(C3,C4,C5,C9,C10,C6,C7,C8)						
15	4	Sun's ecliptic longitude (deg)	124.1873516	=SunLong(C3,C4,C5,C9,C10,C6,C7,C8)						
16	5	RA (deg)	126.515956	=ECRA(C15,0,0,0,0,0,C12,C13,C14)						
17	6	RA (hours)	8.434397066	=DDDh(C16)						
18	7	dec (deg)	19.21381243	=ECDec(C15,0,0,0,0,0,C12,C13,C14)						

Figure 56. Finding the position of the Sun by a more precise method.

48 Calculating the Sun's distance and angular size

Having found the true anomaly, v , by the method of Sections 46 or 47, we can easily calculate the Sun–Earth distance, r , and the Sun's angular size (i.e. its angular diameter), θ . The formulas are:

$$r = r_0 \left(\frac{1 - e^2}{1 + e \cos v} \right),$$

$$\theta = \theta_0 \left(\frac{1 + e \cos v}{1 - e^2} \right),$$

where r_0 is the semi-major axis, θ_0 is the angular diameter when $r = r_0$, and e is the eccentricity of the orbit. These constants are given in Table 7. Continuing the example of Section 47 we can find r and θ for the Sun on Greenwich date 27 July 1988 at 0 h UT.

Method	Example
1. Find the true anomaly, v (§§46 or 47).	$v = 201.443\ 110$ degrees
2. Find $f = \frac{1+e \cos v}{(1-e^2)}$. (See Table 7 for e .)	$f = 0.984\ 726$
3. Then $r = \frac{r_0}{f}$. (See Table 7 for r_0 .)	$r = 1.519\ 189 \times 10^8$ km
4. And $\theta = f\theta_0$. (See Table 7 for θ_0 .)	$\theta = 0^\circ\ 31'\ 30''$

The *Astronomical Almanac* gives $\theta = 0^\circ\ 31'\ 30''$ and, in general, we should be within a few arcseconds of the correct value. It is interesting to note that the Sun's light took r/c seconds to reach us, where $c = 3 \times 10^5$ km s⁻¹. In this case the light travel time was 506 seconds, during which interval the Sun moved

about 21 arcseconds. Strictly speaking, we should subtract this from the calculated position to find the Sun's apparent position.

Spreadsheet Sundist (Figure 57) also shows how to make this calculation. We have cheated a bit by using a function, called `SunTrueAnomaly` (row 15), to find the Sun's true anomaly without going through the fuss of solving Kepler's equation etc. separately. This function takes the following arguments: the local civil time in hours, minutes and seconds, the daylight saving correction and time zone offset in hours, and the local calendar date as day, month and year. We have also used the function `SunEcc` (row 17; defined in Section 46) to get the eccentricity of the Sun's apparent orbit about the Earth.

We have provided spreadsheet functions `SunDist` and `SunDia` to make these calculations more simply. The functions return the values of the Sun's distance from the Earth in astronomical units (multiply by r_0 to get kilometres), and the Sun's angular diameter in decimal degrees respectively. Both functions take the same arguments as `SunTrueAnomaly` (see above). Thus, having saved a copy, you could modify the spreadsheet `Sundist` as follows: delete rows 12 to 20 inclusive, and insert the following spreadsheet formulas into cells H3 to H6:

=1.495985E8*`SunDist`(C3,C4,C5,C9,C10,C6,C7,C8)[†]

=`DDDeg`(`SunDia`(C3,C4,C5,C9,C10,C6,C7,C8))

=`DDMin`(`SunDia`(C3,C4,C5,C9,C10,C6,C7,C8))

=`DDSec`(`SunDia`(C3,C4,C5,C9,C10,C6,C7,C8)).

	A	B	C	D	E	F	G	H	I	J
1	Sun's distance and angular size									
2										
3	Input	local civil time (hour)	0	Output		Sun's dist (km)	1.519201E+08	= <code>ROUND</code> (C19,-2)		
4		local civil time (min)	0			Sun's ang size (deg)	0	= <code>DDDeg</code> (C20)		
5		local civil time (sec)	0			Sun's ang size (min)	31	= <code>DDMin</code> (C20)		
6		local date (day)	27			Sun's ang size (sec)	29.93	= <code>DDSec</code> (C20)		
7		local date (month)	7							
8		local date (year)	1988							
9		daylight saving	0							
10		zone correction	0							
11										
12	1	Gyear	27	= <code>LctGDay</code> (C3,C4,C5,C9,C10,C6,C7,C8)						
13	2	Gmonth	7	= <code>LctGMonth</code> (C3,C4,C5,C9,C10,C6,C7,C8)						
14	3	Gyear	1988	= <code>LctGYear</code> (C3,C4,C5,C9,C10,C6,C7,C8)						
15	4	true anomaly (deg)	-158.556892	= <code>SunTrueAnomaly</code> (C3,C4,C5,C9,C10,C6,C7,C8)						
16	5	true anomaly (rad)	-2.767339817	= <code>RADIANS</code> (C15)						
17	6	eccentricity	0.01671392	= <code>SunEcc</code> (C12,C13,C14)						
18	7	f	0.984718087	=(1+C17* <code>COS</code> (C16))/(1-C17* <code>C17</code>)						
19	8	r (km)	151920130.3	=149598500/C18						
20	9	theta (deg)	0.524980784	=C18*0.533128						

Figure 57. Finding the Sun's distance and angular size.

[†]1.495985E8 is the number $1.495\,985 \times 10^8$

49 Sunrise and sunset

In Section 33 we found how to calculate the rising and setting times of any celestial object whose equatorial coordinates were known. We have calculated the right ascension and declination of the Sun (Sections 46 and 47) so that we can apply the same method to find the times of sunrise and sunset. The problem is complicated, however, by the fact that the Sun is in continual motion along the ecliptic, and its equatorial coordinates are therefore continuously changing. The values of α and δ we have calculated are correct only for the time we have chosen. (In the example of Section 46, this time was the midnight between 26 July and 27 July. By the time the Sun had risen the next morning it had moved by about a quarter of a degree from its midnight position, and by sunset about three quarters of a degree.) Provided that we do not require high accuracy in our calculations, we can ignore the Sun's motion and simply take the position at midday as correct for both sunrise and sunset. The results are then within a few minutes of their correct values.

Further refinements include taking account of refraction by the Earth's atmosphere (Section 37) and geocentric parallax (Section 38). We must also consider the finite diameter of the Sun's disc; times of sunrise and sunset are usually quoted as those appropriate to the upper limb.

For our example, let us calculate the times of sunrise and sunset (upper limb) over a level horizon at sea-level on 10 March 1986, as observed from Boston, Massachusetts, at longitude 71.05° W and latitude 42.37° N. We shall take the Sun's angular diameter to be 0.533 degrees, its horizontal parallax to be 8.79 arcseconds, and the refraction due to the atmosphere as 34 arcminutes and, having added on half of the Sun's angular diameter and a small correction for parallax, we arrive at a total vertical shift at the horizon of the upper limb of 0.833 333 degrees. The time zone correction (Section 9) is -5 hours.

Method	Example
1. Calculate the Sun's position at midday (§§46 or 47; we've used SunPos1).	$\alpha_{\odot} = 23.375999$ hours $\delta_{\odot} = -4.034153$ degrees
2. Calculate the corresponding rising and setting times (§33; we've used RiseSet).	$UT_r = 11\text{h } 6\text{m}$ $UT_s = 22\text{h } 43\text{m}$
3. Convert these times to local civil times (§10).	$LCT_r = \mathbf{6h } \mathbf{6m}$ $LCT_s = \mathbf{17h } \mathbf{43m}$

The *Old Farmer's Almanac* (see the Bibliography on page 208) lists these times as $EST_r = 6\text{h } 5\text{m}$ am, and $EST_s = 5\text{h } 45\text{m}$ pm. In general, we should be within a few minutes of the actual times using this approximate method.

The spreadsheet shown in Figure 58, called **SunRS**, performs this calculation rather more precisely. It uses the five spreadsheet functions called **SunriseLCT**, **SunsetLCT**, **SunriseAz**, **SunsetAz** and **eSunRS**, which calculate the local civil times of sunrise and sunset, their azimuths, and a status word respectively. Each has the same seven arguments, namely the local calendar date as day, month, year, the daylight saving correction and time zone offset (both in hours), and the geographical longitude and latitude (both in degrees). These functions start out by performing the calculation based on the Sun's position at midday, but then use the calculated times of sunrise and sunset to find new positions for the Sun at those times, before recalculating the times of sunrise and sunset using the new Sun positions. The result should be accurate to within a minute of time. If an error is detected, such that the Sun never rises or never sets, as can happen near the poles (try a latitude of $+ \text{ or } - 89^\circ$), the function **eSunRS** returns the status word **** never rises**, or **** circumpolar**, respectively, otherwise it returns with **OK**. If not **OK**, the other functions return with -99 , a

disallowed value of time (0 to 24 hours) or angle (0 to 360 degrees). The result of eSunRS is used to gate the writing of the output values in cells H3 to H8 so that numerical results only appear if they are valid. You can see that the times of rising and setting calculated by spreadsheet SunRS agree with the values given in the *Old Farmer's Almanac* in this example.

	A	B	C	D	E	F	G	H	I	J	K
1	Sunrise and sunset										
2											
3	<i>Input</i>	local date (day)	10		<i>Output</i>	local sunrise (hour)	6	=IF(C13="OK",DHHour(C14), "")			
4		local date (month)	3			local sunrise (min)	5	=IF(C13="OK",DHMin(C14), "")			
5		local date (year)	1986			local sunset (hour)	17	=IF(C13="OK",DHHour(C15), "")			
6		daylight saving (hours)	0			local sunset(min)	45	=IF(C13="OK",DHMin(C15), "")			
7		zone correction (hours)	-5			azimuth of sunrise (deg)	94.83	=IF(C13="OK",ROUND(C16,2), "")			
8		geographical long (deg)	-71.05			azimuth of sunset (deg)	265.43	=IF(C13="OK",ROUND(C17,2), "")			
9		geographical lat (deg)	42.37			status	OK	=C13			
10											
11	1	local sunrise (hours)	6.0854363	=SunriseLCT(C3,C4,C5,C6,C7,C8,C9)							
12	2	local sunset (hours)	17.7427973	=SunsetLCT(C3,C4,C5,C6,C7,C8,C9)							
13	3	Sun rise/set status	OK	=eSunRS(C3,C4,C5,C6,C7,C8,C9)							
14	4	adjusted sunrise (hours)	6.0937693	=C11+0.008333							
15	5	adjusted sunset (hours)	17.7511303	=C12+0.008333							
16	6	azimuth of sunrise (deg)	94.8266327	=SunriseAz(C3,C4,C5,C6,C7,C8,C9)							
17	7	azimuth of sunset (deg)	265.430869	=SunsetAz(C3,C4,C5,C6,C7,C8,C9)							

Figure 58. Calculating the circumstances of sunrise and sunset.

50 Twilight

Whenever the Sun is less than a certain amount below the horizon, after sunset or before sunrise, the light scattered by the upper atmosphere illuminates the Earth. The intensity of the scattered sunlight falls sharply as the Sun dips lower below the horizon, and is almost negligible by the time the Sun's zenith angle reaches 108° , i.e. 18° below the horizon. The period after sunset or before sunrise during which the Sun's zenith angle is less than an agreed amount is called **twilight**: 96° for **civil twilight**, 102° for **nautical twilight**, and 108° for **astronomical twilight**.

We can calculate the duration of morning or evening twilight quite simply. We first find the hour angles, H , of the Sun at rising or setting by

$$\cos H = -\tan \phi \tan \delta,$$

where ϕ is the geographic latitude and δ is the Sun's declination. Then we calculate its hour angle, H' , at the point when its zenith angle is $\theta_t = 96, 102$, or 108 degrees using

$$\cos H' = \frac{\cos \theta_t - \sin \phi \sin \delta}{\cos \phi \cos \delta}.$$

Then the duration of twilight in sidereal hours is simply

$$t = \frac{H' - H}{15} \text{ hours.}$$

We should multiply this by 0.9973 to obtain the equivalent time interval in terms of normal (UT) hours.

During the course of one year the Sun's declination ranges from about -23.5° to $+23.5^\circ$. Latitudes north of $+48.5^\circ$ or south of -48.5° will therefore experience a twilight which lasts all night during the summer.

For example, on the latitude 60° N the twilight lasts all night from about 23 April until 22 August. When this is so, the value of $\cos H'$ lies outside its allowed range of -1 to $+1$, and your calculator should respond with 'error' if you attempt inverse cos. For example, let us calculate the beginning of morning astronomical twilight and the end of evening astronomical twilight on 7 September 1979 for an observer at latitude 52° N and longitude 0° .

Method	Example
1. Calculate the declination of the Sun; its value at midday will do as this is not a very precise calculation. We have used SunPos1.	$\delta_\odot = 6.189592$ degrees
2. Find the Sun's hour angle at setting from $H = \cos^{-1} \{-\tan \phi \tan \delta\}$.	$H = 97.979045$ degrees
3. Find the Sun's hour angle at $z = 108^\circ$ from $H' = \cos^{-1} \left\{ \frac{\cos 108 - \sin \phi \sin \delta}{\cos \phi \cos \delta} \right\}$.	$H' = 130.066840$ degrees
4. Calculate $t = \frac{H' - H}{15}$ hours and multiply by 0.9973 to convert to interval of UT.	$t = 2.133411$ UT hours
5. Add or subtract this from the time of sunset or sunrise respectively to find the end of evening twilight or the start of morning twilight. We used SunRS.	$UT_s = 18.582658$ hours $UT_r = 5.338170$ hours evening twilight ends at 20h 43m UT morning twilight begins at 3h 12m UT

The *Astronomical Ephemeris* gives these times as 3h 17m and 20h 37m, having taken due account of the Sun's changing coordinates throughout the day, as well as refraction and parallax.

You can make a more-accurate estimate of the time of the start or end of twilight using the spreadsheet Twilight shown in Figure 59. To do this, we need to take account of the change in the Sun's position between the start of morning twilight and end of evening twilight, in effect treating the problem in the same way as for sunrise and sunset. In the spreadsheet we have used the spreadsheet functions TwilightAMLCT, TwilightPMLCT and eTwilight to carry out the calculation behind the scenes, returning respectively the local civil times of the start of morning twilight, the end of evening twilight, both in hours, and a status word (really a string) of OK, ** lasts all night, or ** Sun too far below horizon. All three functions take the same eight arguments which are the local date as day, month, year, the daylight saving correction and time zone offset in hours, the geographical longitude and latitude in degrees (W and S are negative), and a switch in the form of the single character C, N, or A to specify whether civil, nautical, or astronomical twilight is to be calculated respectively. You can see that the result produced by the spreadsheet agrees exactly with the *Astronomical Ephemeris* in this case.

	A	B	C	D	E	F	G	H	I	J	K
1	Morning and evening twilight										
2											
3	<i>Input</i>	local date (day)	7		<i>Output</i>	am twilight begins (hour)	3	=IF(C14="OK",DHHour(C15),"")			
4		local date (month)	9			am twilight begins (min)	17	=IF(C14="OK",DHMin(C15),"")			
5		local date (year)	1979			pm twilight ends (hour)	20	=IF(C14="OK",DHHour(C16),"")			
6		daylight saving (hours)	0			pm twilight ends(min)	37	=IF(C14="OK",DHMin(C16),"")			
7		zone correction (hours)	0			status	OK	=C14			
8		geographical long (deg)	0								
9		geographical lat (deg)	52								
10		twilight type (C, N, A)	A								
11											
12	1	start of am twilight (hours)	3.2844351 =TwilightAMLCT(C3,C4,C5,C6,C7,C8,C9,C10)								
13	2	end of pm twilight (hours)	20.6228 =TwilightPMLCT(C3,C4,C5,C6,C7,C8,C9,C10)								
14	3	twilight status	OK =eTwilight(C3,C4,C5,C6,C7,C8,C9,C10)								
15	4	adjusted am start time	3.2927681 =C12+0.008333								
16	5	adjusted pm end time	20.631133 =C13+0.008333								

Figure 59. Calculating twilight.

51 The equation of time

The apparent motion of the Sun along the plane of the ecliptic is not regular. This is rather surprising at first because we are used to thinking of the Sun as a time-keeper by which we can set our watches. In fact, it is really quite a bad time-keeper by quartz-crystal watch standards; it can at any moment in the year be as much as 16 minutes out compared with a regular clock whose time increases at a uniform rate. The Sun's non-uniform motion is caused by two effects:

- (a) The Earth's orbit is not circular but elliptical. Its speed therefore varies throughout the year, being maximum at perihelion and minimum at aphelion. Viewed from the Earth, the Sun's speed in its apparent orbit varies from a maximum at perigee to a minimum at apogee.
- (b) The Earth's axis is tilted at an angle to the perpendicular of the plane of the ecliptic. The angle is the same as the obliquity of the ecliptic, $\epsilon = 23^\circ 26'$ (Section 27). The Earth acts as a huge gyroscope keeping its rotation axis in a fixed direction in space, making the Sun's altitude at noon vary throughout the year from a maximum at midsummer to a minimum at midwinter. This variation in altitude has a small effect on the time of transit of the Sun.

To take account of the Sun's apparent aberrations from perfect time-keeping we imagine a fictitious Sun, called the **mean Sun**, which moves at a uniform rate along the equator. Noon is defined to be the instant when the mean Sun crosses the meridian, and two successive passages of the mean Sun across it define the length of the day. Time measured by the mean Sun corresponds to UT.

The difference between the real Sun time and the mean Sun time is called the **equation of time**. Hence

$$\Delta t = \text{RST} - \text{MST},$$

where Δt is the value of the equation of time, MST is the mean Sun time and RST is the real Sun time. It is plotted in Figure 60.

We can calculate the equation of time on any day quite easily by first finding the Sun's right ascension at noon and then, remembering that the right ascension is the sidereal time at transit, converting the right ascension to UT. The result is the UT at which the real Sun transits; by subtracting 12h 00m from this, the UT at which the mean Sun transits, we have the value of the equation of time.

For example, what was the value of the equation of time on 27 July 2010? (Remember that noon is July 27.5.)

<i>Method</i>	<i>Example</i>
1. Calculate the right ascension of the Sun in decimal hours. We have used SunPos2 (§47) to do this.	$\alpha_{\odot} = 8.446350$ hours
2. Taking this as the GST, convert it to UT (§13).	UT = 12.108755 hours
3. Subtract 12 and convert to hours, minutes and seconds (§8). This is the value of the equation of time.	$\Delta t = \mathbf{6m\ 32s}$

If you have a sundial, you will need the equation of time to convert the sundial's reading into mean time (UT).

Figure 61 shows the spreadsheet for finding the value of the equation of time. We have also provided a spreadsheet function called **EqOfTime** which calculates this value in hours for the Greenwich date specified

in the three arguments as day, month and year. Using this function, you could delete rows 7–10 of the spreadsheet of Figure 61 and insert the following two spreadsheet formulas into cells H3 and H4:

=DHMin(EqOfTime(C3,C4,C5))

=DHSec(EqOfTime(C3,C4,C5)).

Remember to save a copy first in case you need it later.

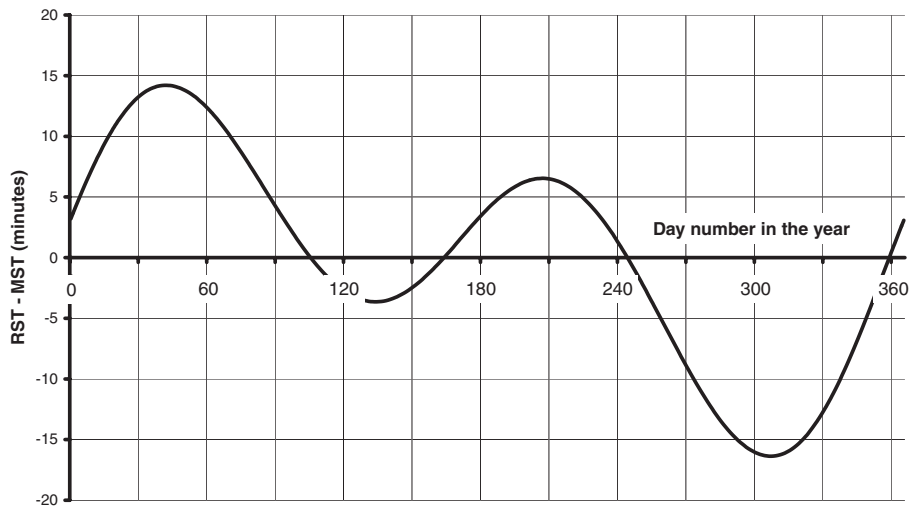


Figure 60. The equation of time. This diagram was made in *Excel* using the functions described in this section.

	A	B	C	D	E	F	G	H	I	J
1		The equation of time								
2										
3	Input	Greenwich date (day)	27		Output	equation of time (min)	6	=DHMin(C10)		
4		Greenwich date (month)	7			equation of time (sec)	31.52	=DHSec(C10)		
5		Greenwich date (year)	2010							
6										
7	1	Sun longitude (deg)	124.3626189	=SunLong(12,0,0,0,0,C3,C4,C5)						
8	2	Sun RA (hours)	8.446349638	=DDDh(ECRA(C7,0,0,0,0,0,C3,C4,C5))						
9	3	equivalent UT (hours)	12.10875487	=GSTUT(C8,0,0,C3,C4,C5)						
10	4	equation of time (hours)	0.108754868	=C9-12						

Figure 61. Finding the value of the equation of time.

52 Solar elongations

The **solar elongation** of a planet (or other celestial object) is the angle between the lines of sight from the Earth to the Sun and from the Earth to the planet. It is quite often necessary to find the value of this angle, as it tells us how close to the Sun we should look to see the planet and hence whether it will be visible. The formula for the solar elongation, ε , is

$$\varepsilon = \cos^{-1} \left\{ \sin \delta_p \sin \delta_{\odot} + \cos (\alpha_p - \alpha_{\odot}) \cos \delta_p \cos \delta_{\odot} \right\} \text{ degrees,}$$

where α_{\odot} and δ_{\odot} are the right ascension and declination of the Sun, and α_p and δ_p are the right ascension and declination of the planet.

On Greenwich date 27 July 2010 at 8 pm UT, the equatorial coordinates of the planet Mercury were found to be $\alpha_p = 10^{\text{h}} 6^{\text{m}} 45^{\text{s}}$ and $\delta_p = 11^{\circ} 57' 27''$. What was the solar elongation?

Method	Example
1. Calculate the right ascension and declination of the Sun in decimal degrees. We have used SunPos2 to do this (§47).	$\alpha_{\odot} = 127.022544$ degrees $\delta_{\odot} = 19.092414$ degrees
2. Convert α_p and δ_p into decimal degrees (§§7 and 21).	$\alpha_p = 151.687500$ degrees $\delta_p = 11.957500$ degrees
3. Find $\varepsilon = \cos^{-1} \left\{ \sin \delta_p \sin \delta_{\odot} + \cos (\alpha_p - \alpha_{\odot}) \cos \delta_p \cos \delta_{\odot} \right\}$.	$\varepsilon = 24.78$ degrees

The spreadsheet for making this calculation is shown in Figure 62. You will see that, in cell C9, we have specified the day part of the date as 27.833333 to represent 8 pm on the 27th. We have not supplied a spreadsheet function to make the same calculation.

	A	B	C	D	E	F	G	H	I	J
1	Solar elongation									
2										
3	Input	RA (hour)	10		Output	Solar elongation (deg)	24.78	=ROUND(C16,2)		
4		RA (min)	6							
5		RA (sec)	45							
6		dec (deg)	11							
7		dec (min)	57							
8		dec (sec)	27							
9		Greenwich date (day)	27.833333							
10		Greenwich date (month)	7							
11		Greenwich date (year)	2010							
12										
13	1	Sun longitude (deg)	124.6810398	=SunLong(0,0,0,0,0,C9,C10,C11)						
14	2	Sun RA (hours)	8.468169748	=DDDH(ECRA(C13,0,0,0,0,0,C9,C10,C11))						
15	3	Sun dec (deg)	19.09242158	=ECDec(C13,0,0,0,0,0,C9,C10,C11)						
16	4	solar elongation (deg)	24.78195627	=Angle(C14,0,0,C15,0,0,C3,C4,C5,C6,C7,C8,"H")						

Figure 62. Finding the solar elongation.

The planets, comets and binary stars

An observer looking up at the night sky from the surface of the Earth sees an unchanging pattern of stars revolving slowly about the pole as the Earth spins on its axis. So great are the distances to the stars that the changing position of the Earth as it travels along its orbit around the Sun causes hardly any movement in the pattern, even in the course of six months. There are a few objects, however, which do appear to move a great deal with respect to this fixed background of stars. The objects are members of our Solar System, the planets, the asteroids and the comets. Eight major planets have been identified so far which, in order of increasing distance from the Sun, are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. (Pluto was originally classified as a planet, but is now considered the largest member of another grouping of objects called the Kuiper belt.) These, together with other members of the Solar System, are all bound by the gravitational field of the Sun so that instead of moving off into space in different directions they are constrained to follow elliptical orbits about it. Their apparent motions in the sky are complicated because they are relatively close to us so that the position of the Earth in its own orbit needs to be taken into account. The next few sections contain methods for calculating the positions, angular sizes, distances, phases and brightnesses of the major planets. There are also sections describing how to calculate the orbit of a comet and the orbit of a binary star.

53 The planetary orbits

Each planet in our Solar System describes an elliptical orbit about the Sun with the Sun at a focus of the ellipse. We discovered how to calculate the Sun–Earth orbit in Sections 44 to 47. This was a particularly simple case since the plane of the orbit defined the plane of the ecliptic; the ecliptic latitude was therefore always zero and the fundamental direction, the first point of Aries, was in the orbital plane. The other planets, however, do not move in the plane of the ecliptic but describe orbits inclined at small angles to it. Figure 63 shows the situation.

The Sun, S , is at the centre of the diagram and you are to imagine that you are looking at the path of a planet around the Sun from a great distance. The orbit of the planet is the small shaded ellipse N_1AP . The perihelion is marked A and the planet's present position is marked P . That part of the orbit which lies above the ecliptic is shown with solid lines, while that lying below it is shown with dashed lines. The large sphere is centred on the Sun and the plane of the planet's orbit is projected to cut the sphere along the circle $N'_1A'P'N'_2$. Here A' is the projection of A onto the sphere, P' the projection of P and so forth. Also shown in the diagram is the plane of the ecliptic $\Upsilon N'_1N'_2$, which contains the direction of Υ , the first point of Aries.

The planet moves along its orbit in the direction of the arrow. The point N_1 where it rises out of the plane of the ecliptic is called the **ascending node**. N_2 , the point where it descends below the plane of the ecliptic, is called the **descending node**. Angles in the orbital plane are measured from the ascending node while longitudes are reckoned from the direction Υ which is not in the orbital plane. Thus the perihelion is at an angle ω to the node (the 'argument' of the perihelion) and the present position of the planet is at an angle $\omega + \nu$. The corresponding longitudes are $\omega + \Omega$ and $\omega + \nu + \Omega$, where Ω is the longitude of the ascending node. Note that longitudes are the sum of two angles in different planes.

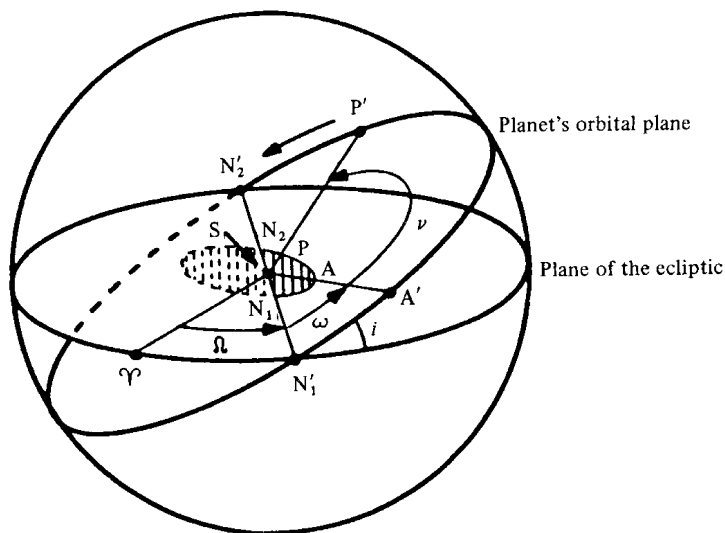


Figure 63. Defining the orbit of a planet.

54 Calculating the coordinates of a planet

Our calculation will proceed in three steps. The first is to calculate the position of the planet in its own orbital plane exactly as we did for the Sun–Earth orbit in Section 46. In the second step we will project the planet's calculated position onto the plane of the ecliptic and hence find its ecliptic longitude and latitude referred to the Sun (**heliocentric coordinates**). The third step will involve transforming from the Sun to the Earth to find the ecliptic coordinates referred to the Earth, from which we can find the right ascension and declination by the method given in Section 27.

As before, we choose our starting point, the epoch, as 2010.0. Having calculated the number of days, D , since the epoch, we find the mean anomaly, M , by the formula

$$M = \frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon - \varpi \text{ degrees,}$$

where T_p is the orbital period of the planet in tropical years, ε is the mean longitude of the planet at the epoch, and ϖ is the longitude of the perihelion. These constants are listed for the planets in our Solar System in Table 8. This table is extracted from a list of **osculating elements** published on the web by the US Naval Observatory (see page 209).

Being osculating elements, they change with time and are valid only over a relatively short period. We can use the values in Table 8 for low-precision calculations, but should use the more-precise spreadsheet of Section 56 for extrapolations into the past or future of more than a few tens of years from 2010, or where higher accuracy is needed.

The mean anomaly refers to the motion of a fictitious planet, P_1 , moving in a circle at constant speed with the same orbital period as the real planet (see Figure 64). We really want to know the value of the true anomaly, v , which is the angle the real planet actually makes with the line joining the Sun to the perihelion. We can find v from M using the equation of the centre:

$$v = M + \frac{360}{\pi} e \sin M \text{ degrees,}$$

where e is the eccentricity of the orbit (Table 8) and $\pi = 3.141\,592\,7$. Once again, this formula is an approximation that is good enough for most purposes; if you wish to make more precise calculations you can find the value of v by solving Kepler's equation via the method of Section 47.

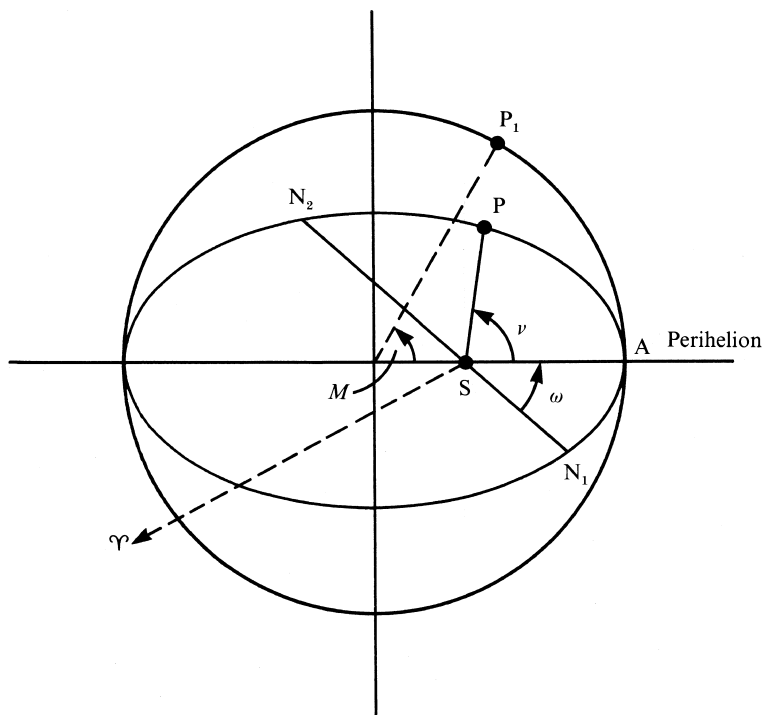


Figure 64. Mean and true anomalies.

	T_p (tropical years)	ε (degrees)	ϖ (degrees)	e	a (AU)	i (degrees)	Ω (degrees)	θ_0 (arcsec)	V_0
Mercury	0.24085	75.5671	77.612	0.205627	0.387098	7.0051	48.449	6.74	-0.42
Venus	0.615207	272.30044	131.54	0.006812	0.723329	3.3947	76.769	16.92	-4.40
Earth	0.999996	99.556772	103.2055	0.016671	0.999985				
Mars	1.880765	109.09646	336.217	0.093348	1.523689	1.8497	49.632	9.36	-1.52
Jupiter	11.857911	337.917132	14.6633	0.048907	5.20278	1.3035	100.595	196.74	-9.40
Saturn	29.310579	172.398316	89.567	0.053853	9.51134	2.4873	113.752	165.60	-8.88
Uranus	84.039492	271.063148	172.884833	0.046321	19.21814	0.773059	73.926961	65.80	-7.19
Neptune	165.84539	326.895127	23.07	0.010483	30.1985	1.7673	131.879	62.20	-6.87

1 AU = 149.6×10^6 km.

T_p : period of orbit; ε : longitude at the epoch; ϖ : longitude of the perihelion; e : eccentricity of the orbit; a : semi-major axis of the orbit; i : orbital inclination; Ω : longitude of the ascending node; θ_0 : angular diameter at 1 AU; V_0 : visual magnitude at 1 AU.

Table 8. Elements of the planetary orbits at epoch 2010.0.

The next step is to calculate the heliocentric longitude, l , and this is simply given by

$$l = v + \varpi,$$

or

$$l = \left(\frac{360}{365.242191} \times \frac{D}{T_p} \right) + \frac{360}{\pi} e \sin \left(\frac{360}{365.242191} \times \frac{D}{T_p} + \varepsilon - \varpi \right) + \varepsilon \text{ degrees.}$$

We also need the length of the radius vector, r , calculated from

$$r = \frac{a(1 - e^2)}{1 + e \cos v},$$

where a is the semi-major axis of the orbit (Table 8).

The above calculations which you have made for the planet have to be repeated for the Earth as well. We shall denote the values derived for the planet by small letters and use capital letters for the Earth's values. Thus, we arrive at the figures for l and r for the planet and L and R for the Earth. In addition, we need the heliocentric latitude of the planet:

$$\psi = \sin^{-1} \{ \sin(l - \varOmega) \sin i \},$$

where i is the inclination of the orbit and \varOmega is the longitude of the ascending node (Table 8). The heliocentric latitude of the Earth is, of course, zero.

Now we need to project our calculations for the planet onto the plane of the ecliptic to find the projected heliocentric longitude, l' , and the projected radius vector, r' . These are given by the formulas

$$l' = \tan^{-1} \{ \tan(l - \varOmega) \cos i \} + \varOmega,$$

$$r' = r \cos \psi.$$

The final step in the process is to refer the calculations to the Earth to find the geocentric ecliptic latitude, β , and longitude, λ , of the planet. Figure 65(a) describes the situation for an **outer planet**, whose orbit lies outside that of the Earth (i.e. Mars, Jupiter, Saturn, Uranus and Neptune), and Figure 65(b) is for an **inner planet** (the inner planets are Mercury and Venus). The plane of the paper represents the plane of the ecliptic. S is the Sun, E is the Earth and P₁ is the position of the planet projected onto the ecliptic. The first point of Aries is taken to be at a distance from the Solar System so large that the directions E Υ and S Υ are parallel. Then by application of a little simple geometry we have for the outer planets

$$\lambda = \tan^{-1} \left\{ \frac{R \sin(l' - L)}{r' - R \cos(l' - L)} \right\} + l' \text{ degrees,}$$

and for the inner planets

$$\lambda = 180 + L + \tan^{-1} \left\{ \frac{r' \sin(L - l')}{R - r' \cos(L - l')} \right\} \text{ degrees,}$$

Figure 66 gives the diagram for calculating the latitude. Again, using simple geometry we find

$$\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda - l')}{R \sin(l' - L)} \right\} \text{ degrees,}$$

true for inner and outer planets alike.

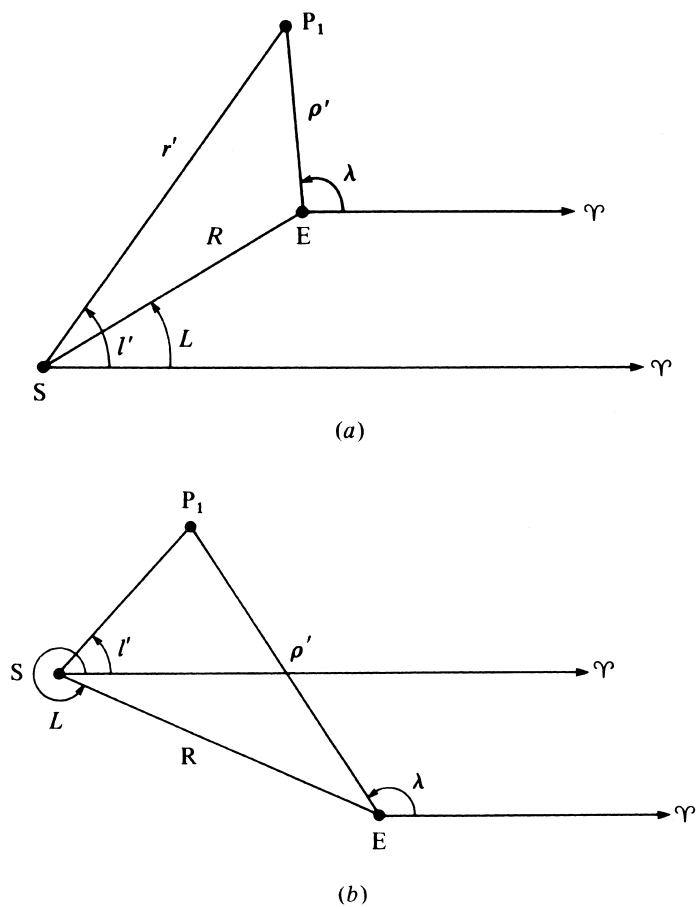


Figure 65. Ecliptic geometry: (a) outer planet, (b) inner planet.

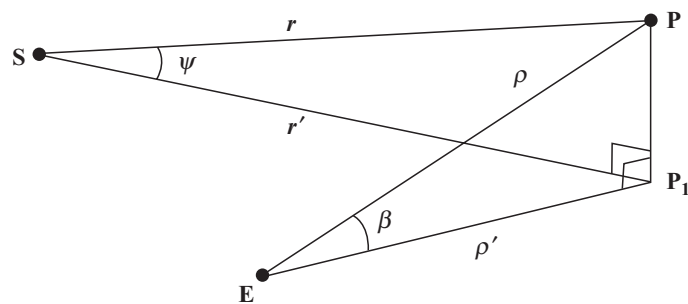


Figure 66. Projecting onto the ecliptic.

Let us consolidate these rather lengthy calculations with two examples, one for an outer planet, Jupiter, and the other for an inner planet, Mercury. For each of these two planets we shall calculate its right ascension and declination on 22 November 2003. For Jupiter (outer planet):

Method	Example
1. Find the number of days since 2010 January 0.0 (§3). The total is D .	$\begin{aligned} 22 \text{ November} &= 304 + 22 \\ &= 326 \\ &- 2557 \\ D &= -2231 \text{ days} \end{aligned}$
<i>For the planet, Jupiter:</i>	
2. Calculate $N_p = \frac{360}{365.242191} \times \frac{D}{T_p}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$N_p = 174.555932 \text{ degrees}$
3. Find $M_p = N_p + \varepsilon - \varpi$.	$M_p = 497.809764 \text{ degrees}$
4. Calculate $v_p = M_p + \frac{360}{\pi} e \sin M_p$ degrees. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$v_p = 141.573600 \text{ degrees}$
5. Find $l_p = v_p + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$l_p = 156.236900 \text{ degrees}$
6. Calculate $r = \frac{a(1-e^2)}{1+e \cos v_p}$.	$r = 5.397121 \text{ AU}$
<i>Now do the calculations for the Earth:</i>	
7. Calculate $N_E = \frac{360}{365.242191} \times \frac{D}{T_E}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$N_E = 321.011952 \text{ degrees}$
8. Find $M_E = N_E + \varepsilon_E - \varpi_E$.	$M_E = 317.363223 \text{ degrees}$
9. Calculate $v_E = M_E + \frac{360}{\pi} e_E \sin M_E$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$v_E = 316.069248 \text{ degrees}$
10. Find $L = v_E + \varpi_E$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$L = 59.274748 \text{ degrees}$
11. Calculate $R = \frac{a_E(1-e_E^2)}{1+e_E \cos v_E}$.	$R = 0.987847 \text{ AU}$

Method (continued)	Example
<i>More calculations for the planet:</i>	
12. Calculate $\psi = \sin^{-1} \{ \sin(l_p - \delta_\odot) \sin i \}$.	$\psi = 1.076\,044$ degrees
13. Find $y = \sin(l_p - \delta_\odot) \cos i$.	$y = 0.825\,313$
14. Find $x = \cos(l_p - \delta_\odot)$.	$x = 0.564\,363$
15. Find $\tan^{-1}(\frac{y}{x})$ and remove the ambiguity by referring to Figure 29, adding or subtracting 180 degrees as indicated by the signs of x and y so that the result lies in the correct quadrant.	$\tan^{-1}(\frac{y}{x}) = 55.634\,991$ y and x both positive so the result is in the right quadrant
16. Add δ_\odot to get l' (check: l' should be nearly equal to l_p).	$l' = 156.229\,991$ degrees
17. Find $r' = r \cos \psi$.	$r' = 5.396\,170$ AU
<i>Combine the calculations:</i>	
18. Calculate $\lambda = \tan^{-1} \left\{ \frac{R \sin(l' - L)}{r' - R \cos(l' - L)} \right\} + l'$. Bring the result into the range 0 to 360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	$\lambda = 166.310\,510$ degrees
19. Find $\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda - l')}{R \sin(l' - L)} \right\}$. This is the planet's geocentric ecliptic latitude.	$\beta = 1.036\,466$ degrees
20. Finally, convert λ and β to right ascension and declination (§27).	$\alpha = 11^{\text{h}} 11^{\text{m}} 14^{\text{s}}$ $\delta = 6^\circ 21' 25''$

The *Astronomical Almanac* gives the apparent coordinates of Jupiter for this day as $\alpha = 11^{\text{h}} 10^{\text{m}} 30^{\text{s}}$ and $\delta = 6^{\circ} 25' 56''$. The error due to our approximation in counting only the first term of the equation of the centre could be reduced by solving Kepler's equation using the method in Section 47. We will see how to make more exact calculations in Section 56. For Mercury (inner planet):

Method	Example
1. We proceed exactly as we did in the previous example, calculating l_p , v_p , r , ψ , l' and r' for Mercury, and L , v_E and R for the Earth.	$l_p = 288.012\,253$ degrees $v_p = 210.400\,253$ degrees $r = 0.450\,657$ AU $\psi = -6.035\,842$ degrees $l' = 287.824\,406$ degrees $r' = 0.448\,159$ degrees $L = 59.274\,748$ degrees $v_E = 316.069\,248$ degrees $R = 0.987\,847$ AU
2. Now calculate $\lambda = 180 + L + \tan^{-1} \left\{ \frac{r' \sin(L-l')}{R-r' \cos(L-l')} \right\}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	$\lambda = 253.929\,758$ degrees
3. Find $\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda-l')}{R \sin(l'-L)} \right\}$.	$\beta = -2.044\,057$ degrees
4. Finally, convert λ and β to right ascension and declination (§27).	$\alpha = \mathbf{16^{\text{h}} 49^{\text{m}} 12^{\text{s}}}$ $\delta = \mathbf{-24^{\circ} 30' 09''}$

The *Astronomical Almanac* gives the apparent coordinates of planet Mercury as $\alpha = 16^{\text{h}} 52^{\text{m}} 02^{\text{s}}$ and $\delta = -24^{\circ} 38' 41''$. We should generally expect an error in α of a few minutes at most and in δ of a quarter of a degree, but the errors may be more for Mercury, for which $e = 0.2$. The inaccuracies arise because we have used only the first term in the equation of the centre, we have not allowed for the light travel time, and because of the slight perturbations to the orbits from other planets in the Solar System (see Section 56). We could reduce the error from the first cause by using the longer method of Section 47; see Figure 68 for a graph of the error incurred by the shorter method.

The spreadsheet for this calculation, called PlanetPos1, Figure 67 (three panels), uses a technique which we have not met previously in this book. The parameters of the orbits of the planets are presented in a table in a separate spreadsheet called Planet data (third panel). This table reproduces most of the data in Table 8, but note that the order of the planets is now alphabetical. The planet name is in column A and is used as the key to the corresponding row of data contained in columns C to I inclusive. If we number the columns in the table, then column 1 contains the planet name, 3 contains the orbital period, 4 the longitude at the epoch and so on. We can obtain any element of the data using the spreadsheet function VLOOKUP (e.g. row 17 of the first panel of Figure 67). This takes four arguments, which are the planet name (upper or lower case, or a mixture), the range of the table from top-left-hand corner to bottom-right-hand corner (e.g. 'Planet data'!A3:I10), the column number containing the required element (e.g. 3 for the orbital period, 4 for the longitude at the epoch), and a switch which is set to TRUE to find either an approximate or an exact match with the planet name in column 1, or FALSE if an exact match is required (as here). Using this formula makes it easy to change the orbital parameters without affecting the main spreadsheet calculation. Simply fill in the Planet data table with new numbers, and the spreadsheet will use those instead. The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.

Type a question for help												
	A	B	C	D	E	F	G	H	I	J	K	L
1	The positions of the planets (approximate method)											
2												
3	Input	local civil time (hour)	0		Output	Jupiter RA (hour)	11	=DHHour(C61)				
4		local civil time (min)	0			Jupiter RA (min)	11	=DHMin(C61)				
5		local civil time (sec)	0			Jupiter RA (sec)	13.8	=DHSec(C61)				
6		daylight saving (hours)	0			Jupiter dec (deg)	6	=DDDeg(C62)				
7		zone correction (hours)	0			Jupiter dec (min)	21	=DDMin(C62)				
8		local date (day)	22			Jupiter dec (sec)	25.1	=DDSec(C62)				
9		local date (month)	11									
10		local date (year)	2003				=CONCATENATE(\$C\$11," RA (hour)")					
11		planet name	Jupiter				=CONCATENATE(\$C\$11," RA (min)")					
12							=CONCATENATE(\$C\$11," RA (sec)")					
13							=CONCATENATE(\$C\$11," dec (deg)")					
14							=CONCATENATE(\$C\$11," dec (min)")					
15							=CONCATENATE(\$C\$11," dec (sec)")					
16												
17	1	planet Tp from table	11.857911	=VLOOKUP(C11,'Planet data'!A3:I10,3,FALSE)								
18	2	planet long from table	337.917132	=VLOOKUP(C11,'Planet data'!A3:I10,4,FALSE)								
19	3	planet peri from table	14.6633	=VLOOKUP(C11,'Planet data'!A3:I10,5,FALSE)								
20	4	planet ecc from table	0.048907	=VLOOKUP(C11,'Planet data'!A3:I10,6,FALSE)								
21	5	planet axis from table	5.20278	=VLOOKUP(C11,'Planet data'!A3:I10,7,FALSE)								
22	6	planet incl from table	1.3035	=VLOOKUP(C11,'Planet data'!A3:I10,8,FALSE)								
23	7	planet node from table	100.595	=VLOOKUP(C11,'Planet data'!A3:I10,9,FALSE)								
24	8	Gdate (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
25	9	Gdate (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
26	10	Gdate (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
27	11	UT (hours)	0	=LctUT(C3,C4,C5,C6,C7,C8,C9,C10)								
28	12	D (days)	-2231	=CDJD(C24+(C27/24),C25,C26)-CDJD(0,1,2010)								
29	13	Np (deg)	-185.4440679	=360*C28/(365.242191*C17)								
30	14	Np (deg)	174.5559321	=C29-360*INT(C29/360)								
31	15	Mp (deg)	497.8097641	=C30+C18-C19								
32	16	Lp (deg)	516.2369	=C30+(360*C20*SIN(RADIANS(C31)))/PI()+C18								

Figure 67. Finding the position of a planet by an approximate method; panels one and two show the main spreadsheet, and the third shows the data table (continued on the next page). The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.

	A	B	C	D	E	F	G	H	I	J	K	L
33	17	L p (deg)	156.2369	=C32-360*INT(C32/360)								
34	18	planet true anomaly (deg)	141.5736	=C33-C19								
35	19	r (AU)	5.397121314	=C21*(1-(C20^2)/(1+C20*COS(RADIANS(C34))))								
36	20	Earth Tp from table	0.999996	=VLOOKUP("Earth",Planet data!A3:I10,3,FALSE)								
37	21	Earth long from table	99.556772	=VLOOKUP("Earth",Planet data!A3:I10,4,FALSE)								
38	22	Earth peri from table	103.2055	=VLOOKUP("Earth",Planet data!A3:I10,5,FALSE)								
39	23	Earth ecc from table	0.016671	=VLOOKUP("Earth",Planet data!A3:I10,6,FALSE)								
40	24	Earth axis from table	0.999985	=VLOOKUP("Earth",Planet data!A3:I10,7,FALSE)								
41	25	N e (deg)	-2198.988048	=360*C28/(365.242191*C36)								
42	26	N e (deg)	321.0119519	=C41-360*INT(C41/360)								
43	27	M e (deg)	317.3632239	=C42+C37-C38								
44	28	L e (deg)	419.2747476	=C42+C37+360*C39*SIN(RADIANS(C43))/PI()								
45	29	L e (deg)	59.27474763	=C44-360*INT(C44/360)								
46	30	Earth true anomaly (deg)	-43.93075237	=C45-C38								
47	31	R (AU)	0.987846892	=C40*(1-(C39^2)/(1+C39*COS(RADIANS(C46))))								
48	32	(L p-Node) (rad)	0.971134357	=RADIANS(C33-C23)								
49	33	psi (rad)	0.018780513	=ASIN(SIN(C48)*SIN(RADIANS(C22)))								
50	34	y	0.825312808	=SIN(C48)*COS(RADIANS(C22))								
51	35	x	0.564363453	=COS(C48)								
52	36	l d (deg)	156.229991	=DEGREES(ATAN2(C51,C50))+C23								
53	37	r d (AU)	5.396169539	=C35*COS(C49)								
54	38	L e-L d (rad)	-1.692188223	=RADIANS(C45-C52)								
55	39	ATAN2 type 1	-1.273466183	=ATAN2(C47-C53*COS(C54),C53*SIN(C54))								
56	40	ATAN2 type 2	0.175938247	=ATAN2(C53-C47*COS(C54),C47*SIN(-C54))								
57	41	A (rad)	0.175938247	=IF(C53<1,C55,C56)								
58	42	lamda (deg)	166.31051	=IF(C53<1,180+C45+DEGREES(C57),DEGREES(C57)+C52)								
59	43	lamda (deg)	166.31051	=C58-360*INT(C58/360)								
60	44	beta (deg)	1.036465596	=DEGREES(ATAN(C53*TAN(C49)*SIN(RADIANS(C59-C52)))/(C47*SIN(-C54))))								
61	45	RA (hours)	11.1871665	=DDDH(ECRA(C59,0,0,C60,0,0,C24,C25,C26))								
62	46	dec (deg)	6.356972099	=ECDDec(C59,0,0,C60,0,0,C24,C25,C26)								

	A	B	C	D	E	F	G	H	I
1	Planet name		Tp	Long	Peri	Ecc	Axis	Incl	Node
2									
3	EARTH		0.999996	99.556772	103.2055	0.01667	0.99999		
4	JUPITER		11.85791	337.91713	14.6633	0.04891	5.20278	1.3035	100.595
5	MARS		1.880765	109.09646	336.217	0.09335	1.52369	1.8497	49.632
6	MERCURY		0.24085	75.5671	77.612	0.20563	0.3871	7.0051	48.449
7	NEPTUNE		165.8454	326.89513	23.07	0.01048	30.1985	1.7673	131.879
8	SATURN		29.31058	172.39832	89.567	0.05385	9.51134	2.4873	113.752
9	URANUS		84.03949	271.06315	172.88483	0.04632	19.2181	0.77306	73.926961
10	VENUS		0.615207	272.30044	131.54	0.00681	0.72333	3.3947	76.769

Figure 67. (Continued.)

55 Finding the approximate positions of the planets

The method of finding the equatorial coordinates of the planets given in the previous section is quite accurate but involves lengthy calculations. An amateur astronomer often only wants to know the approximate position of a planet so that he or she knows where to look for it in the sky, and does not want to have to spend 20 minutes beforehand submerged in a sea of figures obtaining the information. In that case it is usually sufficient to assume that the planets describe circular orbits about the Sun which all lie in the plane of the ecliptic. This leads to considerable simplifications in the calculations.

The heliocentric longitude, l , does not have to be corrected by the equation of the centre so that we may write

$$l = \frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon \text{ degrees.}$$

We repeat this calculation for the Earth as before (giving L). Since the orbits are assumed to be circular with the Sun at the centre, the radius vector is constant. Hence

$$r = a.$$

The heliocentric (and therefore the geocentric) latitude of the planet is zero since we have assumed that the orbit lies in the ecliptic plane. Our final calculation is therefore

$$\lambda = \tan^{-1} \left\{ \frac{\sin(l-L)}{a - \cos(l-L)} \right\} + l,$$

for the outer planets and

$$\lambda = 180 + L + \tan^{-1} \left\{ \frac{a \sin(L-l)}{1 - a \cos(L-l)} \right\},$$

for the inner planets, since we have assumed that $R = 1$ (the Earth's orbital radius is taken to be unity). This is the geocentric longitude of the planet from which the right ascension and declination can be found using the formulas of Section 27 (remember $\beta = 0$):

$$\alpha = \tan^{-1} \{ \tan \lambda \cos \varepsilon \},$$

$$\delta = \sin^{-1} \{ \sin \lambda \sin \varepsilon \},$$

where ε here is the obliquity of the ecliptic (about 23.5 degrees; see Section 27). In some cases it may even be possible to ignore the fact that the plane of the ecliptic is inclined at an angle to the plane of the equator and to write

$$\alpha = \lambda.$$

As an example, we will calculate again the coordinates of Jupiter on 22 November 2003 using this approximate method.

Method	Example
1. Find the number of days since 2010 January 0.0 (§3). The total is D .	22 November = 304 + 22 = 326 – 2 557 D = –2 231 days
2. Calculate $l = \left(\frac{360}{365.242\,191} \times \frac{D}{T_p} \right) + \varepsilon$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l = 152.47 degrees
3. Repeat step 2 for the Earth to find L .	L = 60.57 degrees
4. Calculate $\lambda = \tan^{-1} \left\{ \frac{\sin(l-L)}{a - \cos(l-L)} \right\} + l$. Bring the result into the range 0–360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	λ = 163.28 degrees
5. Finally, convert λ and β ($= 0$) to right ascension and declination (§27).	α = 10h 58m δ = 6° 34'

56 Perturbations in a planet's orbit

Throughout the calculations to find the coordinates of a planet (Section 54), we assumed that its motion was controlled entirely by the gravitational field of the Sun so that the influences of other members of the Solar System were negligible. This is true to quite a high accuracy, but for more precision we need to take account of these **perturbations**, especially for the orbits of Jupiter and Saturn where the effects can be as large as 1 degree in longitude. The usual method of doing so is to apply a series of correction terms to the quantities calculated in Section 54. We have to make similar adjustments for the Moon in Section 65. Here, we shall consider only the most important terms in the orbits of Jupiter and Saturn where the corrections amount to more than about 0.04 degrees in longitude.

We must first calculate the time, T , in Julian centuries since the epoch 1900 January 0.5. This is given by

$$T = \frac{\text{JD} - 2415020.0}{36525},$$

where JD is the Julian date (Section 4).[†] Then we calculate the quantities:

$$A = \frac{T}{5} + 0.1,$$

$$P = 237.47555 + 3\,034.9061T \text{ degrees},$$

$$Q = 265.91650 + 1\,222.1139T \text{ degrees},$$

$$V = 5Q - 2P, \text{ and}$$

$$B = Q - P.$$

[†]Note that this definition of T differs from that used in other parts of the book.

The principal terms for Jupiter and Saturn are then:

$$\text{Jupiter : } \Delta l = (0.3314 - 0.0103A) \sin V - 0.0644A \cos V \text{ degrees.}$$

$$\begin{aligned} \text{Saturn : } \Delta l = & (0.1609A - 0.0105) \cos V + (0.0182A - 0.8142) \sin V - 0.1488 \sin B \\ & - 0.0408 \sin 2B + 0.0856 \sin B \cos Q + 0.0813 \cos B \sin Q \text{ degrees.} \end{aligned}$$

The value of Δl must be added to the mean longitude l before proceeding with the calculation of Section 54.

Let us now recalculate the position of Jupiter on 22 November 2003, solving Kepler's equation properly (Section 47) and allowing for these principal terms of perturbation.

Method	Example
1. Calculate the Julian date (§4).	JD = 2 452 965.5 days
2. Find $T = \frac{\text{JD} - 2415020.0}{36525}$.	T = 1.038 891 centuries
3. Find $A = \left(\frac{T}{5}\right) + 0.1$.	A = 0.307 778 centuries
4. Calculate $P = 237^\circ.47555 + 3034^\circ.9061T$.	P = 3 390.412 183 degrees
5. Find $Q = 265^\circ.91650 + 1222^\circ.1139T$.	Q = 1 535.559 632 degrees
6. Find $V = 5Q - 2P$.	V = 896.973 792 degrees
7. Calculate $\Delta l = (0.3314 - 0.0103A) \sin V - 0.0644A \cos V$.	Δl = 0.037 121 degrees
8. Now proceed as in the example of §54 to find M_p .	M_p = 497.809 764 degrees
9. Use the method of §47 to find v_p .	E_p = 2.436 915 radians v_p = 141.407 886 degrees
10. Find $l_p = v_p + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l_p = 156.071 186 degrees
11. Add Δl to get a better estimate of l_p .	l_p = 156.108 307 degrees
12. Calculate L , v_E and R for the Earth also using the method of §47.	E_E = 5.527 600 radians v_E = 316.049 185 degrees L = 59.254 685 degrees R = 0.987 847 AU
13. Now proceed with the calculations of §54 to find α and δ .	r = 5.396 627 AU ψ = 0.018 751 degrees l' = 156.101 386 degrees r' = 5.395 678 AU λ = 166.188 415 degrees β = 1.035 198 degrees α = 11h 10m 47s δ = 6° 24' 12"

The error incurred in considering only the first term of the equation of the centre is plotted as a function of the mean anomaly, M , in Figure 68 for two values of the eccentricity, e .

The spreadsheet for finding the positions of the planets by a more precise method is called PlanetPos2 and is shown in Figure 69. We have used a full numerical model of the orbits of the planets in the two spreadsheet functions PlanetLong and PlanetLat which return, respectively, the named planet's ecliptic longitude and latitude in degrees. These functions appear in rows 20 and 21. They each take nine arguments, namely the local civil time expressed as hours, minutes, and seconds, the daylight saving correction and zone time offset in hours, the local calendar date expressed as day, month, year, and the full name of the planet as a string in upper or lower case. The example shown is for Jupiter on the 22 November 2003, and you can see that this method has provided the position correct within about 1 second in right ascension, and within a few arcseconds in declination. The formulas in cells G3 to G8 are shown in cells H10 to H15 to save space.

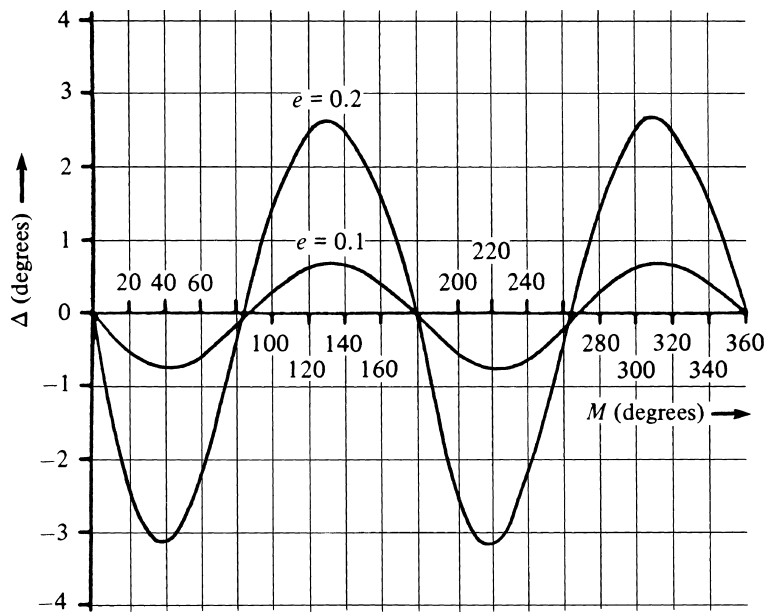


Figure 68. The error, Δ , incurred by taking $v = M + (360/\pi) e \sin M$ as an approximation to elliptical motion. The true anomaly should be $v - \Delta$.

	A	B	C	D	E	F	G	H	I	J	K
1	The positions of the planets (more precise method)										
2											
3	<i>Input</i>	local civil time (hour)	0		<i>Output</i>	Jupiter RA (hour)	11	=DHour(C22)			
4		local civil time (min)	0			Jupiter RA (min)	10	=DHMin(C22)			
5		local civil time (sec)	0			Jupiter RA (sec)	30.99	=DDSec(C22)			
6		daylight saving (hours)	0			Jupiter dec (deg)	6	=DDDeg(C23)			
7		zone correction (hours)	0			Jupiter dec (min)	25	=DDMin(C23)			
8		local date (day)	22			Jupiter dec (sec)	49.46	=DDSec(C23)			
9		local date (month)	11								
10		local date (year)	2003					=CONCATENATE(\$C\$11," RA (hour)")			
11		planet name	Jupiter					=CONCATENATE(\$C\$11," RA (min)")			
12								=CONCATENATE(\$C\$11," RA (sec)")			
13								=CONCATENATE(\$C\$11," dec (deg)")			
14								=CONCATENATE(\$C\$11," dec (min)")			
15								=CONCATENATE(\$C\$11," dec (sec)")			
16											
17	1	Greenwich date (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)							
18	2	Greenwich date (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)							
19	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)							
20	4	planet ecl long (deg)	166.1186259	=PlanetLong(C3,C4,C5,C6,C7,C8,C9,C10,C11)							
21	5	planet ecl lat (deg)	1.035218287	=PlanetLat(C3,C4,C5,C6,C7,C8,C9,C10,C11)							
22	6	planet RA (hours)	11.17527609	=DDDH(ECRA(C16,0,0,C17,0,0,C8,C9,C10))							
23	7	planet dec (deg)	6.430406407	=ECDec(C16,0,0,C17,0,0,C8,C9,C10)							

Figure 69. Finding the position of a planet by a more precise method.

57 The distance, light-travel time and angular size of a planet

During the course of our calculations in Section 54 to determine the position of a planet we found the distances r and R of the planet and the Earth, respectively, from the Sun. We can quite easily use these values together with the heliocentric longitudes l and L to calculate the planet's distance, ρ , from the Earth. The situation is drawn in Figure 70. The planet, P, does not lie in the plane of the ecliptic, so its heliocentric latitude, ψ , must be taken into account. The formula is

$$\rho^2 = R^2 + r^2 - 2Rr \cos(l - L) \cos \psi.$$

It is usual to express r and R in **astronomical units** (AU) where 1 AU is the semi-major axis of the Earth's orbit. ρ is then the distance of the planet from the Earth measured in AU.

Having calculated ρ , it is then an easy matter to find the light-travel time, τ , the time taken for the light to reach us from the planet. When we view a planet now, we see it in the position it occupied τ hours ago, given by

$$\tau = 0.1386\rho \text{ hours,}$$

where ρ is expressed in AU.

We can also find the apparent angular diameter, θ , of the planet given by

$$\theta = \frac{\theta_0}{\rho},$$

where ρ is again expressed in AU and θ_0 is the angular diameter of the planet when it is at 1 AU from the Earth. Values of θ_0 are given in Table 8.

We shall calculate the distance, the light-travel time and the apparent angular diameter of Jupiter on 22 November 2003.

Method	Example
1. Find r , R , l_p , L and ψ as in §54.	$r = 5.397\,121$ AU $R = 0.987\,847$ AU $l_p = 156.236\,900$ degrees $L = 59.274\,748$ degrees $\psi = 1.076\,044$ degrees
2. Calculate $\rho^2 = R^2 + r^2 - 2Rr \cos(l_p - L) \cos \psi$.	$\rho^2 = 31.397\,037$ AU ²
3. Take the square root to find ρ .	$\rho = \mathbf{5.603}$ AU
4. Multiply by 0.1386 to find τ ; convert to minutes and seconds (§8).	$\tau = \mathbf{46m\,36s}$
5. Find θ from $\theta = \frac{\theta_0}{\rho}$.	$\theta = \mathbf{35.1}$ arcsec

The *Astronomical Almanac* quotes $\rho = 5.60$ AU, $\tau = 46m\,34s$ and $\theta = 35.0''$ for Jupiter on this day.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

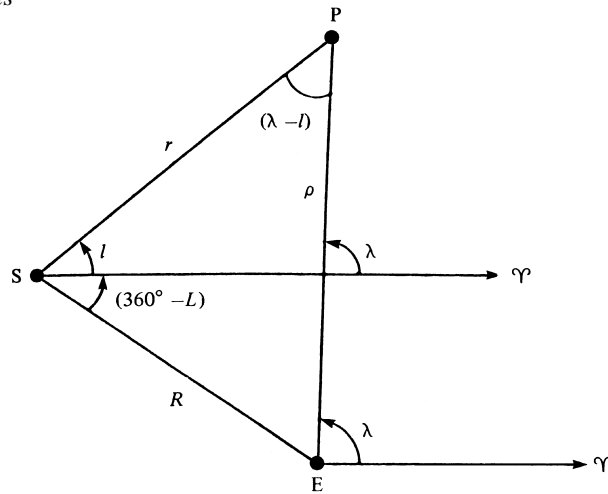


Figure 70. Finding the distance of a planet.

58 The phases of the planets

At any point in the orbit of a planet, the hemisphere which faces towards the Sun is brightly illuminated while the other half of the planet's surface is dark. The fraction of the surface that we can see from the Earth, however, is that part lying on the hemisphere facing the Earth which usually overlaps both the bright and the dark sides. We are presented therefore with a view of the planet's disc that is not uniformly illuminated but which contains a bright segment, the rest of the disc being dark and usually invisible. As the relative positions of the Earth, the planet and the Sun vary, so the area of the visible disc that is illuminated changes. The **phase** is defined to be the fraction of the visible disc that is illuminated.

In Figure 70, the angle $(\lambda - l)$ at P is the solar elongation of the Earth as measured at the planet. We represent this angle by d . Thus

$$d = \lambda - l.$$

The phase, F , is related to d by the formula

$$F = \frac{1}{2} (1 + \cos d).$$

F always lies in the range 0 to 1. When $F = 0$, the whole of the dark side of the planet is towards the Earth. This can only happen for the inner planets Mercury and Venus. When $F = 1$, the whole of the bright side faces the Earth.

We shall find the phases of Mercury and Jupiter on 22 November 2003 as our example.

Method	Example
1. Calculate $d = \lambda - l$ using the method outlined in §54 to find λ and l .	Mercury: $d_1 = -34.082^\circ$ Jupiter: $d_2 = 10.736^\circ$
2. Find $F = \frac{1}{2}(1 + \cos d)$.	Mercury: $F_1 = \mathbf{0.91}$ Jupiter: $F_2 = \mathbf{0.99}$

The *Astronomical Almanac* gives phase values of 0.90 and 0.99 for Mercury and Jupiter respectively.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

59 The position-angle of the bright limb

Figure 71 shows the appearance of a planet whose phase is about $F = 0.7$. The dashed outline is of that part of the disc which is invisible, and the line NS is the projection of the Earth's axis onto the disc. The terminator, the line dividing night from day, is the curve AB. Position-angles are measured anticlockwise from the north. Thus points A and B are at position-angles θ_1 and θ_2 . The point C, halfway between A and B on the circumference of the disc, is the midpoint of the bright side and it defines the position-angle, χ , of the bright limb. Hence

$$\chi = \frac{1}{2}(\theta_1 + \theta_2).$$

We can easily calculate χ provided we know the equatorial coordinates of the planet (α, δ) and of the Sun $(\alpha_\odot, \delta_\odot)$. Then

$$\chi = \tan^{-1} \left\{ \frac{\cos \delta_\odot \sin(\alpha_\odot - \alpha)}{\cos \delta \sin \delta_\odot - \sin \delta \cos \delta_\odot \cos(\alpha_\odot - \alpha)} \right\}.$$

For example, what was the position-angle of the bright limb of Mercury on 22 November 2003? The Sun's coordinates were $\alpha_\odot = 15^{\text{h}} 48^{\text{m}} 13^{\text{s}}$, $\delta_\odot = -19^\circ 59' 32''$, and Mercury's coordinates were $\alpha = 16^{\text{h}} 52^{\text{m}} 02^{\text{s}}$, $\delta = -24^\circ 38' 41''$.

Method	Example
1. Find the right ascension and declination of the planet (§54).	$\alpha = 16^{\text{h}} 52^{\text{m}} 02^{\text{s}}$ $\delta = -24^{\circ} 38' 41''$
2. Find the coordinates of the Sun (§46).	$\alpha_{\odot} = 15^{\text{h}} 48^{\text{m}} 13^{\text{s}}$ $\delta_{\odot} = -19^{\circ} 59' 32''$
3. Convert both sets of coordinates to decimal form (§§7 and 21).	$\alpha = 16.867222$ hours $\delta = -24.644722$ degrees $\alpha_{\odot} = 15.803611$ hours $\delta_{\odot} = -19.992222$ degrees
4. Find $\Delta\alpha = \alpha_{\odot} - \alpha$. Convert to degrees by multiplying by 15 (§22).	$\Delta\alpha = -15.954165$ degrees
5. Find $y = \cos \delta_{\odot} \sin \Delta\alpha$.	$y = -0.258304$
6. Find $x = \cos \delta \sin \delta_{\odot} - \sin \delta \cos \delta_{\odot} \cos \Delta\alpha$.	$x = 0.066018$
7. Find $\chi = \tan^{-1} \left(\frac{y}{x} \right)$. Remove the ambiguity from taking inverse tan using the signs of x and y , referring to Figure 29, and adding or subtracting 180 if necessary to bring the result into the correct quadrant.	$\chi = -75.663043$ degrees (already in correct quadrant; add 360 to bring into the range 0–360) $\chi = \mathbf{284.3 \text{ degrees}}$

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

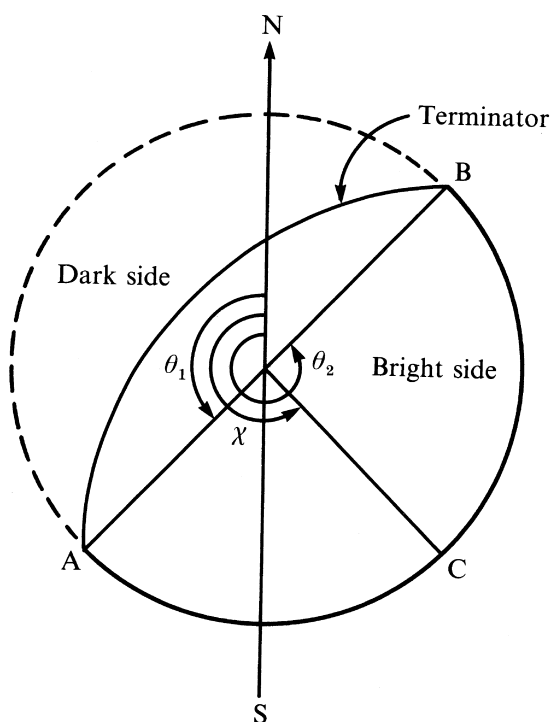


Figure 71. The position-angle of the bright limb.

60 The apparent brightness of a planet

Our calculations so far have given us the position, the solar elongation (Section 52), the distance from the Earth, the apparent angular diameter, the phase, and the position-angle of the bright limb of a planet. We need only add the apparent brightness to the list to obtain all the important parameters of the planet's **visual aspect**.

Brightness is usually measured in **magnitudes**, m , on a non-linear scale such that decreasing brightness goes with increasing magnitude. The brightest stars have a magnitude of about 1 while the faintest stars just visible with the unaided eye are of magnitude 6. The ratio in the light power flux between one magnitude and the next is about 2.5. The Sun, very bright at the Earth, has a visual magnitude of -26.74 while the Moon's magnitude at opposition is -12.73 . The planets range from about $m = -4$ for Venus at its most brilliant to $m = +14$ for Pluto (strictly no longer classified as a planet) at its brightest.

The variation in a planet's brightness is caused by several factors. First the Sun's light flux on the planet varies inversely as the square of its distance, r , from the Sun. Then the amount of that light reradiated towards the Earth depends on the phase, F , and a 'brightness factor', V_0 , the latter being a measure of the reflectivity of the planet combined with the area of the planet's disc. The larger the planet's area, the more light it intercepts from the Sun and hence the more it radiates towards the Earth. Finally, the light flux received from the planet varies inversely as the square of the planet's distance, ρ , from the Earth.

We can obtain an approximate value for the apparent magnitude of a planet from the formula

$$m = 5 \log_{10} \left(\frac{r\rho}{\sqrt{F}} \right) + V_0,$$

where r and ρ are measured in AU. The values of V_0 are listed in Table 8.

As an example, let us calculate the apparent magnitude of Jupiter on 22 November 2003.

Method	Example
1. Find the values of r , ρ and F using the methods given in §§54, 57 and 58.	$r = 5.397$ AU $\rho = 5.603$ AU $F = 0.99$
2. Calculate $m = 5 \log_{10} \left(\frac{r\rho}{\sqrt{F}} \right) + V_0$. r and ρ must both be expressed in AU.	$m = -2$

The value of m given in the *Astronomical Almanac* for Jupiter on 22 November 2003 is $m = -1.9$. In general, our calculations should be correct to within a magnitude or so. No account has been taken of **atmospheric extinction** (see Section 43), which can increase the apparent magnitude of a star or planet near the horizon by 2 or 3. Nonetheless, our calculations will provide a fair guide of what to expect.

The distance, light time, angular diameter, phase, position-angle of the bright limb, and the approximate magnitude of a planet are all parts of its visual aspect (i.e. its appearance when viewed from Earth) and these calculations are swept up into one spreadsheet called PlanetVis, Figure 72. As in PlanetPos1 (Section 54), we make use of the spreadsheet function VLOOKUP to extract data from a table contained on a second spreadsheet (rows 22 and 31). We have also defined three additional spreadsheet functions to make life easier. The first of these to appear in the spreadsheet (row 20) is PlanetDist, taking nine arguments which are the local civil time expressed as hours, minutes, and seconds, the daylight saving and time zone offsets in hours, the local calendar date expressed as day, month, year, and the planet full name as a string

(upper or lower case, or a combination). This function returns the Earth–planet distance in AU. The next new spreadsheet function to appear is called **PlanetHLong1** (row 23) which returns the planet’s heliocentric orbital longitude in degrees. It takes the same nine arguments as **PlanetDist**. The third new spreadsheet function is called **PlanetRVect** (row 30) and, as its name suggests, it returns the distance of the planet from the Sun (i.e. the length of its radius vector) in AU; it has the same nine arguments as **PlanetDist**.

	A	B	C	D	E	F	G	H	I	J
1	The visual aspects of the planets									
2										
3	Input	local civil time (hour)	0		Output	Jupiter ...	=CONCATENATE(\$C\$11," ...")			
4		local civil time (min)	0			distance (AU))	5.59829	=ROUND(C20,5)		
5		local civil time (sec)	0			ang dia (arcsec)	35.1	=ROUND(C22,1)		
6		daylight saving (hours)	0			phase	0.99	=ROUND(C23,2)		
7		zone correction (hours)	0			light time (hour)	0	=DHHour(C21)		
8		local date (day)	22			light time (min)	46	=DHMin(C21)		
9		local date (month)	11			light time (sec)	33.32	=DHSec(C21)		
10		local date (year)	2003			pos angle bright limb (deg)	113.2	=ROUND(C29,1)		
11		planet name	Jupiter			approximate magnitude	-2	=ROUND(C31,1)		
12										
13	1	Greenwich date (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
14	2	Greenwich date (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
15	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
16	4	planet ecl long (deg)	166.1186259	=PlanetLong(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
17	5	planet ecl lat (deg)	1.035218287	=PlanetLat(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
18	6	planet RA (rad)	2.925680439	=RADIANS(ecra(C16,0,0,C17,0,0,C8,C9,C10))						
19	7	planet dec (rad)	0.112231764	=RADIANS(ECDec(C16,0,0,C17,0,0,C8,C9,C10))						
20	8	planet dist (AU)	5.598285365	=PlanetDist(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
21	9	light travel time (hours)	0.775922352	=C20*0.1386						
22	10	angular diameter (arcsec)	35.14290308	=VLOOKUP(C11,'Planet data'!A3:D9,3,FALSE)/C20						
23	11	phase	0.992275914	=0.5*(1+COS(RADIANS(C16-PlanetHLong1(C3,C4,C5,C6,C7,C8,C9,C10,C11))))						
24	12	Sun ecl long (deg)	239.2631516	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)						
25	13	Sun RA (rad)	4.137341758	=RADIANS(ECRA(C24,0,0,0,0,C13,C14,C15))						
26	14	Sun dec (rad)	-0.348952785	=RADIANS(ECDec(C24,0,0,0,0,C13,C14,C15))						
27	15	y	0.879777546	=COS(C26)*SIN(C25-C18)						
28	16	x	-0.376753175	=COS(C19)*SIN(C26)-SIN(C19)*COS(C26)*COS(C25-C18)						
29	17	chi (deg)	113.1823938	=DEGREES(ATAN2(C28,C27))						
30	18	radius vector (AU)	5.395337237	=PlanetRVec(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
31	19	approximate magnitude	-1.991212985	=5*LOG10(C30*C20/SQRT(C23))+VLOOKUP(C11,'Planet data'!A3:D9,4,FALSE)						

	A	B	C	D	E	F	G	H	I	J	K	L
1	Planet name		Theta0	V0								
2												
3	JUPITER		196.74	-9.4								
4	MARS		9.36	-1.52								
5	MERCURY		6.74	-0.42								
6	NEPTUNE		62.2	-6.87								
7	SATURN		165.6	-8.88								
8	URANUS		65.8	-7.19								
9	VENUS		16.92	-4.4								

Figure 72. Calculating some visual aspects of a planet. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

61 Comets

In earlier sections we discovered how to calculate the orbit of any solid body moving around a central massive object, and we applied the method to the major planets of our Solar System. All we needed to know were the orbital elements of each planet. Likewise, we can calculate the position of a periodic comet given its orbital elements but the method needs to be modified slightly for two reasons:

- (i) The longitude of the comet is not usually specified at a particular epoch. Rather, the epoch is given when the comet is at perihelion, the point of its closest approach to the Sun.
- (ii) The eccentricity, e , of a comet is usually much more than 0.1 so that the equation of the centre does not apply. Instead, we have to solve Kepler's equation properly.

The orbital elements of some periodic comets are given in Table 9. Note that, as in the case of the planetary elements, we have specified ϖ , the longitude of the perihelion. Sometimes the **argument** of the perihelion is given instead. It has the symbol ω (very confusing) and is related to ϖ by $\varpi = \omega + \Omega$.

We begin, as before, by finding the mean anomaly, M , of the comet given by the formula

$$M = \frac{360}{365.242\,191} \times \frac{D}{T_p} + \varepsilon - \varpi,$$

where D is the number of days since the epoch, T_p is the orbital period in years, ε is the longitude of the comet at the epoch and ϖ is the longitude of the perihelion. In this case, however, the epoch is the moment of perihelion so that $\varepsilon = \varpi$. Further, we do not need to specify the date in terms of days since the epoch. Rather, we can work in decimal years. Hence, M may be found from

$$M = \frac{360Y}{T_p},$$

where Y is the number of tropical years since perihelion.

Next we have to solve Kepler's equation

$$E - e \sin E = M,$$

where e is the eccentricity and E is the eccentric anomaly. Both E and M are expressed in radians. A method of doing this was given in Section 47, *Routine R2*, in which the eccentricity was assumed to be less than 0.1 so that the first guess at the solution, $E = M$, was good enough for an accurate solution to be reached after only one or two iterations. Here, the eccentricity is much larger and although the routine would always converge eventually, many iterations might be needed. We can speed up the process if we begin with an approximate solution that is better than $E = M$. Kepler's graphs, Figure 73, are provided for this purpose. Given any value of e between 0 and 1 and the value of M (expressed in radians), you choose the corresponding value of Δ from the graphs. Then, in place of the first guess $E = E_0 = M$, use instead $E = E_0 + \Delta$ and proceed with the rest of *Routine R2* as before. You should find that only two or three iterations are needed whatever the values of e and M .

Alternatively, you may like to use the nomogram of Figure 74 (kindly provided by Mr S. J. Garvey) to find Δ . Place a ruler across the diagram joining the value of M (in radians) on the right-hand vertical scale with the value of e on the left-hand vertical scale. The point of intersection with the curve gives the magnitude of Δ/e . Multiply this by e to find Δ and give it the sign shown on the right-hand scale. For

example, the line joining $M = 5.6$ with $e = 0.46$ cuts the curve at $|\Delta/e| = 0.9$. Thus $|\Delta| = 0.9 \times 0.46 = 0.41$ and its sign is negative, giving

$$\Delta = -0.41.$$

(Vertical bars either side of a quantity signify the absolute value.) Having found E , we can calculate the true anomaly, v , from

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

(remember that you have found E in radians), and then carry on with the rest of the calculations of Section 54. If we find that r' is less than R , we must use the formula at the end which is appropriate for an **inner planet**, while if r' is greater than R , we must use the formula for an **outer planet**. In these calculations, bear in mind that the epoch for the comet and the epoch for the Earth are usually different.

Comet name	P	ϖ (degrees)	Ω (degrees)	T_p (years)	a (AU)	e	i (degrees)
Encke	1974.32	160.1	334.2	3.30	2.209	0.847	12.0
Temple 2	1972.87	310.2	119.3	5.26	3.024	0.549	12.5
Haneda-Campos	1978.77	12.016	131.700	5.37	3.066	0.641 52	5.805
Schwassmann–Wachmann 2	1974.70	123.3	126.0	6.51	3.489	0.386	3.7
Borrelly	1974.36	67.8	75.1	6.76	3.576	0.632	30.2
Whipple	1970.77	18.2	188.4	7.47	3.821	0.351	10.2
Oterma	1958.44	150.0	155.1	7.88	3.958	0.144	4.0
Schaumasse	1960.29	138.1	86.2	8.18	4.054	0.705	12.0
Comas Sola	1969.83	102.9	62.8	8.55	4.182	0.577	13.4
Schwassmann–Wachmann 1	1974.12	334.1	319.6	15.03	6.087	0.105	9.7
Neujmin 1	1966.94	334.0	347.2	17.93	6.858	0.775	15.0
Crommelin	1956.82	86.4	250.4	27.89	9.173	0.919	28.9
Olbers	1956.46	150.0	85.4	69.47	16.843	0.930	44.6
Pons–Brooks	1954.39	94.2	255.2	70.98	17.200	0.955	74.2
Halley	1986.112	170.011 0	58.154 0	76.008 1	17.943 5	0.967 3	162.238 4

P: epoch of the perihelion; ϖ : longitude of the perihelion; Ω : longitude of the ascending node; T_p : period of the orbit; a : semi-major axis of the orbit; e : eccentricity of the orbit; i : inclination of the orbit.

Table 9. The orbital elements of some periodic comets.

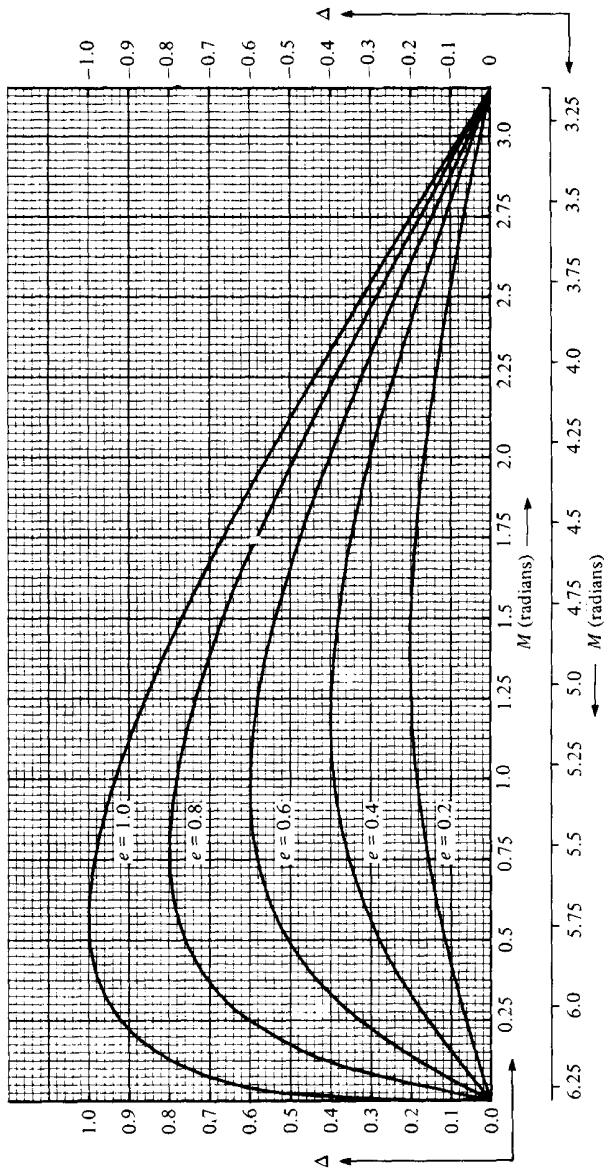


Figure 73. Kepler's graphs. Use the left-hand and upper M scales for values of M (in radians) between 0 and 3.14, and the right-hand and lower M scales for values of M between 3.14 and 6.28.

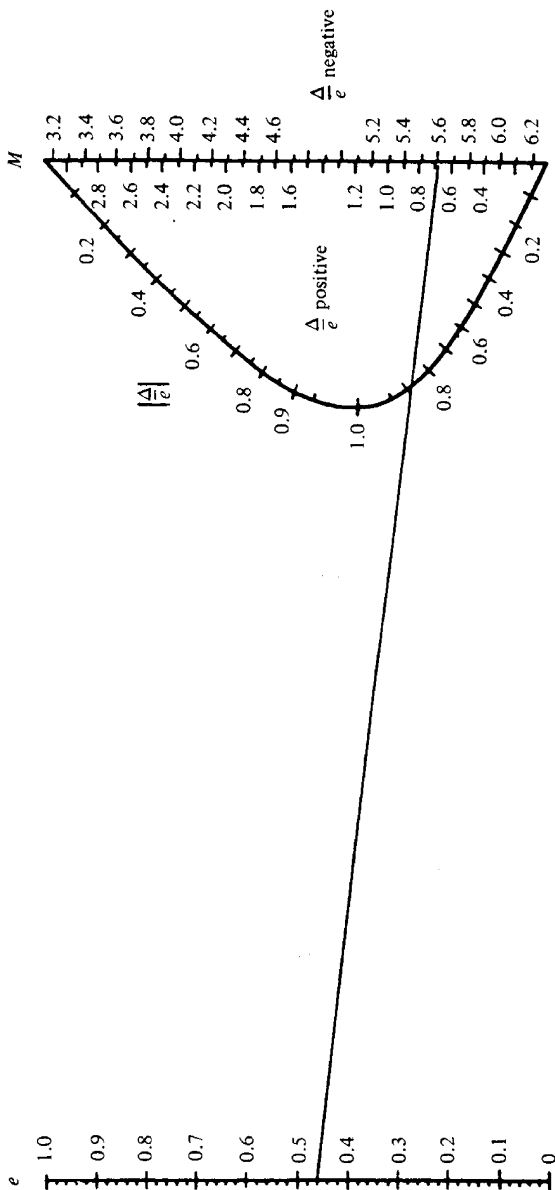


Figure 74. Nomogram to calculate Δ . Vertical bars around Δ/e signify its absolute value.

The method of finding the position of a comet is best clarified by an example. Let us calculate the position of comet Halley at the beginning of the year 1984.

Method	Example
<i>The calculations for the comet</i>	
1. Find the number of years since the epoch.	Y = 1984.0 – 1986.112 = –2.112
2. Find $M_c = \frac{360Y}{T_p}$ degrees. Subtract multiples of 360 to bring the result back into the range 0 to 360, or if negative add 360.	M_c = 349.996 856 degrees
3. Convert M_c to radians by multiplying by $\frac{\pi}{180}$ ($\pi = 3.141\,592\,7$).	M_c = 6.108 598 radians
4. Now we solve Kepler's equation by the routine R2 (§47). First guess (from Kepler's graphs).	E_0 = $M + (-0.8)$ = 5.31 radians
5. Find a more accurate solution using R2.	E = 5.307 696 radians
6. Calculate $\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$ (all angles in radians).	$\tan \frac{v}{2}$ = –4.114 741
7. Take inverse tan and double to get v .	v = –152.680 599 degrees
8. Find $l = v + \varpi$.	l = 17.330 401 degrees
9. Find $r = \frac{a(1-e^2)}{1+e \cos v}$.	r = 8.210 480 AU
10. Calculate $\psi = \sin^{-1} \{\sin(l - \varpi) \sin i\}$.	ψ = –11.503 378 degrees
11. Find $y = \sin(l - \varpi) \cos i$.	y = 0.622 572
12. Find $x = \cos(l - \varpi)$.	x = 0.756 726
13. Find $\tan^{-1}(\frac{y}{x})$ and remove the ambiguity by comparing the signs of x and y with Figure 29. If not in the correct quadrant, add or subtract 180 or 360.	$\tan^{-1}(\frac{y}{x})$ = 39.444 727 degrees
14. Add ϖ to find l' .	l' = 97.598 727 degrees
15. Find $r' = r \cos \psi$.	r' = 8.045 555 AU
<i>Now do the calculations for the Earth</i>	
16. Find $N_E = \frac{360}{365.242\,191} \times \frac{D}{T_E}$, where D is the number of days since 1990.0 (see Table 3). Subtract multiples of 360 to bring the result into the range 0 to 360.	D = –2192 N_E = –2160.452 587 = 359.547 413 degrees
17. Calculate $M_E = N_E + \varepsilon - \varpi$ (see Table 8).	M_E = 356.182 293 degrees
18. Find $L = N_E + \frac{360}{\pi} e \sin(M_E) + \varepsilon$ ($\pi = 3.141\,592\,7$). If the result is more than 360, subtract 360. If the result is negative, add 360.	L = 98.823 197 degrees
19. Calculate $v_E = L - \varpi$.	v_E = –3.945 223 degrees
20. Find $R = \frac{a(1-e^2)}{1+e \cos v_E}$.	R = 0.983 325 AU
21. If r' is less than R , calculate λ by using equation (a); if r' is more than R , use equation (b): (a) $\lambda = 180 + L + \tan^{-1} \left\{ \frac{r' \sin(L-L')}{R-r' \cos(L-L')} \right\}$, (b) $\lambda = \tan^{-1} \left\{ \frac{R \sin(l'-L)}{r'-R \cos(l'-L)} \right\} + l'$. Add 360 to the result if negative.	$r' > R$; use equation (b) λ = 97.428 255 degrees
22. Find $\beta = \tan^{-1} \left\{ \frac{r' \tan \psi \sin(\lambda-l')}{R \sin(l'-L)} \right\}$.	β = –13.052 959 degrees
23. Calculate the right ascension and declination using the method given in §27.	α = 6h 29m δ = 10° 12'
24. Find the distance using the formula given in §57.	ρ = 8.13 AU

The *Astronomical Almanac* gives these values as $\alpha = 6^{\text{h}} 29^{\text{m}}$, $\delta = 10^{\circ} 13'$ and $\rho = 7.2$ AU. We must not expect great precision when dealing with comets as the orbital elements change all the time because of the perturbations to the comet's orbit by the gravitational fields of the planets.

The spreadsheet for this calculation, called Comets, is shown in Figure 75. Once again, we have made use of the spreadsheet function VLOOKUP (e.g. row 16) to look up data in a table contained on a separate page (here simply called Table). We have also provided the spreadsheet formula called TrueAnomaly (row 20) which solves Kepler's equation and returns the value of the true anomaly in radians. Its two arguments are the mean anomaly in radians, and the eccentricity.

The Positions of Elliptical Comets									
1	local civil time (hour)	0	Output	Comet Halley:	=CONCATENATE("Comet ",C11,"")				
2	local civil time (min)	0		RA (hour)	6 =DHour(C36+0.008333)				
3	local civil time (sec)	0		RA (min)	29 =DHMin(C36+0.008333)				
4	daylight saving (hours)	0		dec (deg)	10 =DDDeg(C37+0.008333)				
5	zone correction (hours)	0		dec (min)	12 =DDMin(C37+0.008333)				
6	local date (day)	0		distance from Earth (AU)	8.13 =ROUND(C38,2)				
7	local date (month)	1							
8	local date (year)	1984							
9	comet name	Halley							
10	Greenwich date (day)	31	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
11	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
12	Greenwich date (year)	1983	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
13	time since epoch (years)	-2.112663097	=(CDJD(C13,C14,C15)-CDJD(0.1,C15))/365.242191+C15-VLOOKUP(C11,Table!A3:I17,3,FALSE)						
14	M c (deg)	-10.00628505	=360*C16/VLOOKUP(C11,Table!A3:I17,6,FALSE)						
15	M c (rad)	6.108542687	=RADIANS(C17-360*INT(C17/360))						
16	eccentricity	0.9673	=VLOOKUP(C11,Table!A3:I17,8,FALSE)						
17	true anomaly (deg)	-152.6844037	=DEGREES(TrueAnomaly(C18,C19))						
18	L c (deg)	17.32659625	=C20+VLOOKUP(C11,Table!A3:I17,4,FALSE)						
19	r (AU)	8.21220141	=VLOOKUP(C11,Table!A3:I17,7,FALSE)*(1+C20*C20)/(1+C20*COS(RADIANS(C20)))						
20	L c-node (rad)	-0.71257262	=RADIANS(C21-VLOOKUP(C11,Table!A3:I17,5,FALSE))						
21	psi (rad)	-0.200787457	=ASIN(SIN(C23)*SIN(RADIANS(VLOOKUP(C11,Table!A3:I17,9,FALSE))))						
22	y	0.622619426	=SIN(C23)*COS(RADIANS(VLOOKUP(C11,Table!A3:I17,9,FALSE)))						
23	x	0.756682447	=COS(C23)						
24	L d (deg)	97.60250101	=DEGREES(ATAN2(C26,C25))+VLOOKUP(C11,Table!A3:I17,5,FALSE)						
25	rd (AU)	8.047216892	=C22*COS(C24)						
26	Earth's longitude Le (deg)	458.732849	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)+180						
27	Earth's radius vector (AU)	0.983304665	=SunDist(C3,C4,C5,C6,C7,C8,C9,C10)						
28	L e-L d (rad)	6.302913601	=RADIANS(C29-C27)						
29	A (rad)	-0.002745942	=IF(C28<C30,ATAN2(C30-C28*COS(C31),C28*SIN(C31)),ATAN2(C28-C30*COS(C31),C30*SIN(C31)))						
30	comet long (deg)	97.44517014	=IF(C28<C30,180+C29+DEGREES(C32),DEGREES(C32)+C27)						
31	comet long (deg)	97.44517014	=C33-360*INT(C33/360)						
32	comet lat (deg)	-13.05363642	=DEGREES(ATAN((C28*TAN(C24)*SIN(RADIANS(C33-C27)))/(C30*SIN(C31))))						
33	comet RA (hours)	6.491251781	=DDDH(eccra(C34,0.0,C35,0.0,C13,C14,C15))						
34	comet dec (deg)	10.19802172	=ECDDec(C34,0.0,C35,0.0,C13,C14,C15)						
35	comet distance (AU)	8.126643925	=SORT(C30*2+C22*2-2*C30*C22*COS(RADIANS(C21-C29))*COS(C24))						

A	B	C	D	E	F	G	H	I
Comet name		Epoch	Peri	Node	Period	Axis	Ecc	Incl
Encke		1974.32	160.1	334.2	3.3	2.209	0.847	12
Temple 2		1972.87	310.2	119.3	5.26	3.024	0.549	12.5
Haneda-Campos		1978.77	12.016	131.7	5.37	3.066	0.64152	5.805
Schwassmann-Wachmann 2		1974.7	123.3	126	6.51	3.489	0.386	3.7
Borrelly		1974.36	67.8	75.1	6.76	3.576	0.632	30.2
Whipple		1970.77	18.2	188.4	7.47	3.821	0.351	10.2
Oterma		1958.44	150	155.1	7.88	3.958	0.144	4
Schaumasse		1960.29	138.1	86.2	8.18	4.054	0.705	12
Comas Sola		1969.83	102.9	62.8	8.55	4.182	0.577	13.4
Schwassmann-Wachmann 1		1974.12	334.1	319.6	15.03	6.087	0.105	9.7
Neujmin 1		1966.94	334	347.2	17.93	6.858	0.775	15
Crommelin		1956.82	86.4	250.4	27.89	9.173	0.919	28.9
Olbers		1956.46	150	85.4	69.47	16.843	0.93	44.6
Pons-Brooks		1954.39	94.2	255.2	70.98	17.2	0.955	74.2
Halley		1986.112	170.011	58.154	76.0081	17.9435	0.9673	162.2384

Figure 75. Finding the position of a comet. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

62 Parabolic orbits

In preceding sections, we have calculated the orbits of members of the Solar System which are gravitationally bound to the Sun, like the planets and the periodic comets. These objects move in (more or less) elliptical orbits with the Sun at a focus of the ellipse, and in the absence of perturbations from other members of the Solar System or from external influences, would continue to move indefinitely along the same elliptical paths. However, some comets do not seem to be bound to the Sun. If unperturbed they would appear once and shoot off into space never to return again. Their orbits are often defined in terms of **parabolic motion** and we have to use a slightly different procedure for calculating their positions, given the parabolic orbital elements:

- t_0 = the epoch of the perihelion (a calendar date);
- q = perihelion distance (AU);
- i = inclination of the orbit (degrees);
- ω = argument of the perihelion (degrees; $\omega = \varpi - \varOmega$); and
- \varOmega = longitude of the ascending node (degrees).

The calculations proceed on much the same lines as for an elliptical orbit; once we have found the true anomaly, v , and radius vector, r , we can use exactly the same method as in Section 61 to calculate the position of the comet. However, the calculations of v and r are slightly different.

First, we find the value of the quantity W (in radians) from

$$W = \frac{0.036491\,1624}{q\sqrt{q}} \times d,$$

where d is the number of days since the comet passed through perihelion. Next we have to solve the equation

$$s^3 + 3s - W = 0.$$

This can be done by means of the iterative method shown in *Routine R3* below. Finally, calculate v and r from

$$v = 2 \tan^{-1} s,$$

$$r = q(1 + s^2).$$

(Note that s is in radians.) For example, the International Astronomical Union issued the following data on comet Kohler (designated 1977m; IAU circular number 3137):

- $t = 1977$ November 10.5659
- $q = 0.990\,662$ AU
- $i = 48.719\,6$ degrees
- $\omega = 163.479\,9$ degrees
- $\varOmega = 181.817\,5$ degrees (hence $\varpi = \omega + \varOmega = 345.297\,4$ degrees).

The values of i , ω and \varOmega quoted here were referred to the standard equinox of 1950.0. Strictly, we should refer all our calculations to the same equinox, but we shall ignore the small error introduced by not doing so. Let us calculate the position of the comet on Christmas Day 1977, assuming no perturbations to its orbit.

Method	Example
1. Calculate the number of days since the epoch of perihelion. This can be done (for example) by subtracting the Julian date (§4) of the epoch from the Julian date of the day in question.	10 Nov. 1977: $JD_1 = 2\,443\,457.5$ 25 Dec. 1977: $JD_2 = 2\,443\,502.5$ $JD_2 - JD_1 = 45$ days Epoch = November 10.5659 $\therefore d = 44.4341$ days
2. Find $W = \frac{0.0364911624}{q\sqrt{q}} \times d$.	$W = 1.644\,432$
3. Solve $s^3 + 3s - W = 0$ by using Routine R3.	First guess $s = 0.548\,144$ $s = 0.505\,171$
4. Find $v = 2 \tan^{-1} s$ and $r = q(1 + s^2)$.	$v = 53.603\,189$ degrees $r = 1.243\,477$ AU
5. Carry on at instruction 8 (ignoring instruction 9) of §61 to find α and δ .	$l = 398.900\,589$ degrees $\psi = -26.944\,536$ degrees $l' = 28.320\,864$ degrees $r' = 1.108\,492$ AU
For the Earth:	$D = -736$ days $N_E = -725.407\,420$ degrees $= 354.592\,580$ degrees $L = 93.120\,810$ degrees $R = 0.983\,503$ AU $r' > R$; use equation (b) $\lambda = 336.099\,109$ degrees $\beta = -26.585\,505$ degrees
Here is the result:	$\alpha = 23^h\,17^m$ $\delta = -33^\circ\,41'$

Comet Kohler was in the constellation of Sculptor on Christmas Day 1977.

Routine R3: To solve the equation $s^3 + 3s - W = 0$.

1. First guess put $s = s_0 = W/3$.
2. Calculate $\delta = s^3 + 3s - W$.
3. If $|\delta| < \varepsilon$, go to step 6.
Otherwise proceed with step 4.
 ε is the required accuracy (e.g. 10^{-6} degree).
4. Calculate $s_1 = \frac{2s^3 + W}{3(s^2 + 1)}$ (note that s^3 can be calculated from $s \times s^2$).
5. Set $s = s_1$ and go to step 2.
6. The current value of s is within $\pm\varepsilon$ of the correct value.

The spreadsheet for finding the position of a comet in a parabolic orbit is called Pcomets and is shown in Figure 76. Once again, we have used the spreadsheet function VLOOKUP to select an item of data from a table (e.g. row 17). In this case, the table contains data for only one comet, but you can easily extend it and add more data. Note that the comets need to be listed alphabetically in column A, and that the second argument (at present Table!A4:I4) needs to be altered so that the reference after the colon is to the bottom right-hand cell of the table. For example, if you add two more comets, the second argument would become Table!A4:I6. Do this for every instance of VLOOKUP in the main spreadsheet. We have also provided a

new spreadsheet function called SolveCubic (row 23) that takes one argument, namely the value of W (in radians) as defined above and that returns the value of s (also in radians).

The positions of parabolic comets												
3	Input	local civil time (hour)	0	Output	Comet Kohler:	=CONCATENATE("Comet ",C11,".")						
4		local civil time (min)	0		RA (hour)	23	=DHHour(C40)					
5		local civil time (sec)	0		RA (min)	17	=DHMin(C40)					
6		daylight saving (hours)	0		RA (sec)	11.53	=DHSec(C40)					
7		zone correction (hours)	0		dec (deg)	-33	=DDDeg(C41)					
8		local date (day)	25		dec (min)	42	=DDMin(C41)					
9		local date (month)	12		dec (sec)	26.42	=DDSec(C41)					
10		local date (year)	1977		distance from Earth (AU)	1.11	=ROUND(C42,2)					
11		comet name	Kohler									
13	1	Greenwich date (day)	25	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
14	2	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
15	3	Greenwich date (year)	1977	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
16	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)								
17	5	perihelion epoch (day)	10.5659	=VLOOKUP(C11,Table!A4:J4,3,FALSE)								
18	6	perihelion epoch (month)	11	=VLOOKUP(C11,Table!A4:J4,4,FALSE)								
19	7	perihelion epoch (year)	1977	=VLOOKUP(C11,Table!A4:J4,5,FALSE)								
20	8	pime since epoch (days))	44.4341	=(C16/365.242191)+CDJD(C13,C14,C15)-CDJD(C17,C18,C19)								
21	9	q (AU)	0.990662	=VLOOKUP(C11,Table!A4:J4,8,FALSE)								
22	10	W	1.644431658	=0.0364911624*C20/(C21*SORT(C21))								
23	11	s	0.505171274	=SolveCubic(C22)								
24	12	true anomaly (deg)	53.60318855	=DEGREES(2*ATAN(C23))								
25	13	r (AU)	1.243476977	=C21*(1+C23^C23)								
26	14	Lc (deg)	398.9005886	=C24+VLOOKUP(C11,Table!A4:J4,6,FALSE)+VLOOKUP(C11,Table!A4:J4,7,FALSE)								
27	12	Lc-node (rad)	3.788814646	=RADIANS(C26-VLOOKUP(C11,Table!A4:J4,7,FALSE))								
28	13	psi (rad)	-0.470270859	=ASIN(SIN(C27)*SIN(RADIANS(VLOOKUP(C11,Table!A4:J4,9,FALSE))))								
29	14	y	-0.397807902	=SIN(C27)*COS(RADIANS(VLOOKUP(Pcomets!C11,Table!A4:J18,9,FALSE)))								
30	15	x	-0.797761937	=COS(C27)								
31	16	Ld (deg)	28.32086331	=DEGREES(ATAN2(C30,C29))+VLOOKUP(C11,Table!A4:J4,7,FALSE)								
32	17	rd (AU)	1.108492064	=C25^COS(C28)								
33	18	Earth's longitude Le (deg)	453.0850136	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)+180								
34	19	Earth's radius vector (AU)	0.983497524	=SunDist(C3,C4,C5,C6,C7,C8,C9,C10)								
35	20	Le-Ld (rad)	7.413532968	=RADIANS(C33-C31)								
36	21	A (rad)	-0.911685096	=IF(C32<C34,ATAN2(C34-C32^COS(C35),C32^SIN(C35)),ATAN2(C32-C34^COS(C35),C34^SIN(-C35)))								
37	22	comet long (deg)	-23.91484493	=IF(C32<C34,180+C33+DEGREES(C36),DEGREES(C36)+C31)								
38	23	comet long (deg)	336.0851551	=C37-360*INT(C37/360)								
39	24	comet lat (deg)	-26.59670863	=DEGREES(ATAN((C32^TAN(C28)*SIN(RADIANS(C37-C31)))/(C34^SIN(-C35))))								
40	25	comet RA (hours)	23.28653691	=DDDH(ECRA(C38,0,0,C39,0,0,C13,C14,C15))								
41	26	comet dec (deg)	-33.70733952	=ECDec(C38,0,0,C39,0,0,C13,C14,C15)								
42	27	comet distance (AU)	1.112466142	=SQRT(C34^2+C25^2-2*C34^C25^COS(RADIANS(C26-C33))*COS(C28))								
Pcomets,Table/												

	A	B	C	D	E	F	G	H	I
1	Comet name		Epoch of peri		Arg peri	Node	Peri dist	Incl	
2			Day	Month	Year				
3									
4	Kohler		10.5659	11	1977	163.4799	181.8175	0.990662	48.7196

Figure 76. Finding the position of a comet in a parabolic orbit. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

You can, if you prefer, simplify the main spreadsheet by using three other spreadsheet functions that we have provided for you. They are PcometLong, PcometLat and PcometDist, returning the geocentric ecliptic longitude (degrees), the geocentric ecliptic latitude (degrees) and the distance of the comet from Earth (AU) respectively. Each of them takes the same 15 arguments, namely the local civil time expressed as hours, minutes and seconds, daylight saving and time zone offsets in hours, the local civil date as day, month and year, the Greenwich epoch of perihelion expressed as day, month and year, the perihelion distance in AU, the inclination of the orbit in degrees, the argument of the perihelion in degrees, and finally the longitude of the ascending node in degrees. You will need to use the VLOOKUP function to obtain the arguments corresponding to the orbital elements (we suggest doing this first), and you will also need rows 40 and 41 to convert the ecliptic coordinates into right ascension and declination, but the spreadsheet should look much less complex as a result of using these functions. One example is shown in Figure 77.

The positions of parabolic comets														
Input	local civil time (hour)	0	Output	Comet Kohler:										
	local civil time (min)	0		RA (hour)	23									
	local civil time (sec)	0		RA (min)	17									
	daylight saving (hours)	0		RA (sec)	11.53									
	zone correction (hours)	0		dec (deg)	-33									
	local date (day)	25		dec (min)	42									
	local date (month)	12		dec (sec)	26.42									
	local date (year)	1977		Distance from Earth (AU)	1.11									
	comet name	Kohler												
1	Greenwich date (day)	25	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)											
2	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)											
3	Greenwich date (year)	1977	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)											
4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)											
5	perihelion epoch (day)	10.5659	=VLOOKUP(C11,Table!A4:I4,3,FALSE)											
6	perihelion epoch (month)	11	=VLOOKUP(C11,Table!A4:I4,4,FALSE)											
7	perihelion epoch (year)	1977	=VLOOKUP(C11,Table!A4:I4,5,FALSE)											
8	q (AU)	0.990662	=VLOOKUP(C11,Table!A4:I4,8,FALSE)											
9	inclination (deg)	48.7196	=VLOOKUP(C11,Table!A4:I4,9,FALSE)											
10	perihelion (deg)	163.4799	=VLOOKUP(C11,Table!A4:I4,6,FALSE)											
11	node (deg)	181.8175	=VLOOKUP(C11,Table!A4:I4,7,FALSE)											
12	comet long (deg)	336.085155	=PcometLong(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)											
13	comet lat (deg)	-26.59670863	=PcometLat(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)											
14	comet RA (hours)	23.28653691	=DDDH(ecra(C24,0,0,C25,0,0,C13,C14,C15))											
15	comet dec (deg)	-33.70733952	=ECDec(C24,0,0,C25,0,0,C13,C14,C15)											
16	comet distance (AU)	1.112466142	=PcometDist(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)											

Figure 77. Finding the position of a comet in a parabolic orbit using the Pcomet spreadsheet functions.

63 Binary-star orbits

Quite often an astronomer sees a pair of stars very close together in the telescope. This apparent closeness may be just because two quite unrelated stars happen to lie near to the same line of sight. Sometimes, however, the stars are actually close to one another in space and they may then form a **binary star** in which each is bound to the other by mutual gravitational attraction. The stars describe elliptical orbits about one another, just as Jupiter describes an elliptical orbit about the Sun. The brighter of the two stars is generally called the **primary** and the fainter is called the **companion**; we shall consider that the companion orbits about the primary which is fixed in space, although really both stars orbit about their common centre of mass.

Figure 78 shows the appearance of a binary star. A is the primary, B the companion, and the line NAS is the observer's meridian through A; AN therefore defines the direction north. The line joining A to B is at position-angle θ (measured anticlockwise as shown) and of length ρ . Provided that we know the elements of the binary orbit, we can calculate the values of θ and ρ and hence we can predict the appearance of the binary star at any time.

A binary-star orbit is drawn in Figure 79. The sphere is centred on the primary star, A, and its companion, B, describes an orbit about it shown by the small hatched ellipse in the centre. The great circle NL'DM' shows where the plane through A perpendicular to the line of sight cuts the sphere. This plane is the plane of the sky as seen from the Earth. The line AN defines the direction north as in Figure 78. The great circle L'P'B'M' shows where the plane of the true binary orbit cuts the sphere. The point L' is the projection of the ascending node, L, onto the sphere, M' the projection of the descending node, M, and P' the projection of the point of closest approach, P, the **periastron**. The companion star is at B. Longitudes are reckoned from the ascending node, L, and the true anomaly, v , is the angle between B and the periastron. We need the following elements to calculate the orbit:

- T = the period of revolution;
- t = the epoch of periastron;
- e = the eccentricity of the orbit;
- a = the semi-major axis of the orbit;
- i = the inclination of the orbit to the plane of the sky;
- Ω = the position-angle of the ascending node; and
- ω = the longitude of the periastron.

All angles are measured in the direction of motion. The elements for some binary stars are listed in Table 10.

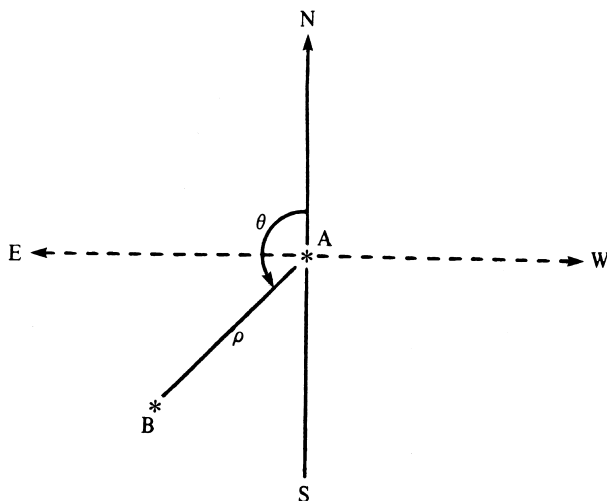


Figure 78. A binary star seen from the Earth.

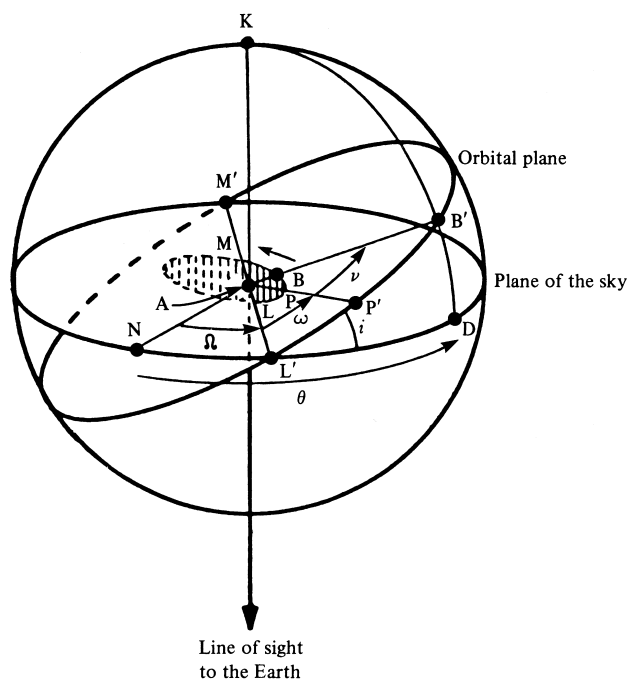


Figure 79. A binary-star orbit.

Name	T_p (years)	t	ω (degrees)	e	a (arcsec)	i (degrees)	Ω (degrees)
η Coronae Borealis	41.623	1934.008	219.907	0.2763	0.907	59.025	23.717
γ Virginis	171.37	1836.433	252.88	0.8808	3.746	146.05	31.78
η Cassiopeiae	480	1889.6	268.59	0.497	11.9939	34.76	278.42
ζ Orionis	1508.6	2070.6	47.3	0.07	2.728	72.0	155.5
α Canis Majoris (Sirius)	50.09	1894.13	147.27	0.5923	7.500	136.53	44.57
δ Geminorum	1200	1437	57.19	0.1100	6.9753	63.28	18.38
α Geminorum (Castor)	420.07	1965.3	261.43	0.33	6.295	115.94	40.47
α Canis Minoris (Procyon)	40.65	1927.6	269.8	0.40	4.548	35.7	284.3
α Centauri	79.920	1955.56	231.560	0.516	17.583	79.240	204.868
α Scorpionis (Antares)	900	1889.0	0.0	0.0	3.21	86.3	273.0

T_p : period; t : epoch of periastron; ω : longitude of periastron; e : eccentricity; a : semi-major axis of orbit; i : inclination of orbit; Ω : position angle of ascending node.

Table 10. The orbital elements of some binary stars.

The calculation of a binary-star orbit proceeds in much the same way as that of a planetary orbit. We first find the mean anomaly, M , from

$$M = \frac{360Y}{T} \text{ degrees,}$$

where Y is the number of years since the epoch of periastron. Next we must solve Kepler's equation

$$E - e \sin E = M \text{ radians,}$$

using the method given in Section 61. The true anomaly, v , and radius vector, r , can then be found from

$$v = 2 \tan^{-1} \left\{ \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right\},$$

and

$$r = a(1 - e \cos E),$$

(remembering that E has been found in radians). Finally, θ is given by

$$\theta = \tan^{-1} \left\{ \frac{\sin(v + \omega) \cos i}{\cos(v + \omega)} \right\} + \oslash \text{ degrees,}$$

and ρ from

$$\rho = \frac{r \cos(v + \omega)}{\cos(\theta - \oslash)} \text{ degrees.}$$

For example, let us calculate the visual aspect of the binary system η Coronae Borealis at the beginning of 1980.

Method	Example
1. Find the number of years since the epoch.	$Y = 1980.0 - 1934.008$ $= 45.992$ years
2. Find $M = \frac{360Y}{P}$. Subtract multiples of 360 to bring the result into the range 0 to 360.	$M = 397.787\,762$ degrees $= 37.787\,762$ degrees
3. Convert to radians by multiplying by $\frac{\pi}{180}$ ($\pi = 3.141\,592\,7$).	$M = 0.659\,521$ radians
4. Solve Kepler's equation $E - e \sin E = M$ by the method outlined in §61.	First guess $E_0 = 0.86$ radians Solution is $E = 0.870\,858$ radians
5. Find $v = 2 \tan^{-1} \left\{ \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right\}$, all angles in radians.	$v = 1.106\,803$ radians
6. Multiply by $\frac{180}{\pi}$ to convert to degrees ($\pi = 3.141\,592\,7$).	$v = 63.415\,137$ degrees
7. Find $r = a(1 - e \cos E)$, remembering that E is expressed in radians.	$r = 0.745\,568$ arcsec
8. Calculate $y = \sin(v + \omega) \cos i$.	$y = -0.500\,814$
9. Calculate $x = \cos(v + \omega)$.	$x = 0.230\,426$
10. Find $\tan^{-1} \left(\frac{y}{x} \right)$ and remove the ambiguity by reference to Figure 29. Add or subtract 180 or 360 to bring the result into the correct quadrant, unless it is already in the correct quadrant.	$\tan^{-1} \left(\frac{y}{x} \right) = -65.292\,744$ $+ 360$ $= 294.707\,256$ degrees
11. Add φ to find θ . Subtract 360 if more than 360. Add 360 if negative.	$\theta = 318.424$ degrees
12. Find $\rho = \frac{r \cos(v + \omega)}{\cos(\theta - \varphi)}$.	$\rho = 0.411$ arcsec

Figure 80 shows the spreadsheet for the binary-star calculation. As before, we have put the orbital elements of some binary stars into a data table on a separate page. This table reproduces the data of Table 10, but it represents only a small fraction of the known binary stars. You can add to the table as you wish. The VLOOKUP function (e.g. row 8) is used to select each item from the table as needed. If you extend the table, remember (i) to order the stars in alphabetical order, and (ii) to make sure that the cell reference after the colon in the second argument is to the bottom right-hand cell of the table. We have defined and used a new spreadsheet function EccentricAnomaly (row 13). This solves Kepler's equation and returns the value of the eccentric anomaly in radians for the two arguments, namely the mean anomaly in radians, and the eccentricity. We have already met the function TrueAnomaly (Section 61), which takes the same pair of arguments but returns the true anomaly.

Binary star orbits										
1	Binary star orbits									
2										
3	Input	Greenwich date (day)	0	Output	binary star eta-Cor:	=CONCATENATE("binary star ",C6,"")				
4		Greenwich date (month)	1		position angle (deg)	318.4	=ROUND(C19,1)			
5		Greenwich date (year)	1980		separation (arcsec)	0.41	=ROUND(C20,2)			
6		binary abbreviated name	eta-Cor							
7										
8	1	Y (years)	45.992	=(C5+(CDJD(C3,C4,C5)-CDJD(0,1,C5))/365.242191)-VLOOKUP(C6,Table!A3:I12,4,FALSE)						
9	2	M (deg)	397.7877616	=360*C8/VLOOKUP(C6,Table!A3:I12,3,FALSE)						
10	3	M (rad)	0.659520856	=RADIANS(C9-360*INT(C9/360))						
11	4	eccentricity	0.2763	=VLOOKUP(C6,Table!A3:I12,6,FALSE)						
12	5	true anomaly (rad)	1.106802937	=TrueAnomaly(C10,C11)						
13	6	r (arcsec)	0.745568165	=(1-C11*COS(EccentricAnomaly(C10,C11)))*VLOOKUP(C6,Table!A3:I12,7,FALSE)						
14	7	TA+peri (rad)	4.944904136	=C12+RADIANS(VLOOKUP(C6,Table!A3:I12,5,FALSE))						
15	8	y	-0.500814365	=SIN(C14)*COS(RADIANS(VLOOKUP(C6,Table!A3:I12,8,FALSE)))						
16	9	x	0.230425722	=COS(C14)						
17	10	A (deg)	-65.29274351	=DEGREES(ATAN2(C16,C15))						
18	11	theta (deg)	-41.57574351	=C17+VLOOKUP(C6,Table!A3:I12,9,FALSE)						
19	12	theta (deg)	318.4242565	=C18-360*INT(C18/360)						
20	13	rho (arcsec)	0.411017766	=C13*COS(C14)/COS(RADIANS(C19-VLOOKUP(C6,Table!A3:I12,9,FALSE)))						

Binary star data table								
Binary name	Period	Epoch of peri	Long peri	Eccentricity	Axis	Incl	PA of node	
alpha Sco	900	1889	0	0	3.21	86.3	273	
alpha-Cen	79.92	1955.56	231.56	0.516	17.583	79.24	204.868	
alpha-CMa	50.09	1894.13	147.27	0.5923	7.5	136.53	44.57	
alpha-Gem	420.07	1965.3	261.43	0.33	6.295	115.94	40.47	
alpha-CMi	40.65	1927.6	269.8	0.4	4.548	35.7	284.3	
delta-Gem	1200	1437	57.19	0.11	6.9753	63.28	18.38	
eta-Cas	480	1889.6	268.59	0.497	11.9939	34.76	278.42	
eta-Cor	41.623	1934.008	219.907	0.2763	0.907	59.025	23.717	
gamma-Vir	171.37	1836.433	252.88	0.8808	3.746	146.05	31.78	
zeta-Ori	1508.6	2070.6	47.3	0.07	2.728	72	155.5	

Figure 80. Calculating a binary-star orbit. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

The Moon and eclipses

Of all the heavenly bodies visible in the night from the Earth, the Moon is the most spectacular. It far outshines even the most brilliant planet, moves so quickly that you can see its motion against the stars, and provides a wealth of detail in the shadowy features of its disc. Yet its motion is the most difficult to predict and it is for that reason we have left it until last. It is, of course, in orbit about the Earth but the Sun and other members of the Solar System perturb that orbit to such an extent that many corrections are needed to calculate the Moon's position accurately.

In the next few sections we use a simple method to find the position of the Moon. The method takes account only of the principle perturbations to the orbit yet gives results which are accurate enough for most purposes. (We have also provided spreadsheet functions which have much higher accuracy.) We calculate the times of moonrise and moonset, the phases of the Moon, and the circumstances of both solar and lunar eclipses. Finally, we show how to construct an astronomical calendar, bringing together the changing positions of all the Solar-System objects over the course of one year onto a single page. The calculations are lengthy but the satisfaction you feel when you predict, for example, the occurrence of a lunar eclipse, cannot be denied.

64 The Moon's orbit

To an Earth-bound observer, the Moon appears to be in orbit about the Earth, making one complete revolution with respect to the background of stars in 27.321 7 days. This period is called the **sidereal month**. During this time the Earth moves on along its own orbit so that the Sun's position changes with respect to the stars. Hence the Moon has some extra distance to make up to regain its position relative to the Sun. The interval defined by the time taken for the Moon to return to the same position relative to the Sun is called the **synodic month** and is equal to 29.530 6 days. The direction of motion of the Moon in its orbit about the Earth is **prograde**; that is, it is in the same sense as that of all the planets about the Sun.

A celestial observer viewing the Solar System from a great distance would not, however, see the Moon making loops in space about the Earth. Rather, he or she would describe the situation by saying that the Moon is in orbit around the Sun, as is the Earth, and that the effect of the Earth's influence is to make the Moon's orbit wiggle a little as the relative positions of Earth and Moon change (Figure 81). This is because the Sun's gravitational force on the Moon is much greater than that of the Earth, even though the latter is nearer. It is hardly surprising that the orbit of the Moon is so complicated to calculate since it is regulated by two bodies, not one, and the two bodies are themselves tied in orbit about each other.

For the purposes of our calculations, we are going to imagine that both the Sun and the Moon are in orbit about the Earth. We have already calculated the position of the Sun by these means in Section 46. We will need those calculations in the next few sections to find the magnitude of some of the corrections to the Moon's orbit.

There are three main effects of the perturbations caused by the Sun on the Moon's apparent orbit round the Earth. The first of these is called **evection** in which the apparent value of the eccentricity of the Moon's orbit varies slightly. The second is due to the variation of the Earth–Sun distance as the Earth travels in its own ellipse about the Sun. This correction is called the **annual equation**. The third inequality takes account of the motion of the Moon in the Sun's gravitational field. When the Moon is on one side of the Earth it is nearer the Sun so that the Sun's gravitational attraction is slightly more than when the Moon is on the other side of the Earth. This correction is called the **variation**. These corrections alone, together with the usual correction called the **equation of the centre**, can make up to 9° difference in the Moon's mean anomaly, so it is important that they be taken into account. We shall make six corrections in all to

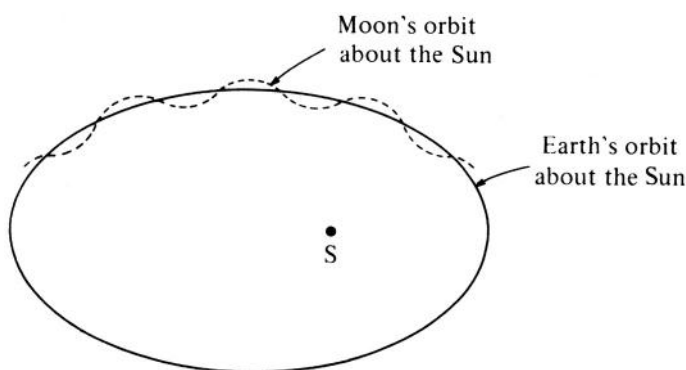


Figure 81. The Moon's orbit, much exaggerated. The Moon is much closer to the Earth than suggested by this diagram; in particular, its orbit is everywhere concave towards the Sun.

find the position of the Moon to within one fifth of a degree. (See spreadsheet MoonPos2 for more precise calculations using spreadsheet functions that employ a better numerical model of the Moon's orbit.)

The apparent motions of the Moon and the Sun about the Earth are drawn in Figure 82. This diagram is similar to that of Figure 63 except that here the Earth is at the centre and both the Sun and the Moon describe ellipses about the Earth. Once again you are to imagine that you are looking at the Solar System from a great distance and, further, that you are moving in such a manner that the Earth appears to be stationary in your view. The large sphere is centred on the Earth, E, and the planes of the orbits of Sun and Moon are projected to cut the sphere along the circles $\Upsilon N'_1 S' N'_2$ and $N'_1 P' m' N'_2$ respectively. S' is the projection of the Sun onto this sphere and its longitude, measured from the first point of Aries, Υ , is denoted by λ_{\odot} . The Moon's orbit is inclined to the ecliptic at an angle i ; N'_1 and N'_2 are the projections of the ascending and descending nodes, P' is the projection of the Moon's perigee, and m' is the projection of the present position of the Moon. The longitude of the ascending node is Ω , the longitude of the perigee is $\Omega + \omega$ and the Moon's true anomaly is ν .

There are two principal effects of the perturbations mentioned above. The first is that the perigee of the Moon's orbit, unlike the (nearly) stationary perihelia of the planets' orbits, advances (prograde) at such a rate that it makes one complete revolution in about 8.85 years. The second is that the line joining the nodes, $N'_1 N'_2$, moves backwards (**retrograde**) around the ecliptic so that it makes one complete revolution in about 18.61 years. Yet another month can be defined by the time it takes the Moon to return to its ascending node. This is the **draconic** or **nodal month** and it is equal to 27.2122 days.

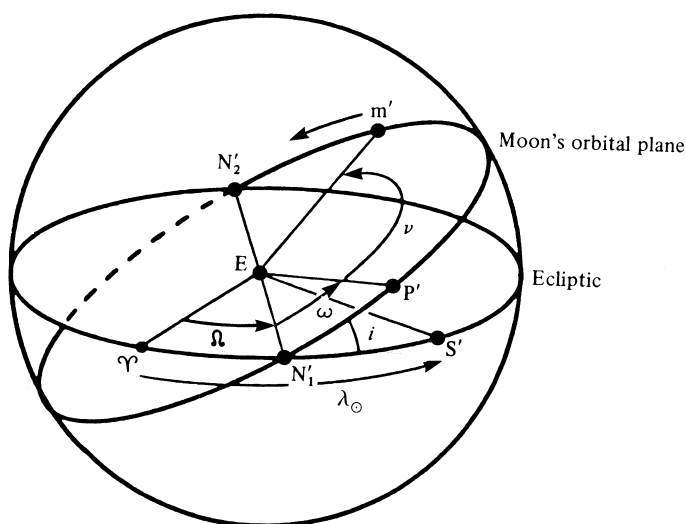


Figure 82. Defining the Moon's orbit.

65 Calculating the Moon's position

The steps involved in the process of finding the position of the Moon are much the same as those involved in calculating the position of a planet, except that (i) correction terms have to be applied at every step and (ii) the longitudes of the ascending node and perigee cannot be regarded as constant. We first determine the Moon's mean anomaly, M_m , which refers to the position of a fictitious Moon in uniform circular motion about the Earth. Then we find the longitude, and, by referring it to the plane of the ecliptic, the geocentric ecliptic coordinates λ_m and β_m . Finally, we convert to right ascension and declination using the method given in Section 27.

Once again we choose the epoch 2010 January 0.0 as our starting point. We calculate the number of days, D , since the epoch to the required date and time, counting the time of day as a fraction of a day. For slightly better accuracy we should use terrestrial time, TT, rather than the universal time, UT (see Section 16). Then we find:

- (a) the Sun's ecliptic longitude, λ_{\odot} , and mean anomaly, M_{\odot} , by the method given in Section 46;
- (b) the Moon's mean longitude, l , given by

$$l = 13.1763966D + l_0;$$

- (c) the Moon's mean anomaly, M_m , given by

$$M_m = l - 0.1114041D - P_0;$$

- (d) the ascending node's mean longitude, N , given by

$$N = N_0 - 0.0529539D.$$

The symbols l_0 , P_0 and N_0 represent the mean longitudes at the epoch.

Next we calculate the corrections for evection, E_v , the annual equation, A_e , and a third correction, A_3 :

$$E_v = 1.2739 \sin(2C - M_m),$$

$$A_e = 0.1858 \sin(M_{\odot}),$$

$$A_3 = 0.37 \sin(M_{\odot}),$$

where $C = l - \lambda_{\odot}$. With these corrections we can find the Moon's corrected anomaly, M'_m :

$$M'_m = M_m + E_v - A_e - A_3.$$

We can now find the correction for the equation of the centre:

$$E_c = 6.2886 \sin(M'_m).$$

Yet another correction term must be calculated:

$$A_4 = 0.214 \sin(2M'_m).$$

Now we can find the value of the Moon's corrected longitude, l' , from

$$l' = l + E_v + E_c - A_e + A_4.$$

The final correction to apply to the Moon's longitude is the variation, V , given by

$$V = 0.6583 \sin 2(l' - \lambda_{\odot}).$$

Then the Moon's true orbital longitude, l'' , is just

$$l'' = l' + V.$$

Referring the longitude to the ecliptic allows us to calculate the ecliptic latitude, β_m , and longitude, λ_m . Thus

$$\lambda_m = \tan^{-1} \left\{ \frac{\sin(l'' - N') \cos i}{\cos(l'' - N')} \right\} + N',$$

and

$$\beta_m = \sin^{-1} \{ \sin(l'' - N') \sin i \},$$

where N' is the corrected longitude of the node, and it is given by

$$N' = N - 0.16 \sin(M_{\odot}).$$

This is a lengthy calculation! Let us illustrate it with an example: what was the position of the Moon on 1 September 2003 at 0h UT? The values of l_0 , P_0 , N_0 , i , and some other parameters of the Moon's orbit are listed in Table 11.

Moon's mean longitude at the epoch	l_0	=	91.929336 degrees
mean longitude of the perigee at the epoch	P_0	=	130.143076 degrees
mean longitude of the node at the epoch	N_0	=	291.682547 degrees
inclination of Moon's orbit	i	=	5.145396 degrees
eccentricity of the Moon's orbit	e	=	0.0549
semi-major axis of Moon's orbit	a	=	384 401 km
Moon's angular diameter at distance a from the Earth	θ_0	=	0.5181 degrees
Moon's parallax at distance a from the Earth	π_0	=	0.9507 degrees

Table 11. Elements of the Moon's orbit, epoch 2010.0.

Method	Example		
1. Find the number of days since 2010 January 0.0 (§3). Remember to count the hours, minutes and seconds as a fraction of a day. The total is D .	1 September	=	243 + 1
		=	244
		–	2 557 days
2. Find λ_{\odot} and M_{\odot} using the method of §46.	D	=	–2313 days
	λ_{\odot}	=	158.171 829 degrees
	M_{\odot}	=	238.533 547 degrees
3. Find $l = 13.176\,396\,6D + l_0$. Adjust to the range 0 to 360 by adding or subtracting multiples of 360.	l	=	214.924 000 degrees
4. Find $M_m = l - 0.111\,404\,1D - P_0$. Adjust to the range 0 to 360.	M_m	=	342.458 607 degrees
5. Find $N = N_0 - 0.052\,953\,9D$. Adjust to the range 0 to 360.	N	=	54.164 917 degrees
6. $E_v = 1.2739 \sin(2C - M_m)$, where $C = l - \lambda_{\odot}$.	E_v	=	0.960 757 degrees
7. Find $A_e = 0.1858 \sin(M_{\odot})$ and $A_3 = 0.37 \sin(M_{\odot})$.	A_e	=	–0.158 477 degrees
	A_3	=	–0.315 590 degrees
8. Find the corrected anomaly: $M'_m = M_m + E_v - A_e - A_3$.	M'_m	=	343.893 432 degrees
9. Calculate $E_c = 6.2886 \sin(M'_m)$.	E_c	=	–1.744 614 degrees
10. Calculate $A_4 = 0.214 \sin(2M'_m)$.	A_4	=	–0.114 077 degrees
11. Find $l' = l + E_v + E_c - A_e + A_4$.	l'	=	214.184 544 degrees
12. Find $V = 0.6583 \sin[2(l' - \lambda_{\odot})]$.	V	=	0.610 256 degrees
13. Hence find the true longitude $l'' = l' + V$.	l''	=	214.794 800 degrees
14. Find $N' = N - 0.16 \sin(M_{\odot})$.	N'	=	54.301 389 degrees
15. Find $y = \sin(l'' - N') \cos i \dots$	y	=	0.332 570
16. \dots and $x = \cos(l'' - N')$.	x	=	–0.942 603
17. Calculate $\tan^{-1}(\frac{y}{x})$. Remove the ambiguity by reference to Figure 29, adding or subtracting 180 or 360 to bring the result into the correct quadrant unless it is already there.	$\tan^{-1}(\frac{y}{x})$	=	–19.433 905
		+	180
		=	160.566 095 degrees
18. Add N' to find λ_m .	λ_m	=	214.867 503 degrees
19. Find $\beta_m = \sin^{-1}\{\sin(l'' - N') \sin i\}$.	β_m	=	1.716 074 degrees
20. Finally, convert λ_m and β_m to right ascension and declination (§27).	α_m	=	14h 12m 42s
	δ_m	=	–11° 31' 38''

The *Astronomical Almanac* gives the apparent coordinates of the Moon at 0h TT as $\alpha = 14\text{h } 12\text{m } 10\text{s}$ and $\delta = -11^\circ 34' 52''$. We may generally expect an error of about a quarter of a degree in ecliptic coordinates (but see MoonPos2 below for a more precise spreadsheet method). This is illustrated in Figure 83 where the error, Δ , between λ_m calculated by this method and that calculated by MoonPos2 is drawn as a function of the date for early 2011.

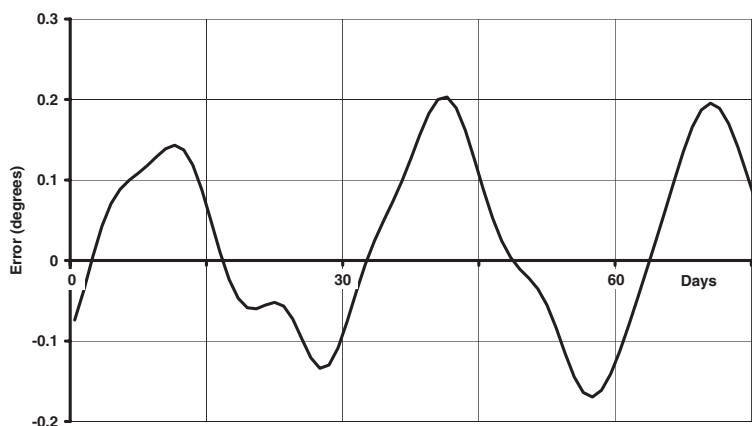


Figure 83. The error, between the ecliptic coordinates of the Moon as calculated by the method given here and those calculated by MoonPos2, for early 2011.

The spreadsheet, labelled MoonPos1 for this calculation, is shown in Figure 84. We have used cells I12 to I15 for the relevant orbital elements of the Moon, rather than having a table on a separate worksheet, since only one body (the Moon) is involved in this case. There is no need, either, to use the VLOOKUP function in this case. We have also cheated a bit by defining new spreadsheet functions SunMeanAnomaly (row 18) and UnwindDeg (e.g. row 19). The former takes eight arguments, which are the local civil time expressed as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the local civil date as day, month, and year. It returns the Sun's mean anomaly for the given instant in radians. The other function, UnwindDeg, takes a single argument of an angle in degrees and returns the equivalent angle in the range 0 to 360 degrees.

We have provided three additional spreadsheet functions called MoonLong, MoonLat, and MoonHP, returning respectively the Moon's geocentric ecliptic longitude, latitude and horizontal parallax (Section 69) in degrees. All three take the same eight arguments as SunMeanAnomaly above. The algorithms behind the functions use a full numerical model to calculate the respective values, and are accurate to within approximately 10, 5 and 0.5 arcseconds respectively over many hundreds of years. These have been used in the spreadsheet MoonPos2 (Figure 85), which also calculates the Earth–Moon distance in kilometres, and the Moon's horizontal parallax in degrees. Note that these are related (row 21) as

$$\text{distance} = \frac{6378.14}{\sin(\text{horizontal parallax})},$$

so in fact the Moon's orbital parameters a and e are not actually required. The number 6378.14 represents the radius of the Earth in kilometres. Also of note is the correction in row 19 for the effects of nutation in longitude.

	A	B	C	D	E	F	G	H	I	J
1	The position of the Moon (approximate method)									
2										
3	<i>Input</i>	local time (hour)	0		<i>Output</i>	Moon's RA (hour)	14 =DHHour(C36)			
4		local time (min)	0			Moon's RA (Min)	12 =DHMin(C36)			
5		local time (sec)	0			Moon's RA (sec)	42.31 =DHSec(C36)			
6		daylight saving (hours)	0			Moon's dec (deg)	-11 =DDDeg(C37)			
7		zone correction (hours)	0			Moon's dec (min)	31 =DDMin(C37)			
8		local date (day)	1			Moon's dec (sec)	38.27 =DDSec(C37)			
9		local date (month)	9							
10		local date (year)	2003							
11										
12	1	Greenwich date (day)	1=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)					L 0	91.92933599	
13	2	Greenwich date (month)	9=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)					P 0	130.1430763	
14	3	Greenwich date (year)	2003=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)					N 0	291.6825466	
15	4	UT (hours)	0=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)					I	5.145396	
16	5	D (days)	-2313=CDJD(C12,C13,C14)-CDJD(0,1,2010)+C15/24							
17	6	Sun long (deg)	158.171829=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)							
18	7	Sun mean anomaly (rad)	4.16319578=SunMeanAnomaly(C3,C4,C5,C6,C7,C8,C9,C10)							
19	8	L m (deg)	214.9240002=UnwindDeg(13.1763966*C16+I12)							
20	9	M m (deg)	342.4586072=UnwindDeg(C19-0.1114041*C16-I13)							
21	10	N (deg)	54.16491734=UnwindDeg(I14-(0.0529539*C16))							
22	11	E v (deg)	0.960757104=1.2739*SIN(RADIANS(2*(C19-C17)-C20))							
23	12	A e (deg)	-0.158477357=0.1858*SIN(C18)							
24	13	A 3 (deg)	-0.315590001=0.37*SIN(C18)							
25	14	M md (deg)	343.8934316=C20+C22-C23-C24							
26	15	E c (deg)	-1.744613565=6.2886*SIN(RADIANS(C25))							
27	16	A 4 (deg)	-0.114077039=0.214*SIN(2*RADIANS(C25))							
28	17	L d (deg)	214.184544=C19+C22+C26-C23+C27							
29	18	V (deg)	0.610255619=0.6583*SIN(2*RADIANS(C28-C17))							
30	19	L dd (deg)	214.7947997=C28+C29							
31	20	N d (deg)	54.30138869=C21-0.16*SIN(C18)							
32	21	y	0.33256969=SIN(RADIANS(C30-C31))*COS(RADIANS(I15))							
33	22	x	-0.942603097=COS(RADIANS(C30-C31))							
34	23	Moon long (deg)	214.8675028=UnwindDeg(DEGREES(ATAN2(C33,C32))+C31)							
35	24	Moon lat (deg)	1.716074358=DEGREES(ASIN(SIN(RADIANS(C30-C31))*SIN(RADIANS(I15))))							
36	25	Moon RA (hours)	14.21175359=DDDH(ECRA(C34,0,0,C35,0,0,C12,C13,C14))							
37	26	Moon dec (deg)	-11.52729751=ECDDec(C34,0,0,C35,0,0,C12,C13,C14)							

Figure 84. Finding the approximate position of the Moon.

	A	B	C	D	E	F	G	H	I	J
1	The position of the Moon (higher precision method)									
2										
3	<i>Input</i>	local time (hour)	0		<i>Output</i>	Moon RA (hour)	14	=DHHour(C22)		
4		local time (min)	0			Moon RA (min)	12	=DHMin(C22)		
5		local time (sec)	0			Moon RA (sec)	10.21	=DHSec(C22)		
6		daylight saving (hours)	0			Moon dec (deg)	-11	=DDDdeg(C23)		
7		zone correction (hours)	0			Moon dec (min)	34	=DDMin(C23)		
8		local date (day)	1			Moon dec (sec)	57.83	=DDSec(C23)		
9		local date (month)	9			Earth-Moon dist (km)	367964	=ROUND(C21,0)		
10		local date (year)	2003			Moon hor parallax (deg)	0.993191	=ROUND(C20,6)		
11										
12	1	Greenwich date (day)	1	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
13	2	Greenwich date (month)	9	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
14	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
15	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)						
16	5	Moon ecliptic longitude (deg)	214.7660758	=MoonLong(C3,C4,C5,C6,C7,C8,C9,C10)						
17	6	Moon elciptic latitude (deg)	1.620130541	=MoonLat(C3,C4,C5,C6,C7,C8,C9,C10)						
18	7	nutatation in longitude (deg)	-0.003683653	=NutatLong(C12,C13,C14)						
19	8	corrected long (deg)	214.7623922	=C16+C18						
20	9	Moon horizontal parallax (deg)	0.993190567	=MoonHP(C3,C4,C5,C6,C7,C8,C9,C10)						
21	10	Earth-Moon distance (km)	367964.4345	=6378.14/SIN(RADIANS(C20))						
22	11	Moon RA (hours)	14.20283692	=DDDH(ECRA(C19,0,0,C17,0,0,C12,C13,C14))						
23	12	Moon dec (deg)	-11.58273038	=ECDeg(C19,0,0,C17,0,0,C12,C13,C14)						

Figure 85. Finding the position of the Moon using a more precise method.

66 The Moon's hourly motions

The calculation which we have to do to find the position of the Moon is a lengthy affair and needs great care in its execution to avoid making mistakes. It may be that you require the position at several different times during one day and, rather than repeat the calculation several times, it is sufficient to find the position once and then extrapolate to the other times using the values for the hourly motions of the Moon in ecliptic latitude and longitude. These motions are given by the formulas

$$\Delta\beta = 0.05 \cos(l'' - N') \text{ degrees/hour,}$$

$$\Delta\lambda = 0.55 + 0.06 \cos(M'_m) \text{ degrees/hour,}$$

where $\Delta\beta$ is the hourly motion in latitude and $\Delta\lambda$ is the hourly motion in longitude. Given a position λ_0 , β_0 at time t_0 , the position t hours later is simply

$$\beta = \beta_0 + \Delta\beta t,$$

$$\lambda = \lambda_0 + \Delta\lambda t.$$

Continuing the previous example, what were the Moon's ecliptic coordinates at 3h 30m TT on 1 September 2003?

Method	Example
1. Write down λ_0 , β_0 , (these are the values of λ_m , β_m at time t_0), l'' , N' , M'_m at t_0 (§65).	$\lambda_0 = 214.867\,503$ degrees $\beta_0 = 1.716\,074$ degrees $l'' = 214.794\,800$ degrees $N' = 54.301\,389$ degrees $M'_m = 343.893\,432$ degrees $t_0 = 0.0$ hours
2. Calculate $\Delta\beta = 0.05 \cos(l'' - N')$ and $\Delta\lambda = 0.55 + 0.06 \cos(M'_m)$.	$\Delta\beta = -0.047\,130$ degrees/hour $\Delta\lambda = 0.607\,645$ degrees/hour
3. Find t in hours: $t = \text{new time} - t_0$, both times expressed in decimal hours.	$t = 3.5$ hours
4. Find the coordinates at the new time: $\beta = \beta_0 + \Delta\beta t$, $\lambda = \lambda_0 + \Delta\lambda t$.	$\beta = \mathbf{1.551\,118 \text{ degrees}}$ $\lambda = \mathbf{216.994\,260 \text{ degrees}}$

We have not provided a spreadsheet for this straightforward calculation because, if you are using a spreadsheet to find the Moon's position, you will probably want to calculate the Moon's position directly for every instance rather than use the calculation given above.

67 The phases of the Moon

The relative positions of the Sun and the Moon as viewed from the Earth change during the course of one month. It is always the hemisphere of the Moon facing towards the Sun which is brightly illuminated but we on the Earth see only that half which faces us. Unless the Moon is in opposition to the Sun, the time of full Moon, our half is not uniformly illuminated but overlaps both the bright and dark sides; hence we see only a segment of the disc. The area of the segment expressed as a fraction of the whole disc is called the **phase**.

The variation of phase with the Moon's position is illustrated in Figure 86, showing a plan view of the Moon's orbit about the Earth, E. The Moon is drawn in five positions marked 1 to 5. At 1, the whole of the dark side is turned towards us so that unless the Moon is illuminated by sufficient **earthshine** it is invisible. This is the new Moon. One week later the Moon has reached position 3 and is said to be in **quadrature**. This is the first quarter. Position 4 is the full Moon, the point of opposition with the Sun. At position 5, the Moon is again in quadrature; this is the third quarter. Between positions 3 and 5 more than half of the Moon's face is illuminated and the Moon is said to be **gibbous**.

In Figure 86, the angle D is called the **age** of the Moon, varying from 0° to 360° as the Moon completes one cycle of its orbit. Sometimes this angle is expressed in days, 1 day being equivalent to about 13° . The phase, F , is given by

$$F = \frac{1}{2} (1 - \cos D).$$

We have already made most of the calculations to find D in Section 65. Referring again to Figure 86, we find

$$D = l'' - \lambda_{\odot} \text{ degrees.}$$

Continuing the example of Section 65, we will find the phase of the Moon at 0h TT on 1 September 2003.

Method	Example
1. Find the values of λ_{\odot} and l'' , using the methods of §§46 and 65.	$\lambda_{\odot} = 158.171\,829$ degrees $l'' = 214.794\,800$ degrees
2. Find $D = l'' - \lambda_{\odot}$.	$D = 56.622\,971$ degrees
3. Calculate $F = \frac{1}{2} (1 - \cos D)$.	$F = \mathbf{0.225}$

The *Astronomical Almanac* reports the fraction of the disc which is illuminated (the phase) as 0.226 for this time.

The spreadsheet MoonPhase, Figure 87, provides a spreadsheet version of this calculation. The relevant rows are 12 to 18, the rest (19 to 26) being to do with calculating the position-angle of the bright limb of the Moon (next section). We have also provided a (slightly more accurate) spreadsheet function to calculate the phase explicitly. It is called MoonPhase and it takes eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. Thus, having saved a copy of the spreadsheet, you could delete rows 12 to 18, and place the following spreadsheet formula in cell H3:

$$=\text{ROUND}(\text{MoonPhase}(\text{C3},\text{C4},\text{C5},\text{C6},\text{C7},\text{C8},\text{C9},\text{C10}),2).$$

You can play a similar trick with rows 19 to 26 (see the next section).

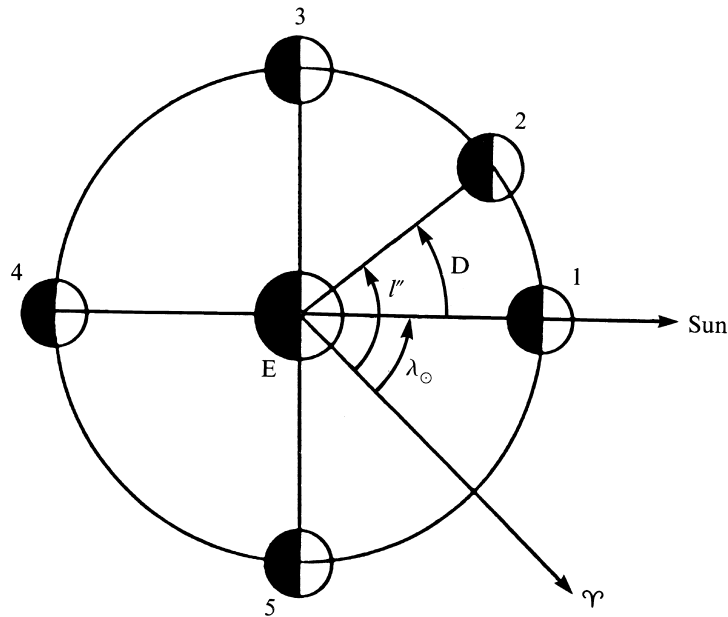


Figure 86. The phases of the Moon.

One of the important steps in calculating an eclipse of the Sun or Moon (Sections 71–74) is to find the times of new Moon and full Moon, that is the moments when the Moon is in **conjunction** or **opposition** to the Sun respectively. To help you to do this, we have provided a spreadsheet called **MoonNewFull** (Figure 88), and spreadsheet functions called **NewMoon** and **FullMoon**. These functions are used in the spreadsheet at rows 16 and 17, and each takes five arguments, namely the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. The functions return the Julian dates of the instances of new Moon and full Moon associated with the lunar cycle (the **lunation**) in progress on the given date. The rest of the spreadsheet, rows 18 to 29, turns the Julian dates into local civil times and calendar dates.

	A	B	C	D	E	F	G	H	I	J
1	The phase and the position angle of the bright limb of the Moon									
2										
3	<i>Input</i>	local time (hour)	0		<i>Output</i>	Moon phase (1 is full)	0.22	=ROUND(C18,2)		
4		local time (min)	0			PA bright limb (deg)	-71.58	=ROUND(C26,2)		
5		local time (sec)	0							
6		daylight saving (hours)	0							
7		zone correction (hours)	0							
8		local date (day)	1							
9		local date (month)	9							
10		local date (year)	2003							
11										
12	1	Greenwich date (day)	1	=LCTGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
13	2	Greenwich date (month)	9	=LCTGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
14	3	Greenwich date (year)	2003	=LCTGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
15	4	Sun longitude (deg)	158.171829	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)						
16	5	MoonLong (deg)	214.7660758	=MoonLong(C3,C4,C5,C6,C7,C8,C9,C10)						
17	6	D (rad)	0.987755945	=RADIANS(C16-C15)						
18	7	Moon phase (0-1)	0.224717717	=(1-COS(C17))/2						
19	8	Moon latitude (deg)	1.620130541	=MoonLat(C3,C4,C5,C6,C7,C8,C9,C10)						
20	9	Sun RA (rad)	2.789425109	=RADIANS(ECRA(C15,0,0,0,0,0,C12,C13,C14))						
21	10	Moon RA (rad)	3.718355856	=RADIANS(ECRA(C16,0,0,0,C19,0,0,C12,C13,C14))						
22	11	Sun dec (rad)	0.148454791	=RADIANS(ECDec(C15,0,0,0,0,0,C12,C13,C14))						
23	12	Moon dec (rad)	-0.20217822	=RADIANS(ECDec(C16,0,0,0,C19,0,0,C12,C13,C14))						
24	13	y	-0.792170114	=COS(C22)*SIN(C20-C21)						
25	14	x	0.263794374	=COS(C23)*SIN(C22)-SIN(C23)*COS(C22)*COS(C20-C21)						
26	15	chi (deg)	-71.58212927	=DEGREES(ATAN2(C25,C24))						

Figure 87. Finding the phase and position-angle of the bright limb of the Moon.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	The times of new Moon and full Moon												
2													
3	Input	daylight saving (hours)	0	Output	new Moon instant:								
4		zone correction (hours)	0		local time (hour)	17	=DHHour(C28)						
5		local date (day)	1		local time (min)	27	=DHMin(C28)						
6		local date (month)	9		local date (day)	27	=UTLcDay(C26,0,0,C6,C7,C19,C20,C21)						
7		local date (year)	2003		local date (month)	8	=UTLcMonth(C26,0,0,C6,C7,C19,C20,C21)						
8					local date (year)	2003	=UTLcYear(C26,0,0,C6,C7,C19,C20,C21)						
9					full Moon instant:								
10					local time (hour)	16	=DHHour(C29)						
11					local time (min)	36	=DHMin(C29)						
12					local date (day)	10	=UTLcDay(C27,0,0,C6,C7,C23,C24,C25)						
13					local date (month)	9	=UTLcMonth(C27,0,0,C6,C7,C23,C24,C25)						
14					local date (year)	2003	=UTLcYear(C27,0,0,C6,C7,C23,C24,C25)						
15													
16	1	JD of new Moon (days)	2452879.227	=NewMoon(C3,C4,C5,C6,C7)									
17	2	JD of full Moon (days)	2452893.192	=FullMoon(3,C4,C5,C6,C7)									
18	3	G date of new Moon (day)	27.72680637	=JDCDay(C16)									
19	4	... integer day	27	=INT(C18)									
20	5	G date of new Moon (month)	8	=JDCMonth(C16)									
21	6	G date of new Moon (year)	2003	=JDCYear(C16)									
22	7	G date of full Moon (day)	10.69174652	=JDCDay(C17)									
23	8	... integer day	10	=INT(C22)									
24	9	G date of full Moon (month)	9	=JDCMonth(C17)									
25	10	G date of full Moon (year)	2003	=JDCYear(C17)									
26	11	UT of new Moon (hours)	17.44335286	=24*(C18-C19)									
27	12	UT of full Moon (hours)	16.60191639	=24*(C22-C23)									
28	13	LCT of new Moon (hours)	17.45168586	=UTLCT(C26+0.008333,0,0,C6,C7,C19,C20,C21)									
29	14	LCT of full Moon (hours)	16.61024939	=UTLCT(C27+0.008333,0,0,C6,C7,C23,C24,C25)									

Figure 88. Finding the instances of new Moon and full Moon.

68 The position-angle of the Moon's bright limb

In Section 59 we saw how to calculate the position-angle, χ , of the bright limb of a planet. χ is defined to be the angle of the midpoint of the illuminated limb measured eastwards from the north point of the disc (see Figure 71). We can do the same for the Moon. χ is given by

$$\chi = \tan^{-1} \left\{ \frac{\cos \delta_{\odot} \sin (\alpha_{\odot} - \alpha_m)}{\cos \delta_m \sin \delta_{\odot} - \sin \delta_m \cos \delta_{\odot} \cos (\alpha_{\odot} - \alpha_m)} \right\},$$

where α_{\odot} , δ_{\odot} and α_m , δ_m are the equatorial coordinates of the Sun and Moon respectively.

For example, what was the position-angle of the Moon's bright limb on 1 September 2003 at 0h TT? The coordinates of the Sun and Moon that day were $\alpha_{\odot} = 10^{\text{h}} 39^{\text{m}} 17^{\text{s}}$, $\delta_{\odot} = 8^{\circ} 30' 21''$, $\alpha_m = 14^{\text{h}} 12^{\text{m}} 10^{\text{s}}$, and $\delta_m = -11^{\circ} 34' 58''$. (These can be calculated by the methods given in Sections 46 and 65.)

Method	Example
1. Convert α_{\odot} and α_m first to decimal hours (§7) and then to degrees (§22).	$\alpha_{\odot} = 10.654\,722$ hours $= 159.820\,833$ degrees $\alpha_m = 14.202\,778$ hours $= 213.041\,667$ degrees
2. Convert δ_{\odot} and δ_m to decimal degrees (§21).	$\delta_{\odot} = 8.505\,833$ degrees $\delta_m = -11.582\,778$ degrees
3. Find $y = \cos \delta_{\odot} \sin (\alpha_{\odot} - \alpha_m) \dots$	$y = -0.792\,170$
4. \dots and $x = \cos \delta_m \sin \delta_{\odot} - \sin \delta_m \cos \delta_{\odot} \cos (\alpha_{\odot} - \alpha_m)$.	$x = 0.263\,794$
5. Find $\chi = \tan^{-1} \left(\frac{y}{x} \right)$. Remove the ambiguity by referring to Figure 29, adding or subtracting 180 to bring the result into the correct quadrant if not already there, according to the signs of x and y .	$\chi = -71.582\,151$ degrees (already in the right quadrant) $\chi = \mathbf{-71.58 \text{ degrees}}$

As mentioned in the previous section, the spreadsheet MoonPhase (Figure 87) also performs this calculation. We have provided a spreadsheet function called MoonPABL and it takes eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. Thus, having saved a copy of the spreadsheet, you could delete rows 19 to 26, and place the following spreadsheet formula in cell H4:

=ROUND(MoonPABL(C3,C4,C5,C6,C7,C8,C9,C10),2).

Note that the angles -71.58 and $360 + (-71.58) = 288.42$ are equivalent to each other (all expressed in degrees).

69 The Moon's distance, angular size and horizontal parallax

During the course of one complete circuit of its orbit, the Moon's distance, ρ , from the Earth varies quite considerably. Its point of closest approach, the perigee, is about 356 000 km from the Earth while the furthest point, the apogee, is at a distance of 407 000 km. We can calculate its distance at any other point quite easily, as it is given by the formula

$$\rho = \frac{a(1 - e^2)}{1 + e \cos(M'_m + E_c)},$$

where M'_m is the corrected anomaly, E_c is the correction for the equation of the centre (defined in Section 65), e is the eccentricity (about 0.054 900) and a is the semi-major axis of the Moon's orbit (see Table 11). We usually wish to express the distance as a fraction of a so we write

$$\rho' = \frac{\rho}{a} = \frac{(1 - e^2)}{1 + e \cos(M'_m + E_c)}.$$

The units of ρ are the same as those of a ; if a is expressed in kilometres, so is ρ .

The Moon's apparent angular diameter, θ , follows directly from the value of ρ' . It is given by

$$\theta = \frac{\theta_0}{\rho'},$$

where θ_0 is the Moon's apparent angular diameter when it is at a distance a from Earth. The value of θ_0 is given in Table 11 (approximately 0.518 1 degrees).

The Moon's horizontal parallax is defined to be the angle subtended at the Moon by the Earth's radius. In Figure 89 it is given by the symbol π (not to be confused with the constant 3.141 592 7). The formula is

$$\pi = \frac{\pi_0}{\rho'},$$

where π_0 is the horizontal parallax at distance a from the Earth (Table 11; about 0.950 7 degrees). Note that we have provided a spreadsheet function, MoonHP (see Section 65) that calculates the value of π directly. From this you can calculate ρ , given the radius of the Earth (6378.14 km):

$$\rho = \frac{6378.14}{\sin(\pi)}.$$

For example, what were the values of ρ' , θ and π on 1 September 1979 at 0h UT?

Method	Example
1. Find M'_m and E_c by the methods in §65.	$M'_m = 343.893\,432$ degrees $E_c = -1.744\,614$ degrees
2. Calculate $\rho' = \frac{\rho}{a} = \frac{1 - e^2}{1 + e \cos(M'_m + E_c)}$.	$\rho' = \mathbf{0.947\,474}$
3. Find $\theta = \frac{\theta_0}{\rho'}$.	$\theta = 0.546\,822$ degrees $= \mathbf{32' \, 49''}$
4. Find $\pi = \frac{\pi_0}{\rho'}$.	$\pi = 1.003\,405$ degrees $= \mathbf{1^\circ \, 00' \, 12''}$

We have provided spreadsheet functions to make these calculations rather more accurately. They are called MoonDist, which returns the distance between the Earth's centre and the Moon's centre in kilometres, MoonSize, which returns the angular diameter of the Moon as seen from the centre of the Earth, in degrees, and MoonHP, which gives us the Moon's horizontal parallax in degrees. Each of them takes the same set of eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours (W negative), and the local calendar date as day, month and year. We have used these in a spreadsheet called MoonDist (Figure 90). The *Astronomical Almanac* gives the values of the distance, angular diameter and horizontal parallax respectively as 367 948 km, 32' 29" and 0° 59' 36". Note that the apparent angular size of the Moon will be slightly different depending on where you are on the Earth, being largest when you are nearest to it, i.e. when the Moon is directly overhead.

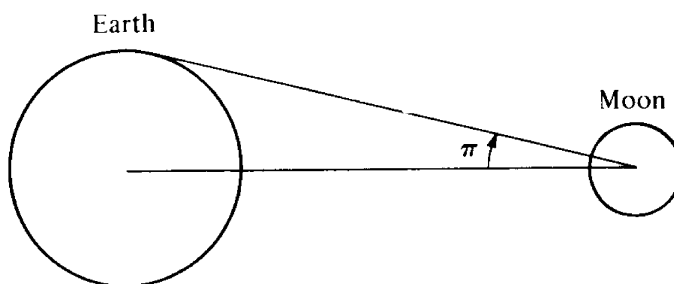


Figure 89. Lunar parallax.

	A	B	C	D	E	F	G	H	I	J	K	
1	The Moon's distance, angular diameter, and horizontal parallax											
2												
3	Input	local time (hour)	0	Output	Earth-Moon dist (km)	367960	=ROUND(C12,-1)					
4		local time (min)	0		ang diameter (deg)	0	=DDDeg(C13+0.008333)					
5		local time (sec)	0		(min)	32	=DDMin(C13+0.008333)					
6		daylight saving (hours)	0		hor parallax (deg)	0	=DDDeg(C14)					
7		zone correction (hours)	0		(min)	59	=DDMin(C14)					
8		local date (day)	1		(sec)	35.49	=DDSec(C14)					
9		local date (month)	9									
10		local date (year)	2003									
11												
12	1	Moon's distance	367964.4345	=MoonDist(C3,C4,C5,C6,C7,C8,C9,C10)								
13	2	Moon's angular diameter	0.541242956	=MoonSize(C3,C4,C5,C6,C7,C8,C9,C10)								
14	3	Moon horizontal parallax	0.993190567	=MoonHP(C3,C4,C5,C6,C7,C8,C9,C10)								

Figure 90. Finding the Moon's distance, diameter and horizontal parallax.

70 Moonrise and moonset

In Section 33 we found how to calculate the times of rising or setting of a star given its equatorial coordinates. We can apply the same method to the Moon to find the times of moonrise and moonset but the problem is complicated by three factors. One is that the Moon is in rapid motion so that its right ascension and declination are continually changing. To find the time of moonset, for instance, we require the coordinates of the Moon at that time; but to find these coordinates we need the time of moonset, and so we go round in a circle.

The second problem is also associated with the Moon's rapidly changing position. In order to find the circumstances of moonrise and moonset at a given location on the Earth for a given calendar date, we need to know the corresponding Greenwich dates. We adopt an iterative scheme here, in which we find the position of the Moon at, say, midday, use that position to calculate the times of moonrise and moonset, then recalculate the Moon's positions at those times, then recalculate the times of moonrise and moonset, and so on until the changes are small enough for us to ignore. We need to take account of the fact that the calendar date might change, and that the conversion from Greenwich sidereal time to UT (an essential step) might be ambiguous (see Section 13). And then there is the fact that the Moon might not rise or set on a particular date!

The third major complication with the Moon is that it is, astronomically speaking, very close to the Earth. The coordinates which we work out are correct for the centre of the Earth, but when we observe from the Earth's surface the apparent coordinates change slightly; this effect is called **parallax** (see Section 38). In the case of the Moon the parallax can be as much as a whole degree. Taking this into account, together with the corrections for atmospheric refraction and the finite size of the Moon's disc (times quoted are for the upper limb), we can find the times of moonrise and moonset correct to within a minute or two of time.

Let us clarify all this with an example: what were the times of moonrise and moonset on 6 March 1986 as observed from Boston, Massachusetts, longitude $71^{\circ} 03' \text{ W}$ and latitude $42^{\circ} 22' \text{ N}$, time zone -5 hours? The *Old Farmer's Almanac* (see Bibliography on page 208) lists these times as EST rising = 4h 20m am and EST setting = 1h 08m pm.

The calculation is lengthy, even for one iteration, so we use spreadsheets straight away. The first of these, Figure 91, is called MoonRiseSet1 and it provides a sort of do-it-yourself iterative method. The local calendar date and a time (called the starting time – cell C11) are first converted to the Greenwich calendar date (rows 14 to 16), and then the Moon's position and horizontal parallax are calculated (rows 17 to 21). The right ascension and declination, and the vertical displacement (row 22), are used to find the local sidereal times (rows 23 to 24) and then the Greenwich sidereal times (rows 26 and 27) of both rising and setting. The times are converted to universal times in rows 28 to 31. Note that by this time we have already acquired two status flags, one to do with rising and setting (row 25) and the other to do with conversion from Greenwich sidereal time to universal time (rows 29 and 31). Finally, the universal times are converted to local calendar dates and times (rows 32 to 39) and the azimuths calculated (rows 40 and 41).

Figure 91 shows the results of carrying out this series of calculations using a starting time of midday, 12 o'clock (cell C11). Comparing the time of moonrise, say, with that given in the *Old Farmer's Almanac*, we see that our result (4h 36m) is some 16 minutes later than the quoted time. We can now realise the power of the spreadsheet by inserting this time back into cell C11 as the starting time. Figure 92 shows what happens. The time of moonrise is now found to be 4h 21m, within a minute of the *Old Farmer's* calculation. We would need to repeat this exercise for the time of moonset, substituting 13.066 7 (13h 4m) into the cell C11.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Moonrise and moonset (manual method)														
2															
3	Input	local date (day)	6		Output	moonrise: (OK: OK)									
4		local date (month)	3			local time (hour)	4								
5		local date (year)	1986			local time (min)	36								
6		daylight saving (hours)	0			local date	6/3/1986								
7		zone correction (hours)	-5			azimuth (deg)	126.14								
8		geog long (deg)	-71.05			moonset: (OK: OK)									
9		geog lat (deg)	42.3667			local time (hour)	13								
10						local time (min)	4								
11		starting time (hours)	12			local date	6/3/1986								
12						azimuth (deg)	233.86								
13															
14	1	G date (day)	6=LCTGDay(C11,0,0,C6,C7,C3,C4,C5)				=CONCATENATE("moonrise:(","C25"," ",C29,")")								
15	2	G date (month)	3=LCTGMonth(C11,0,0,C6,C7,C3,C4,C5)				=DHHour(C32+0.008333)								
16	3	G date (year)	1986=LCTGYear(C11,0,0,C6,C7,C3,C4,C5)				=DHMin(C32+0.008333)								
17	4	Moon long (deg)	297.2909149=MoonLong(C11,0,0,C6,C7,C3,C4,C5)				=CONCATENATE(TEXT(C33,"##"),",",TEXT(C34,"##"),",",TEXT(C35,"###"))								
18	5	Moon lat (deg)	-5.127456775=MoonLat(C11,0,0,C6,C7,C3,C4,C5)				=ROUND(C40,2)								
19	6	Moon HP (rad)	0.016952465=RADIANS(MoonHP(C11,0,0,C6,C7,C3,C4,C5))				=CONCATENATE("moonset:(","C25"," ",C31,")")								
20	7	Moon RA (hours)	20.030675=DDH(E CRA(C17,0,0,C18,0,0,C14,C15,C16))				=DHHour(C36+0.008333)								
21	8	Moon dec (deg)	-25.7304396=ECDec(C17,0,0,C18,0,0,C14,C15,C16)				=DHMin(C36+0.008333)								
22	9	vertical displacement (deg)	-0.139979829=DEGREES(0.27249*SIN(C19)+0.0098902-C19)				=CONCATENATE(TEXT(C37,"##"),",",TEXT(C38,"##"),",",TEXT(C39,"###"))								
23	10	local sid time rising (hours)	15.7846104=RSLSTR(C20,0,0,C21,0,0,C22,C9)				=ROUND(C41,2)								
24	11	local sid time setting (hours)	0.276739591=RSLSTS(C20,0,0,C21,0,0,C22,C9)												
25	12	rising & setting status	OK=eRS(C20,0,0,C21,0,0,C22,C9)												
26	13	GST rising (hours)	20.52127706=LSTGST(C23,0,0,C8)												
27	14	GST setting (hours)	5.013406257=LSTGST(C24,0,0,C8)												
28	15	UT rising (hours)	9.599378064=GSTUT(C26,0,0,C14,C15,C16)												
29	16	UT rising status	OK=eGSTUT(C26,0,0,C14,C15,C16)												
30	17	UT setting (hours)	18.06832006=GSTUT(C27,0,0,C14,C15,C16)												
31	18	UT setting status	OK=eGSTUT(C27,0,0,C14,C15,C16)												
32	19	LCT rising (hours)	4.599378068=UTLCT(C28,0,0,C6,C7,C14,C15,C16)												
33	20	LCT rising date (day)	6=UTLCDay(C28,0,0,C6,C7,C14,C15,C16)												
= * * * MoonriseSet1 /															
34	21	LCT rising date (month)	3=UTLCMonth(C28,0,0,C6,C7,C14,C15,C16)												
35	22	LCT rising (year)	1986=UTLCYear(C28,0,0,C6,C7,C14,C15,C16)												
36	23	LCT setting (hours)	13.06832006=UTLCT(C30,0,0,C6,C7,C14,C15,C16)												
37	24	LCT setting date (day)	6=UTLCDay(C30,0,0,C6,C7,C14,C15,C16)												
38	25	LCT setting date (month)	3=UTLCMonth(C30,0,0,C6,C7,C14,C15,C16)												
39	26	LCT setting date (year)	1986=UTLCYear(C30,0,0,C6,C7,C14,C15,C16)												
40	27	azimuth rising (deg)	126.1440993=RSAR(C20,0,0,C21,0,0,C22,C9)												
41	28	azimuth setting (deg)	233.8559007=RSAS(C20,0,0,C21,0,0,C22,C9)												
= * * * MoonriseSet1 /															

Figure 91. A manual method of calculating moonrise and moonset.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Moonrise and moonset (manual method)														
2															
3	Input	local date (day)	6		Output	moonrise: (OK: OK)									
4		local date (month)	3			local time (hour)	4								
5		local date (year)	1986			local time (min)	21								
6		daylight saving (hours)	0			local date	6/3/1986								
7		zone correction (hours)	-5			azimuth (deg)	127.3								
8		geog long (deg)	-71.05			moonset: (OK: OK)									
9		geog lat (deg)	42.3667			local time (hour)	12								
10						local time (min)	42								
11		starting time (hours)	4.6			local date	6/3/1986								
12						azimuth (deg)	232.7								
= * * * MoonriseSet1 /															

Figure 92. Another iteration in the calculation of moonrise.

We have also provided spreadsheet formulas to carry all this out behind the scenes. Each of them makes many iterations, and should generally work, though they may break down in particularly pathological circumstances. The formulas are MoonRiseLCT, MoonRiseLCDay, MoonRiseLCMonth, MoonRiseLCYear, MoonRiseAz, eMoonRise for moonrise, returning, respectively, the local civil time of moonrise in hours, the local calendar date, month and year, the azimuth, and a status string telling us about any error conditions. The string is set to OK if everything is fine, but if not it will tell us about Greenwich sidereal time conversion uncertainties as well as conditions under which the Moon never rises or never sets. Each function takes the same seven arguments, namely the local calendar date as day, month, year (but note that the functions may give you the result for the day before or after), the daylight saving and time zone offsets in hours (W negative), and the geographical longitude (W negative) and latitude (S negative) in degrees. There is an equivalent set of functions for moonset, identical in every respect except that the word Set replaces the word Rise in the name, e.g. MoonSetAz etc. A spreadsheet which uses these functions, called MoonRiseSet2, is shown in Figure 93. You can see that it delivers results which are very close to those quoted in the almanac.

Note that in Figures 91 to 93, the contents of cells H4 to H12 are shown below in cells H15 to H23 in order to save space. The contents of cell G3 is shown in cell H14. Some of these cells use the CONCATENATE function which joins together strings from various parts of the spreadsheet. Thus, in cell H6, we have

=CONCATENATE(TEXT(C22,##),"",TEXT(C23,##),"",TEXT(C24,####))

to produce the date in the format dd/mm/yyyy. The function TEXT converts a number (first argument) into a text string containing the number of digits identified in the second argument by the identifiers ## (for two digits) and #### for four digits. Then CONCATENATE produces just one string with all its arguments joined together with no spaces.

Moonrise and moonset (automatic method)															
Input	local date (day)	6	Output	moonrise: (OK)											
	local date (month)	3		local time (hour)	4										
	local date (year)	1986		local time (min)	21										
	daylight saving (hours)	0		local date	6/3/1986										
	zone correction (hours)	-5		azimuth (deg)	127.34										
	geog long (deg)	-71.05		moonset: (OK)											
	geog lat (deg)	42.3667		local time (hour)	13										
				local time (min)	8										
				local date	6/3/1986										
				azimuth (deg)	234.05										
1	local time of moonrise (hours)	4.345838	=MoonRiseLCT(C3,C4,C5,C6,C7,C8,C9)	=CONCATENATE("moonrise: ("&C15,"")											
2	local moonrise status	ok	=eMoonRise(C3,C4,C5,C6,C7,C8,C9)	=DHHour(C14+0.008333)											
3	local date of moonrise (day)	6	=MoonRiseLCDay(C3,C4,C5,C6,C7,C8,C9)	=DHMin(C14+0.008333)											
4	local date of moonrise (month)	3	=MoonRiseLCMonth(C3,C4,C5,C6,C7,C8,C9)	=CONCATENATE(TEXT(C16,"##"),",",TEXT(C17,"##"),",",TEXT(C18,"####"))											
5	local date of moonrise (year)	1986	=MoonRiseLCYear(C3,C4,C5,C6,C7,C8,C9)	=ROUND(C19,2)											
6	local azimuth (deg)	127.3362	=MoonRiseAz(C3,C4,C5,C6,C7,C8,C9)	=CONCATENATE("moonset: ("&C21,"")											
7	local time of moonset (hours)	13.1254	=MoonSetLCT(C3,C4,C5,C6,C7,C8,C9)	=DHHour(C20+0.008333)											
8	local moonset status	ok	=eMoonSet(C3,C4,C5,C6,C7,C8,C9)	=DHMin(C20+0.008333)											
9	local date of moonset (day)	6	=MoonSetLCDay(C3,C4,C5,C6,C7,C8,C9)	=CONCATENATE(TEXT(C22,"##"),",",TEXT(C23,"##"),",",TEXT(C24,"####"))											
10	local date of moonset (month)	3	=MoonSetLCMonth(C3,C4,C5,C6,C7,C8,C9)	=ROUND(C25,2)											
11	local date of moonset (year)	1986	=MoonSetLCYear(C3,C4,C5,C6,C7,C8,C9)												
12	local azimuth (deg)	234.0493	=MoonSetAz(C3,C4,C5,C6,C7,C8,C9)												

Figure 93. An automatic method of calculating moonrise and moonset using spreadsheet functions.

71 Eclipses

Both the Earth and the Moon cast long shadows into space. The Earth's shadow lies exactly in the plane of the ecliptic opposite the Sun whereas that of the Moon may be above or below the ecliptic depending on the position of the Moon (Figure 94). The shadows are always present but we are usually unaware of them since we cannot see them from the Earth. Occasionally, however, one of the bodies passes through the shadow of the other and then we observe an eclipse: if the Moon passes through the Earth's shadow it is an eclipse of the Moon, or a **lunar eclipse**; when the Moon casts its shadow upon the Earth, we see the Sun partially or totally obscured and it is then a **solar eclipse**.

An eclipse of the Moon can only happen at full Moon and an eclipse of the Sun at new Moon. We do not see an eclipse on every such occasion, however, since the Moon's orbit does not lie in the plane of the ecliptic. Only when the Moon is also near one of its nodes can an eclipse occur.

A lunar eclipse begins with the **penumbral phase** when the Moon enters the penumbra of the Earth's shadow, and the Moon's disc becomes a little fainter. You probably wouldn't notice this unless you were looking for it. As the Moon enters the umbra the **partial phase** begins; when it has all moved inside the umbra the Sun's light is entirely cut off and the **total phase** begins. The only light reaching the Moon is then that refracted round the edges of the Earth, giving the Moon a coppery hue. The Earth's shadow extends well beyond the Moon's orbit so that it is always possible for a total lunar eclipse to occur, if other circumstances are favourable (Figure 95(a)).

A solar eclipse begins with the partial phase when the Earth enters the penumbra of the Moon's shadow. We see a 'bite' missing out of the Sun's disc and as the eclipse progresses the size of the bite increases. If you are favourably situated, you will see the Moon eventually cover the whole Sun. The eclipse is then total. Since the Moon is so much smaller than the Earth, its umbra extends a much shorter distance into space, in fact only just far enough to reach the Earth when the conditions are right (Figure 95(b)). The tip of the umbra casts a small shadow on the face of the Earth which moves across it as the Moon and Sun change their relative positions. Never is the umbra sufficiently large to engulf the whole Earth. Consequently any total eclipse can only be seen along a narrow strip of the Earth's surface.

Sometimes, however, the umbra does not reach the Earth at all (Figure 95(c)). In this case an **annular eclipse** can occur with the Moon not quite obscuring the whole of the Sun's disc at maximum eclipse but leaving a ring of light round its edge.

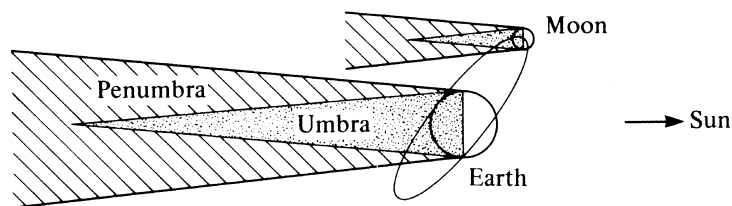


Figure 94. Shadows cast by the Moon and the Earth.

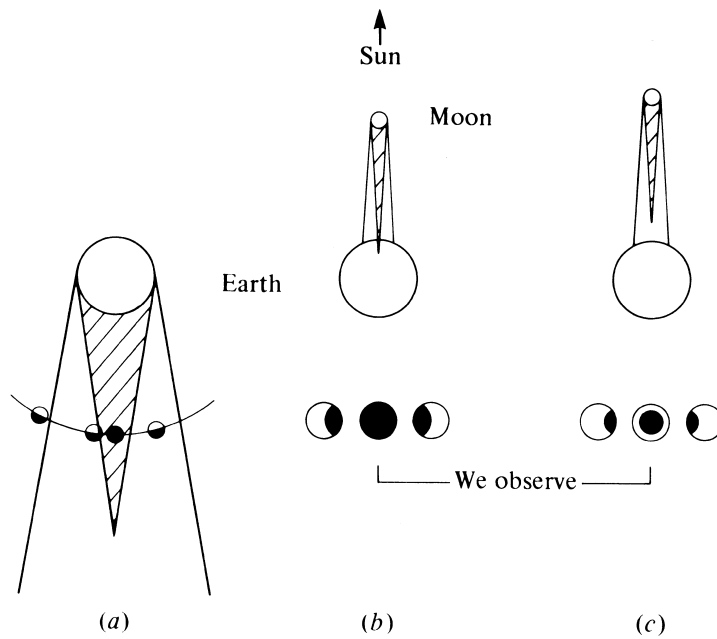


Figure 95. (a) Lunar eclipse, (b) total solar eclipse and (c) annular solar eclipse.

72 The 'rules' of eclipses

Here is a summary of the most important 'rules' which appear to govern the occurrence of eclipses.

- (a) A lunar eclipse can only occur at full Moon and a solar eclipse at new Moon. There is not an eclipse every month.
- (b) At least two solar eclipses occur every year, and never more than five. There is a maximum of three lunar eclipses in a year. The highest total number of eclipses in a year, lunar and solar, is seven.
- (c) Eclipses tend to go in pairs or threes: solar–lunar–solar. A lunar eclipse is always preceded or followed by a solar eclipse (with two weeks in between them).
- (d) The pattern of eclipses tends to recur in cycles of 18 years 11 days and 8 hours, the so-called 'Saros' cycle. The pattern is not repeated exactly.
- (e) At the moment of greatest eclipse the Sun and Moon are nearly in opposition or conjunction. If the angle between the line of nodes and the Sun or Moon is greater than $12^{\circ} 15'$ a total lunar eclipse is not possible, while if it is less than $9^{\circ} 30'$ a lunar eclipse *must* occur. If the angle is more than $18^{\circ} 31'$ a solar eclipse cannot happen, while if it is less than $15^{\circ} 31'$ a solar eclipse *must* occur.
- (f) In a lunar eclipse, the total phase can last for a maximum time of 1 hour 40 minutes, and the umbral phase, partial–total–partial, for a maximum time of 3 hours 40 minutes. The maximum time of total solar eclipse (at the equator) is 7 minutes 40 seconds and an annular eclipse can last at most for 12 minutes and 24 seconds.

A total solar eclipse is a special event in the history of the Earth, for the Moon is gradually moving away from the Earth as tidal forces are continuously transferring angular momentum from the Earth to the Moon's orbit. Right now we live at a particular time when the Sun and the Moon just happen to have more-or-less the same angular size in the sky. A very long time in the future will see the Moon and Earth locked together so that the period of the Moon's orbit around the Earth is exactly the same as the time for the Earth to spin once on its axis. The length of the day will then be nearly two months of present time and the Moon will be much smaller than the Sun in the sky so that total solar eclipses will never occur.

73 Calculating a lunar eclipse

There are two questions to be asked before proceeding with the calculation of an eclipse. The first is, ‘Is an eclipse likely to occur?’ If the answer is yes, the second is, ‘Will I be able to see it?’ You may predict an eclipse of the Moon at a certain time, but if the Moon hasn’t risen or has already set you won’t be able to see it!

First, then, to spot the likely time of occurrence of a lunar eclipse. From rule (a) above there must be a full Moon, that is the angle $\lambda_m - \lambda_{\odot} = 180^\circ$. From rule (e) the angle between the line of nodes and the Moon must be within $12^\circ 15'$ of 0° or 180° at that time. This is the angle $l'' - N'$ in the calculations of Section 65.

For example, is there a lunar eclipse associated with the full Moon on 4 April 2015? In order to find out, we need to calculate the time of opposition, that is when the Moon and Sun are opposite each other in ecliptic longitude, and then we need to find the Moon’s distance from the line of nodes. Using spreadsheet MoonNewFull (Section 67), we see that the UT of the full Moon on this date is 12.118 660 h (12h 07m). Hence we find, using the spreadsheet MoonPos1 of Section 65, that

$$\lambda_{\odot} = 14.407 \text{ degrees,}$$

$$\lambda_m = 194.400 \text{ degrees,}$$

(hence $\lambda_m - \lambda_{\odot} = 179.993$ degrees), and

$$l'' = 194.419 \text{ degrees,}$$

$$N' = 189.824 \text{ degrees,}$$

(hence $l'' - N' = 4.595$ degrees). We see that $\lambda_m - \lambda_{\odot}$ is very nearly 180° so that we are almost at the point of full Moon, and that $l'' - N' = 4.595$ degrees, well within the limit of $12^\circ 15'$. There is indeed a total lunar eclipse that day.

We have also provided a spreadsheet function called LEOccurrence which tests the angle between the Moon and the nearest node and returns with one of the string messages Lunar eclipse certain, Lunar eclipse possible, or No lunar eclipse depending on its magnitude. The function takes five arguments which are the daylight saving and time zone offsets in hours, and the local calendar date as day, month, year. It tests the full Moon of the lunation in progress on the date you specify. Use the function FullMoon to find the time and date of the full Moon. Spreadsheet LunarEclipseOccurrence, Figure 96, shows how it goes.

Having predicted the occurrence of an eclipse, the second question is, can you see it? The answer obviously depends on your position on the Earth. We can calculate, using Section 70, the times of moonrise and moonset on the day of the eclipse for an observer on the Greenwich meridian at latitude 52° N. We find that the Moon does not rise until 18h 49m UT while the eclipse is in progress at 10h 41m UT. Our observer cannot therefore see the eclipse. However, repeating the calculation for someone in Sydney, Australia, shows that moonrise is at 17h 39m local time, in time zone +10 hours, while the eclipse is in progress at midday+10 = 22h local time, giving Sydney dwellers an excellent view if the night is clear.

To calculate the circumstances of an eclipse we need to know the Moon’s position at some particular time near to full Moon, its hourly motions in longitude and latitude, its angular distance from the Sun (the angle $\lambda_m - \lambda_{\odot}$), its angular diameter, and the angular radius of the Earth’s shadow at the distance of the Moon’s orbit. This last is given by the following simple formulas with sufficient accuracy for our purposes:

$$S_p = \text{radius of penumbra} = \pi + 0.27 \text{ degrees,}$$

$$S_u = \text{radius of umbra} = \pi - 0.27 \text{ degrees,}$$

	A	B	C	D	E	F	G	H	I	J	K	L
1		Lunar eclipse occurrence										
2												
3	<i>Input</i>	local date (day)	1		<i>Output</i>	Lunar eclipse certain		=C19				
4		local date (month)	4			... on 4/4/2015		=CONCATENATE(C16,"/",C17,"/",C18)				
5		local date (year)	2015									
6		daylight saving (hours)	0									
7		zone correction (hours)	10									
8												
9	1	Julian date of full Moon	2457117.005			=FullMoon(C6,C7,C3,C4,C5)						
10	2	G date of full Moon (day)	4.50494417			=JDCDay(C9)						
11	3	... integer day	4			=INT(C10)						
12	4	G date of full Moon (month)	4			=JDCMonth(C9)						
13	5	G date of full Moon (year)	2015			=JDCYear(C9)						
14	6	UT of full Moon (hours)	12.11866009			=(C10-C11)*24						
15	7	local civil time (hours)	22.11866008			=UTLCT(C14,0,0,C6,C7,C11,C12,C13)						
16	8	local civil date (day)	4			=UTLCDay(C14,0,0,C6,C7,C11,C12,C13)						
17	9	local civil date (month)	4			=UTLCMonth(C14,0,0,C6,C7,C11,C12,C13)						
18	10	local civil date (year)	2015			=UTLCYear(C14,0,0,C6,C7,C11,C12,C13)						
19	11	eclipse occurrence?				Lunar eclipse certain		=LEOccurrence(C6,C7,C3,C4,C5)				

Figure 96. Predicting a lunar eclipse.

where π is the Moon's horizontal parallax (Section 69). Let us now calculate the circumstances of the lunar eclipse on 4 April 2015. First, we write down all the details:

2015 April 4 at 12h 07m TT:

$$\begin{aligned}
 \lambda_m &= 194.400 \text{ degrees;} \\
 \beta_m &= 0.412 \text{ degrees;} \\
 \lambda_{\odot} &= 14.407 \text{ degrees;} \\
 \Delta\lambda &= 0.500 \text{ degrees/hour;} \\
 \Delta\beta &= 0.050 \text{ degrees/hour;} \\
 \pi &= 0.907 \text{ degrees;} \\
 S_p = \pi + 0.27 &= 1.177 \text{ degrees;} \\
 S_u = \pi - 0.27 &= 0.637 \text{ degrees;} \\
 \theta_m &= 0.494 \text{ degrees.}
 \end{aligned}$$

Add to this the fact that the Sun is also moving in longitude at a rate of $360/(365.2422 \times 24)$ degrees/hour, that is at 0.041 degrees/hour.

Next, we calculate the precise point of opposition. This is the moment when the angle $\lambda_m - \lambda_{\odot} = 180^\circ$. At 12h 07m TT the angle is $194.400 - 14.407 = 179.993$ degrees, just 0.007 degrees short of 180° . The Earth's shadow, always directly opposite the Sun, moves at the same rate and in the same direction as the Sun. The Moon is moving at a rate of 0.500 degrees/hour, also in the same direction, and the Earth's shadow at 0.041 degrees/hour. Thus the Moon is catching up on the shadow at a rate of $0.500 - 0.041 = 0.459$ degrees/hour, so that it takes $0.007/0.459$ hours to catch up by 0.007 degrees, that is almost 1 minute. Opposition (in ecliptic coordinates) is therefore at 12h 08m TT.

Now we are in a position to construct the **eclipse diagram**, Figure 97. Draw a horizontal line to represent

the plane of the ecliptic. Below this, draw a parallel line and, choosing a suitable scale (say 1 hour = 2 cm), mark it in hours. Find the position on the scale corresponding to the moment of opposition and mark it (point P) on the ecliptic line. (This is near the moment of maximum eclipse.) Next draw circles to scale centred on P to represent the umbra and the penumbra. Their radii should correspond with the angular radii of the shadows they represent. For example, we found above that $S_p = 1.177$ degrees. The difference between the hourly motions in longitude of the Earth's shadow and the Moon was 0.459 degrees/hour. Thus, on our scale of 1 hour: 2 cm, 0.459 degrees also equals 2 cm. Hence the scale factor between degrees and centimetres is $1 \text{ degree} = 2/0.459 = 4.357 \text{ cm}$, and we should draw a circle of radius $1.177 \times 4.357 = 5.129 \text{ cm}$ to represent the penumbra. The radius of the circle marking the umbra should be $0.637 \times 4.357 = 2.775 \text{ cm}$.

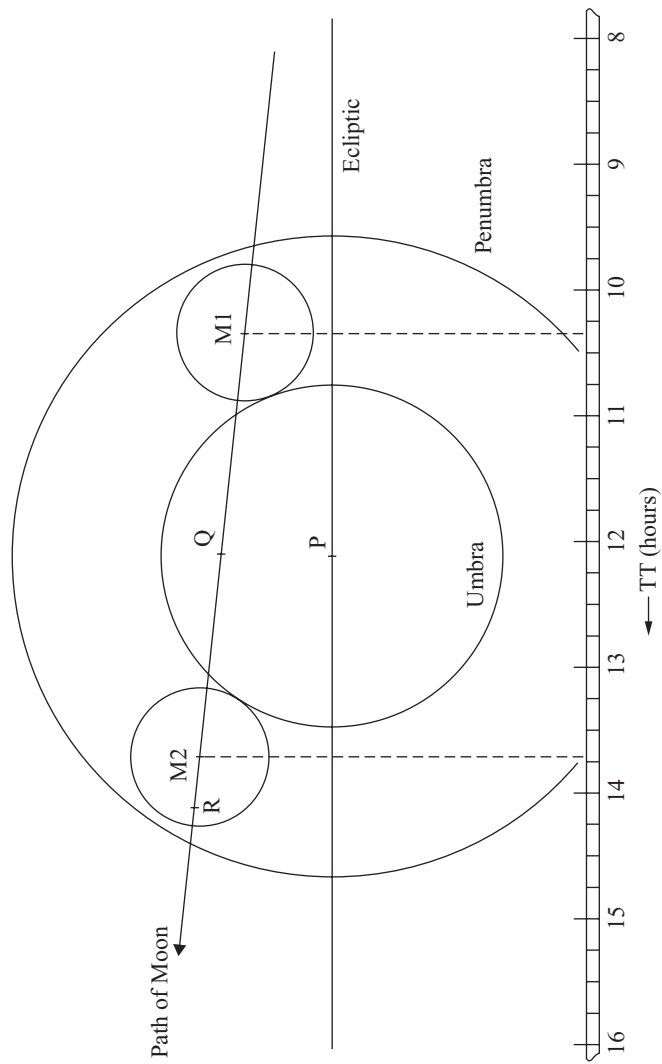


Figure 97. The lunar eclipse of 4 April 2015.

Now we are ready to plot the path of the Moon through the Earth's shadow. First, we mark the position, Q, of the Moon at 12h 07m TT for which we have calculated its ecliptic coordinates. This position lies at $\beta_m = 0.412$ degrees above the ecliptic, corresponding to $0.412 \times 4.357 = 1.795$ cm on the scale of our diagram. Then we calculate the Moon's position at some other time, say 2 hours later at 14h 07m TT. Mark this point R. You can use the spreadsheet MoonPos1 to find the Moon's ecliptic latitude (giving $\beta_m = 0.502$ degrees). Alternatively, you can use the Moon's hourly motions to find its approximate position as

$$\beta_m = \beta_0 + \Delta\beta t = 0.412 + (0.050 \times 2) = 0.512 \text{ degrees.}$$

The difference of 0.01 degrees between the two calculations corresponds to a difference of about 4 mm on the diagram, so for best accuracy you should always use the spreadsheet results if you have them. Using 1 degree = 4.357 cm, we need to mark point R at $0.502 \times 4.357 = 2.187$ cm above the ecliptic line at the 14h 07m on our horizontal timescale. Joining Q and R with a straight line gives the path of the Moon.

Finally, we simply have to draw circles centred on the line RQ to represent the Moon at any point. The radii of these circles must correspond with the calculated angular radius of the Moon, $\theta_m = 0.247$ degrees or 1.08 cm on the scale of the diagram. We mark two such positions: M1 represents the point where the Moon enters the umbra, and M2 where the Moon leaves the umbra. The corresponding times may be found from the scale below. They are:

Calculated
M1 at 10h 22m
M2 at 13h 44m

We have provided spreadsheet functions to calculate these times automatically with higher precision than you can obtain from using pencil and paper. The functions are

UTFirstContactLunarEclipse,
 UTStartUmbralLunarEclipse,
 UTStartTotalLunarEclipse,
 UTMxLunarEclipse,
 UTEndTotalLunarEclipse,
 UTEndUmbralLunarEclipse,
 UTLastContactLunarEclipse, and
 MagLunarEclipse.

Each of these takes the same set of five arguments, namely the local calendar date expressed as day, month, year, and the daylight saving and time zone offsets in hours. All but the last of them return the universal times, respectively, of first contact (when the Moon first touches the penumbra), the start of the umbral phase (M1 on our diagram), the start of the total phase (M2), mid-eclipse (when the eclipse is at its maximum), the end of the total phase (M3), the end of the umbral phase (M4) and the end of the eclipse (when the Moon leaves the penumbra). The last function, MagLunarEclipse, returns the **magnitude** of the eclipse, that is the fraction of the lunar diameter obscured by the Earth's shadow at the moment of greatest eclipse, measured

along the common diameter. Its value is 1 or greater if the eclipse is total. These functions will return a value of -99 if the particular phenomenon does not occur. For example, `UTEndUmbralLunarEclipse` returns -99 if there is no umbral phase.

We have drawn all these functions together in the spreadsheet `LunarEclipseCircumstances` (Figure 98). This calculates the local civil date and universal times of an eclipse, if one occurs, for the lunation in progress on the specified local calendar date. We have used time zone 10 for Sydney, Australia. It would be a small task to modify the spreadsheet so that local civil times were displayed instead of universal times. Try it for yourself. You will need to use the function `UTLCT`. Note that the contents of cells G3, and H3 to H11, are shown respectively in cells H13 to H22 in order to save space.

The circumstances of a lunar eclipse															
1	Input	local date (day)	1	Output	Lunar eclipse certain on	4/4/2015									
2		local date (month)	4		UT start of pen. phase	9:0									
3		local date (year)	2015		UT start of umbral phase	10:16									
4		daylight saving (hours)	0		UT start of total phase	11:55									
5		zone correction (hours)	10		UT of mid eclipse	12:1									
6					UT end of total phase	12:7									
7					UT end of umbral phase	13:46									
8					UT end of pen. phase	15:1									
9					eclipse magnitude	1.005									
10															
11															
12															
13	1	Julian date of full Moon	2457117.005	=FullMoon(C6,C7,C3,C4,C5)		=CONCATENATE(C23," on ")									
14	2	G date full Moon (day)	4.50494417	=JDCDay(C13)		=CONCATENATE(C20,"/".C21,"/".C22)									
15	3	... integer day	4	=INT(C14)		=IF(C25=-99,"",CONCATENATE(DHHour(C25+0.008333),"",DHMin(C25+0.008333)))									
16	4	G date full Moon (month)	4	=JDCMonth(C13)		=IF(C27=-99,"",CONCATENATE(DHHour(C27+0.008333),"",DHMin(C27+0.008333)))									
17	5	G date full Moon (year)	2015	=JDCYear(C13)		=IF(C29=-99,"",CONCATENATE(DHHour(C29+0.008333),"",DHMin(C29+0.008333)))									
18	6	UT of full Moon (hours)	12.11866009	=C14-C15*24		=IF(C24=-99,"",CONCATENATE(DHHour(C24+0.008333),"",DHMin(C24+0.008333)))									
19	7	local civil time (hours)	22.11866008	=UTLCT(C18,0,0,C6,C7,C15,C16,C17)		=IF(C30=-99,"",CONCATENATE(DHHour(C30+0.008333),"",DHMin(C30+0.008333)))									
20	8	local civil date (day)	4	=UTLCTDay(C18,0,0,C6,C7,C15,C16,C17)		=IF(C28=-99,"",CONCATENATE(DHHour(C28+0.008333),"",DHMin(C28+0.008333)))									
21	9	local civil date (month)	4	=UTLCTMonth(C18,0,0,C6,C7,C15,C16,C17)		=IF(C26=-99,"",CONCATENATE(DHHour(C26+0.008333),"",DHMin(C26+0.008333)))									
22	10	local civil date (year)	2015	=UTLCTYear(C18,0,0,C6,C7,C15,C16,C17)		=IF(C31=-99,"",ROUND(C31,3))									
23	11	eclipse occurrence?	eclipse certain	=LEOccurrence(C6,C7,C3,C4,C5)											
24	12	UT max eclipse	12.01218263	=UTMaxLunarEclipse(C3,C4,C5,C6,C7)											
25	13	UT first contact	9.003272842	=UTFirstContactLunarEclipse(C3,C4,C5,C6,C7)											
26	14	UT last contact	15.02109241	=UTLastContactLunarEclipse(C3,C4,C5,C6,C7)											
27	15	UT start umbral phase	10.26520797	=UTStartUmbralLunarEclipse(C3,C4,C5,C6,C7)											
28	16	UT end umbral phase	13.75915728	=UTEndUmbralLunarEclipse(C3,C4,C5,C6,C7)											
29	17	UT start total phase	11.91406936	=UTStartTotalLunarEclipse(C3,C4,C5,C6,C7)											
30	18	UT end total phase	12.11029589	=UTEndTotalLunarEclipse(C3,C4,C5,C6,C7)											
31	19	eclipse magnitude	1.005100347	=MagLunarEclipse(C3,C4,C5,C6,C7)											

Figure 98. Calculating the circumstances of a lunar eclipse.

74 Calculating a solar eclipse

A solar eclipse is rather more difficult to calculate than a lunar eclipse. If you look up a solar eclipse in the *Astronomical Almanac* you will find a map of the world showing the path and duration of the eclipse at each point; we shall not attempt such detail here. Our simple calculations will be made for just one location but will give a good guide of what to expect.

Once again we need to answer the question ‘Is an eclipse likely?’ Rule (a) in Section 72 tells us that we have to be at new Moon, that is the angle $\lambda_m - \lambda_\odot$ equals 0° (or equivalently 360°). Rule (e) tells us that the angle between the Sun or the Moon and a node must be within $18^\circ 31'$ of 0° or 180° at that time; this is the angle $\lambda_\odot - N'$ or $l'' - N'$. We can find the time of new Moon using spreadsheet *NewMoon* (Section 67), but we have also provided a spreadsheet function called *SEOccurrence* which tests the angle between the Moon and the nearest node and returns with one of the string messages “Solar eclipse certain”, “Solar eclipse possible”, or “No solar eclipse” depending on its magnitude. The function takes five arguments which are the daylight saving and time zone offsets in hours, and the local calendar date as day, month, year. It tests the new Moon of the lunation in progress on the date you specify. Spreadsheet *SolarEclipseOccurrence*, Figure 99, shows how to use both of these functions to search for a solar eclipse. Note that the contents of cells G3 and H4 are shown in cells H6 and H7 respectively, moved here in order to save space.

We see from Figure 99 that there is a solar eclipse on 20 March 2015. We shall illustrate the method by working out the circumstances of that eclipse, as observed by someone on longitude 0° and at latitude $68^\circ 39' \text{ N}$.

First, we must work through the calculations of Sections 65, 66 and 69. The results for the time of new Moon (Figure 99), 9.648 h = 9h 39 m TT (taken to be equal to UT) on 20 March 2015 are as follows (we have used spreadsheet functions to find these values):

	A	B	C	D	E	F	G	H	I	J	K
1	Solar eclipse occurrence										
2											
3	Input	local date (day)	1	Output	Solar eclipse certain						
4		local date (month)	4		... on 20/3/2015						
5		local date (year)	2015								
6		daylight saving (hours)	0		=C19						
7		zone correction (hours)	0		=CONCATENATE(C16,"'",C17,"'",C18)						
8											
9	1	Julian date of new Moon	2457101.902	=NewMoon(C6,C7,C3,C4,C5)							
10	2	G date of new Moon (day)	20.40201849	=JDCDay(C9)							
11	3	... integer day	20	=INT(C10)							
12	4	G date of new Moon (month)	3	=JDCMonth(C9)							
13	5	G date of new Moon (year)	2015	=JDCYear(C9)							
14	6	UT of new Moon (hours)	9.648443766	=(C10-C11)*24							
15	7	local civil time (hours)	9.648443766	=UTLCT(C14,0,0,C6,C7,C11,C12,C13)							
16	8	local civil date (day)	20	=UTLCDay(C14,0,0,C6,C7,C11,C12,C13)							
17	9	local civil date (month)	3	=UTLCMonth(C14,0,0,C6,C7,C11,C12,C13)							
18	10	local civil date (year)	2015	=UTLCYear(C14,0,0,C6,C7,C11,C12,C13)							
19	11	eclipse occurrence?	Solar eclipse certain	=SEOccurrence(C6,C7,C3,C4,C5)							

Figure 99. Predicting a solar eclipse.

$\lambda_{\odot} = 359.461$ degrees;
 $\lambda_m = 359.473$ degrees;
 $(\lambda_m - \lambda_{\odot} = +0.012$ degrees);
 $\beta_m = 0.964$ degrees;
 $\Delta\lambda = 0.608$ degrees/hour;
 $\Delta\beta = -0.049$ degrees/hour;
 $\theta_m = 0.556$ degrees;
 $\pi = 1.021$ degrees; and
 hourly motion of the Sun = 0.041 degrees/hour.

We now have to take account of geocentric parallax. The coordinates λ_m and β_m that we have just calculated are those that would be observed at the centre of the Earth. We are observing from the surface of the Earth at longitude 0° and at latitude $68^\circ 39' \text{ N}$, and we see slightly different ecliptic coordinates which can be calculated as follows (values from spreadsheet functions):

Method	Example
1. Transform λ_m and β_m to equatorial coordinates (§27).	$\alpha_m = 23.942$ hours $\delta_m = 0.675$ degrees
2. Find the apparent right ascension and declination after allowing for the effects of geocentric parallax (§39). Use the value of π for P .	$\alpha'_m = 23.957$ hours $\delta'_m = -0.274$ degrees
3. Convert α'_m and δ'_m back to ecliptic coordinates (§28).	$\lambda'_m = \mathbf{359.301 \text{ degrees}}$ $\beta'_m = \mathbf{0.005 \text{ degrees}}$

Next we calculate the precise moment of conjunction in ecliptic coordinates when $\lambda'_m - \lambda_{\odot} = 0^\circ$. Strictly, we ought to apply the correction for parallax to the Sun's coordinates as well, but we shall ignore this small correction here. At 09h 39m TT, $\lambda'_m - \lambda_{\odot} = -0.160$ degrees so that the Moon has still a little distance to catch up with the Sun. Its speed in longitude is $\Delta\lambda = 0.608$ degrees/hour so it is gaining on the Sun at $0.608 - 0.041 = 0.567$ degrees/hour. The difference of 0.160 degrees is made up in $0.160/0.567$ hours = 0.282 hours = 17 minutes. Conjunction therefore occurs at 09h 56m TT. At this moment the Sun's longitude is $359.461 + (0.282 \times 0.041) = 359.473$ degrees.

Now we are ready to construct the eclipse diagram (Figure 100). We proceed exactly as we did for the lunar eclipse, drawing two horizontal lines, one to represent the ecliptic and the lower one to represent time. Choosing a suitable scale (say 2 cm = 1 hour) we mark off the lower line in hours such that the time of conjunction is roughly in the middle of the diagram. Next we find point P on the ecliptic corresponding to conjunction and we draw a circle centred on P of the correct radius to represent the Sun. In this calculation, we assume that the angular radius of the Sun is 0.268 degrees. (We can find this value using spreadsheet SunDist, Section 48.) The relative motion between Sun and Moon in this case is 0.567 degrees/hour. Thus

deduced from a more-accurate program shows that we are within a few minutes of the correct results. Even our comparatively simple method allows us to make quite accurate predictions of what is surely the heavens' most awe-inspiring phenomenon.

As in the previous section, we have provided spreadsheet functions to make these calculations more accurately and with a lot less effort. The functions are:

UTFirstContactSolarEclipse,
UTMaxSolarEclipse,
UTLastContactSolarEclipse, and
MagLunarEclipse.

The first three functions return the universal times of first contact (i.e. the moment when the limbs of the Sun and Moon first intersect with each other), mid-eclipse (which may or may not be total), and last contact (when the limbs no longer intersect). The last function provides the magnitude of the solar eclipse, defined to be the fraction of the solar diameter obscured by the Moon at the moment of greatest eclipse, measured along the common diameter. Total eclipses have magnitudes greater than 1, as in this case. All four functions take the same seven arguments, namely the local calendar date as day, month, year, the daylight saving and time zone offsets in hours, and the geographical longitude and latitude in degrees (W and S negative). The functions return the value -99 if an eclipse does not occur.

A spreadsheet called SolarEclipseCircumstances (Figure 101) shows how to make use of these functions. For our observer on the Greenwich meridian, the universal times are also the local times. For other observers, it would be useful to display local civil times, and you can do this quite easily using the spreadsheet function UTLCT. Note that the contents of cells G3, and H3 to H7, are shown respectively in cells H9 to H14 in order to save space.

The circumstances of a solar eclipse									
Input	local date (day)	20	Output	Solar eclipse certain on	20/3/2015				
	local date (month)	3		UT of first contact	8:55				
	local date (year)	2015		UT of mid eclipse	9:57				
	daylight saving (hours)	0		UT of last contact	10:58				
	zone correction (hours)	0		magnitude of eclipse	1.016				
	geog longitude (deg)	0							
	geog latitude (deg)	68.65							
1	Julian date of new Moon	2457101.902	=NewMoon(C6,C7,C3,C4,C5)						
2	G date of new Moon (day)	20.40201849	=JDCDay(C11)						
3	... integer day	20	=INT(C12)						
4	G date new Moon (month)	3	=JDCMonth(C11)						
5	G date of new Moon (year)	2015	=JDCYear(C11)						
6	UT of new Moon (hours)	9.648443766	=(C12-C13)*24						
7	local civil time (hours)	9.648443766	=UTLCT(C16,0.0,C6,C7,C13,C14,C15)						
8	local civil date (day)	20	=UTLCDay(C16,0.0,C6,C7,C13,C14,C15)						
9	local civil date (month)	3	=UTLCMonth(C16,0.0,C6,C7,C13,C14,C15)						
10	local civil date (year)	2015	=UTLCYear(C16,0.0,C6,C7,C13,C14,C15)						
11	eclipse occurrence?	Solar eclipse certain	=SEOccurrence(C6,C7,C3,C4,C5)						
12	UT max eclipse	9.947296821	=UTMaxSolarEclipse(C3,C4,C5,C6,C7,C8,C9)						
13	UT first contact	8.920277911	=UTFirstContactSolarEclipse(C3,C4,C5,C6,C7,C8,C9)						
14	UT last contact	10.97431573	=UTLastContactSolarEclipse(C3,C4,C5,C6,C7,C8,C9)						
15	magnitude	1.016286817	=MagSolarEclipse(C3,C4,C5,C6,C7,C8,C9)						

Figure 101. Calculating the circumstances of a solar eclipse.

75 The Astronomical Calendar

It is often useful to have a chart that shows, at a glance, the relative configurations of the Sun, Moon and planets and the likely times of occurrence of eclipses. The astronomical calendar is just such a chart, displaying the right ascension of each heavenly body for every day in the year; the chart for 2015 is drawn in Figure 102.

It is convenient (though not essential) to construct the chart on graph paper. Mark the vertical axis in days (1 to 365 or 366) on a scale to make best use of the paper, and the horizontal axis in hours (0 to 24) such that time increases towards the left; this is a convention often adopted by astronomers as the chart then more nearly represents the relative positions of the bodies in the sky as seen from the Earth. Lines representing the times of the Sun and midnight may now be drawn. To do so it is necessary only to calculate the right ascension (RA) of the Sun on two days of the year several months apart by the method of Section 46, and to join the points by a straight line. Where the line goes off the edge of the chart (as at A and B in Figure 102) it should be continued from the points exactly opposite (A' and B'). The resulting lines should slope down towards the right; mark them with the symbol \odot to represent the Sun. The tracks of midnight, marked by the symbol \bullet , are parallel to those of the Sun but displaced by 12 hours on the RA scale.

Next, the track of the Moon should be marked in. Again, this can be done by calculating the Moon's right ascension on two days (a week or so apart) every month using the method of Section 65 and joining the points by straight lines. However, the calculations are lengthy and somewhat tedious unless you have a programmable calculator, or you use spreadsheets, so you may like to cheat a bit. The position of the Moon is given for every hour of the year in the *Astronomical Almanac* which you can consult in your local library, but it can also be deduced from the information given in most diaries, the dates of new Moon and full Moon. We know that when the Moon is full it is in opposition to the Sun and, conversely, it is in conjunction with the Sun at new Moon. We have already marked the tracks of conjunction (\odot) and opposition (\bullet) so that we can easily plot the Moon's position from the dates of new Moon and full Moon. For example, our diary (in 2009) indicated that new and full Moons occurred on 22 July and 6 August, so if we were making an astronomical calendar for 2009 we would be able to deduce that the right ascension of the Moon was the same as that of the Sun on 22 July and the same as midnight on 6 August. Join these two points with a straight line to mark the track of the Moon.

Next we must mark in the tracks of the Moon's ascending node (Ω) and descending node (Υ). The mean longitude of the former is given by the value of N in Section 65, and of the latter by $N + 180$. Find these values on two days separated by six months or so and convert to right ascension by the method of Section 27 (setting $\beta = 0$). Join each pair of points by a straight line.

We are now in a position to make predictions about eclipses. As explained in Section 72, an eclipse can only occur when the Moon is near one of its nodes at full Moon (lunar eclipse) or new Moon (solar eclipse). We must therefore find points on the chart where the tracks of the Moon, Sun or midnight, and either node pass close to one another. In Figure 102 these points are marked '+' together with the dates on which the eclipses occur as follows:

20 March 2015:	eclipse of the Sun;
4 April 2015:	eclipse of the Moon;
13 September 2015:	eclipse of the Sun;
28 September 2015:	eclipse of the Moon.

To complete the chart, we can mark on the tracks of the major planets: Mercury (♿), Venus (♀), Mars (♂), Jupiter (♃), Saturn (♄), Uranus (♅) and Neptune (♆). Their positions can be calculated by the method given in Section 54.

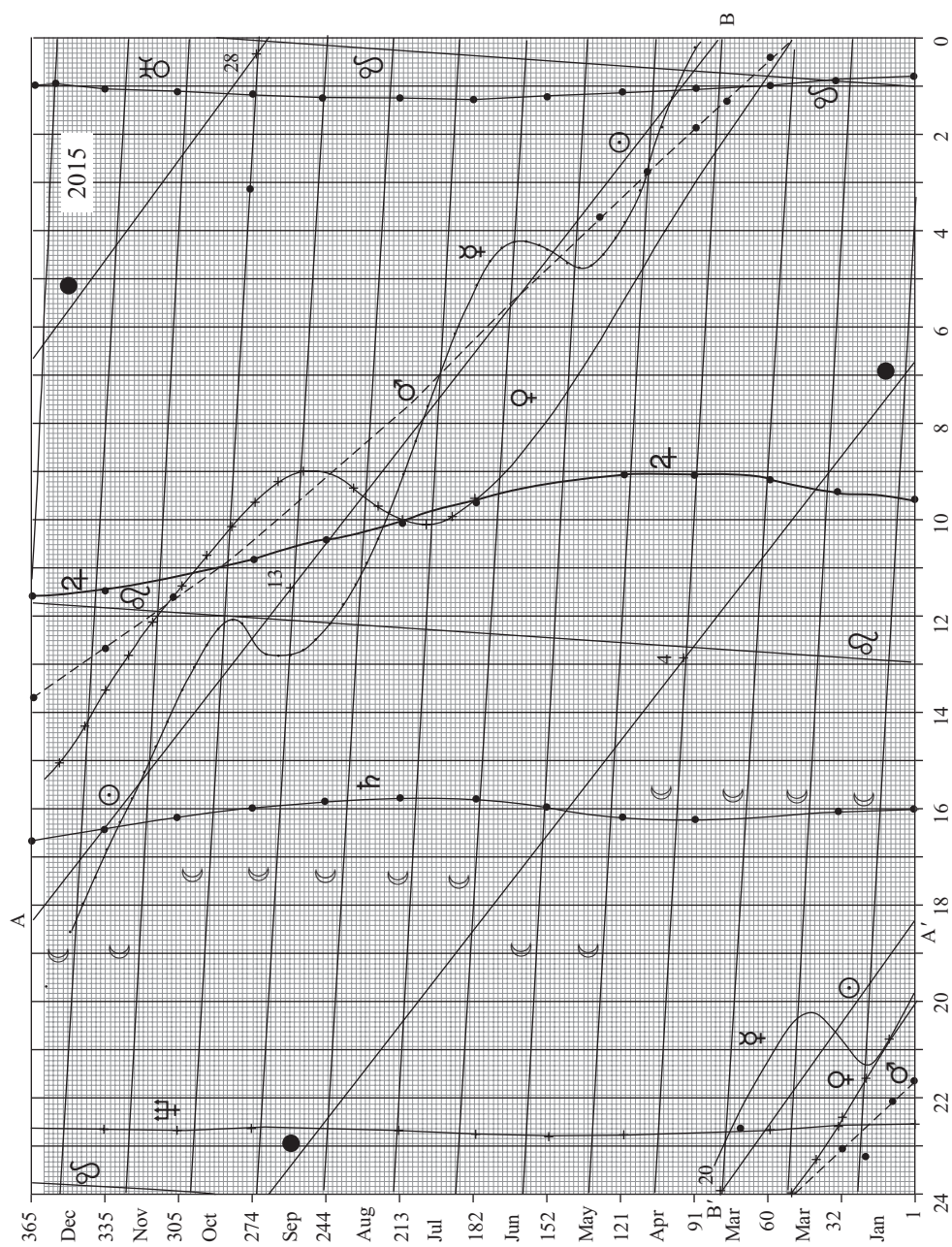


Figure 102. The astronomical calendar for 2015.

Glossary of terms

- aberration:** the apparent angular displacement of a celestial object from its geometric position, caused by the motion of the observer with respect to the object, and the finite speed of light.
- age of Moon:** the angle between the Sun and the Moon measured at the Earth.
- altitude:** the angle up from the horizon.
- annual equation:** a correction of the Moon's orbital motion due to the variation of the Sun–Earth distance as the Earth travels in its own ellipse about the Sun.
- anomaly:** the angle at the focus or the centre of an orbital ellipse between the major axis and the orbiting body or its projection. The **eccentric anomaly**, E , is defined in Figure 55 (page 108), while the **mean anomaly**, M , and **true anomaly**, v , are defined for the Sun in Figure 53 (page 104).
- apastron:** the point in an orbit about a star that is furthest from the star.
- aphelion:** the point in an orbit about the Sun most distant from the Sun.
- apogee:** the point in an orbit about the Earth most distant from the Earth.
- Astronomical Almanac:** a collection of tables predicting the positions and circumstances of astronomical phenomena. This title replaced both the *American Ephemeris and Nautical Almanac* and the *Astronomical Ephemeris*, beginning with the 1981 edition.
- Astronomical Ephemeris:** see *Astronomical Almanac*.
- astronomical latitude:** the angle between the astronomical zenith and the equator.
- astronomical unit:** approximately equal to the length of the semi-major axis of the Earth's orbit about the Sun, 1.496×10^{11} metres.
- atmospheric refraction:** the apparent shift in the position of a celestial object due to the bending of light rays by the atmosphere.
- azimuth:** the angle round from the north point measured on the horizon in the sense NESW.
- binary star:** a pair of stars bound together by their mutual gravitational attraction, both in orbit about their common centre of mass.
- calendar:** system of accounting the days in the year. The **Julian calendar**, introduced by Julius Caesar, divides the year into 365 days except for every fourth year which has 366. The **Gregorian calendar**, introduced by Pope Gregory XIII (1502–1585) in 1582 and accepted in England in 1752, is the one generally in use in the West today. It reduced the errors in the Julian calendar by removing three days every four centuries; if the year ends in two noughts it is only a leap year if it is divisible by 400. So, for example, 2000 was a leap year, but 1700, 1800 and 1900 were not. 2100 will not be a leap year either.

- celestial sphere:** an imaginary sphere, usually centred on the Earth, of arbitrarily large radius on the surface of which the stars can be considered to be fixed.
- circumpolar stars:** stars whose angular distances from the north or south celestial pole are sufficiently small that they never dip below the horizon.
- comet:** a diffuse member of the Solar System, usually with a highly elongated orbit, which becomes visible near the Sun. It has a bright head and one or more diffuse tails of variable length.
- companion star:** the fainter of the pair of stars in a visual-**binary star** system.
- conjunction:** the moment when two celestial bodies occupy the same position in the sky or share a common coordinate when viewed from a particular place. Thus **heliocentric conjunction**, and **conjunction in right ascension**.
- coordinate systems:** frames of reference by means of which the position of any point can be uniquely specified. In astronomy, the systems take their names from the fundamental planes on which they are based. Thus the **ecliptic coordinate system** measures longitude round from the first point of Aries, Υ , in the plane of the ecliptic and latitude northwards from it. The **equatorial coordinate system** measures right ascension round from Υ in the plane of the Earth's equator, and declination northwards from it. In the **horizon coordinate system**, the azimuth is measured round from the north point in the sense NESW and the altitude is the angle up from the horizon. The **galactic coordinate system** specifies position by longitude measured in the galactic plane round from the direction of the galactic centre and by latitude measured perpendicular to the plane. **Heliographic coordinates** enable the position of an object on the surface of the Sun to be specified with respect to the solar equator and a fundamental meridian assumed to rotate at a uniform rate. **Selenographic coordinates** define positions on the surface of the Moon with respect to the lunar equator and the mean **sub-Earth point**.
- coordinated universal time (UTC):** the time scale available from broadcast time signals. It differs from **International atomic time (TAI)** by a whole number of seconds, and is maintained within 0.9 s of universal time (strictly UT1) by the insertion of leap seconds, usually at the ends of June or December.
- culmination:** the moment at which a celestial body crosses the observer's meridian. Circumpolar stars cross the meridian above the horizon twice in one day, giving **upper culmination** and **lower culmination**.
- day:** the interval between two successive transits across the observer's meridian of a fixed star (**sidereal day**), of the Sun (**solar day**), or of a fictitious body called the mean Sun which moves at a uniform rate along the equator (**mean solar day**).
- daylight saving time:** see **time**.
- declination:** in the equatorial coordinate system, the angle measured perpendicular to the equator (north positive, south negative).
- dynamical time:** the family of time scales introduced in 1984 that replaces **ephemeris time**. See **time**.
- earthshine:** light reflected from the Earth which sometimes illuminates the dark portion of the Moon's disc, making it visible.
- eccentricity:** a measure of the degree of elongation of an ellipse, equal to the ratio of the distance of the focus from the centre to the length of the **semi-major axis**.
- eclipse:** the passage of the Moon through the Earth's shadow (**lunar eclipse**) or parts of the Earth through the Moon's shadow (**solar eclipse**). If, at the moment of greatest eclipse, the Moon or Sun is only partly obscured it is a **partial eclipse**; if completely obscured it is a **total eclipse**. If during a solar eclipse the

Moon obscures the central part of the Sun's disc but leaves an unobscured ring around its edge, then it is an **annular eclipse**.

ecliptic: the plane containing the orbit of the Earth about the Sun.

ellipse: a type of regular closed curve, oval in shape, of which a circle is a special case. It is traced by a point moving in such a manner that it keeps constant the sum of its distances from two fixed points, each of which is called a **focus** of the ellipse. The longest diameter of the ellipse, which goes through both foci and the centre, is called the **major axis**, the portion from the centre to the curve in either direction being called the **semi-major axis**.

ephemeris time (ET): see **time**.

epoch: a particular moment specified as the reference point from which time is measured. The dates 1950.0 (strictly 1950 January 0.923) and 2000.0 (2000 January 1.5) are often used as standard epochs.

equation of the centre: a relation between the true and mean anomalies which is an approximation to Kepler's equation. In its simplest form it is

$$v = M + 2e \sin M,$$

where v and M are expressed in radians, useful for values of e less than about 0.1.

equation of the equinoxes: apparent sidereal time minus mean sidereal time, taking account of the effect of **nutation** on the positions of the **equinoxes**.

equation of time: the difference between the real solar time and the mean solar time.

equator: the plane through the centre of the Earth which is perpendicular to the spin axis.

equinox: the moment at which the Sun crosses the celestial equator. This occurs on about 21 March when its right ascension is zero (the **vernal equinox**) and about 22 September when its right ascension is 12 h (the **autumnal equinox**). The positions of the equinoxes on the celestial sphere lie along the line of the intersection of the planes of the equator and the ecliptic.

evection: a correction to the Moon's orbital motion taking account of slight variations in the apparent value of the eccentricity of its orbit.

extinction: the attenuation and colouring of light as it travels through a medium; in particular, **atmospheric extinction**.

figure of the Earth: the true shape of the Earth. It is often approximated by a **spheroid of revolution**, a geometrical shape in which any cross-section parallel to the equator is a circle, while any cross-section through the north-south axis is an ellipse with the minor axis coincident with the diameter joining the north and south poles.

first point of Aries: the position on the celestial sphere of the **vernal equinox**.

focus of an ellipse: see **ellipse**.

geocentric coordinates: coordinates measured with respect to the centre of the Earth. Hence the **geocentric latitude** is the angle between the equator and a point on the surface of the Earth, as measured at the centre of the Earth.

geocentric parallax: the angle subtended at a heavenly body by the centre of the Earth and the point of observation on the Earth's surface.

geostationary satellite: a body orbiting the Earth in the plane of the equator in such a direction and at such a height that its orbital period equals 1 day so that it keeps constant position with respect to the Earth's surface.

GPS time: an atomic time kept by the US Naval Observatory and broadcast by the satellites of the global positioning system. GPS time was equal to UTC on 1980 January 6 0.0, but, unlike UTC, is not adjusted by the insertion of leap seconds. Hence GPS time is equal, in June 2011, to UTC + 15 seconds (kept to within a microsecond) and is the time you can extract from your GPS navigation device.

gravity: the mutual force of attraction between any two bodies which is proportional to the product of their masses and inversely proportional to the square of their separation.

great circle: any circle drawn on the surface of a sphere whose centre is the same as that of the sphere.

Greenwich mean time (GMT): this is ambiguous and is not now used in the *Astronomical Almanac*. Its meaning in civil life is usually the same as UTC, though previously it has been used to mean UT. Before 1925 it was reckoned from Greenwich mean noon (12 h UT).

Greenwich meridian: that half of the great circle on the surface of the Earth passing through the north and south poles and through the reference point in Greenwich, England. It is taken as the line of longitude 0° .

horizontal parallax: the geocentric parallax when the celestial body is on the observer's horizon; hence **equatorial horizontal parallax** when the observer is also on the equator.

hour angle: the difference between the local sidereal time and the right ascension.

inclination of orbit: the angle between the plane of the orbit and the plane of the ecliptic.

inner planet: a planet whose semi-major axis is smaller than that of the Earth; that is the planets Mercury and Venus.

international atomic time (TAI): see **time**.

Julian date: the number of Julian days that have elapsed since the fundamental epoch Greenwich mean noon of 1 January 4713 BC. For 2010 January 0.0 its value is 2 455 196.5. The **Julian day number** is the integer part of the Julian date. See also **modified Julian date (MJD)**.

Kepler's equation: the relation between the mean and eccentric anomalies, M and E ,

$$E - e \sin E = M,$$

where the angles are expressed in radians.

latitude: the coordinate expressing the angle (north positive, south negative) perpendicular to a fundamental plane, hence **ecliptic latitude** and **galactic latitude**. On the Earth, the **geographical latitude** is measured with respect to the equator. The ecliptic latitude can be measured either at the Earth (**geocentric**) or at the Sun (**heliocentric**).

librations: variations in the orientation of the Moon's surface with respect to an observer on the Earth.

light time: the time it takes light signals from a celestial body to reach an observer.

longitude: the coordinate expressing the angle round from a fixed direction measured in a fundamental plane, hence **ecliptic longitude** and **galactic longitude**. On the Earth, the **geographical longitude** is measured at the equator. The ecliptic longitude can be measured either at the Earth (**geocentric**) or at the Sun (**heliocentric**).

lunation: the period between two successive new Moons.

luni-solar precession: the slow retrograde motion of the first point of Aries along the equator caused by the combined effects of the Sun and the Moon on the slightly non-spherical Earth.

magnitude: (i) the unit defined on a logarithmic scale which measures the brightness of a celestial object considered as a point.

- (ii) **in a lunar eclipse**, the fraction of the lunar diameter obscured by the shadow of the Earth at the moment of greatest eclipse, measured along the common diameter.
- (iii) **in a solar eclipse**, the fraction of the solar diameter obscured by the Moon at the moment of greatest eclipse, measured along the common diameter.

mean Sun: a fictitious heavenly body that moves at a uniform rate along the equator making one complete circuit in the same time (1 year) as the real Sun takes to make a complete circuit.

meridian: that half of a great circle which is terminated at the north and south poles. On the Earth a meridian is a line of longitude. On the celestial sphere, the meridian which passes through the zenith is called the **observer's meridian**.

modified Julian date (MJD): the number of Julian days elapsed since 1858 November 17.0.

month: the period taken by the Moon to make one complete circuit of its orbit from reference point to reference point. The **draconic month** or **nodal month** takes the ascending node as the reference and is equal to 27.212 2 mean solar days. The **sidereal month** is reckoned against the background of stars and is equal to 27.321 7 mean solar days. The Sun is used as the reference for the **synodic month** of 29.530 6 mean solar days, and the perigee for the **anomalistic month** of 27.554 6 mean solar days.

nadir: the point on the celestial sphere diametrically opposite the **zenith**.

node: a point on the celestial sphere where the great circle representing the orbit cuts the great circle representing the plane of the ecliptic. The point where the orbiting body is moving from below (south of) to above the ecliptic is called the **ascending node**; the other is the **descending node**.

noon: the instant at which the Sun crosses the observer's meridian.

north celestial pole: the point at which the projection of the Earth's rotation axis through the north pole intersects the celestial sphere.

nutation: a small periodic wobbling motion of the Earth's rotation axis.

obliquity of the ecliptic: the angle at which the plane of the ecliptic is inclined to the plane of the equator.

opposition: the moment when two celestial bodies occupy opposite positions in the sky, or have longitudes different by 180° , when viewed at a particular place.

orbit: the path through space taken by a body gravitationally attracted to another body.

orbital elements: the quantities which need to be known in order to specify an orbit uniquely.

osculating elements: the elements describing the elliptical orbit followed by a body if all perturbing influences vanish. Since perturbations disturb the true orbit of any member of the Solar System, the osculating elements are constantly changing.

outer planet: those planets having semi-major axes larger than that of the Earth. The major outer planets are Mars, Jupiter, Saturn, Uranus and Neptune. In 2006, Pluto was reclassified as a dwarf planet by the International Astronomical Union.

parabolic orbit: an orbit in which the velocity at any point is equal to the escape velocity.

parallax: the amount by which the apparent position of a celestial object shifts as the point of observation is changed.

penumbra: the outer portion of a shadow where the light is only partially cut off.

periastron: the point in an orbit about a star that is nearest to the star.

perigee: the point in an orbit about the Earth which is nearest the Earth.

perihelion: the point of closest approach to the Sun in an orbit about the Sun.

period of orbit: the time taken by the orbiting body to make one complete circuit.

- perturbations:** deviations from true elliptical motion caused by the gravitational fields of other members of the Solar System.
- phase:** (i) **of Moon or planet:** the fraction of the area of the disc which is illuminated. When the dark side of the Moon faces the Earth, the phase is zero and it is **new Moon**. At the **first quarter** and the **third quarter**, the phase is equal to a half and the Moon is in **quadrature**. **Full Moon** has a phase equal to one. Whenever the phase is greater than a half, the Moon is described as **gibbous**.
 (ii) **of an eclipse:** the stage of a lunar or solar eclipse during which the eclipsed body is partly obscured (**partial phase**) or totally obscured (**total phase**). During a lunar eclipse, the Moon is in the penumbra of the Earth's shadow during the **penumbral phase** and partially or totally in the umbra during the **umbral phase**. The partial and total phases occur during the umbral phase.
- planet:** a solid body in closed orbit about a star. In our own Solar System, the major planets are (in order of increasing distance from the Sun) Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. Pluto, formerly recognised as a planet, was reclassified as a dwarf planet in 2006.
- polar distance:** the angle on the celestial sphere from the celestial pole.
- pole:** the point on a sphere which is perpendicular to a given plane. Hence **pole of the ecliptic** and **pole of the equator** (each has two poles called north and south poles for short).
- position-angle:** a celestial angle measured from 0° to 360° eastwards from the north.
- precession:** see **luni-solar precession**.
- primary star:** the brighter of the pair of stars in a visual-binary system.
- prograde motion:** motion in the same sense as that of all the planets about the Sun. When looking down on the Solar System from the north celestial pole, prograde motion is counter-clockwise.
- radian:** a natural unit used to measure angles, equal to $1/2\pi$ revolutions and $180/\pi$ degrees.
- radius vector:** the line joining the principal focus to the position of the orbiting body on its orbital ellipse.
- reflectivity of planet:** a measure of a planet's ability to reflect sunlight; a factor affecting its apparent brightness.
- refraction:** see **atmospheric refraction**.
- retrograde motion:** motion in the opposite sense to that of all the planets about the Sun. When looking down on the Solar System from the north celestial pole, retrograde motion is clockwise.
- right ascension:** in the equatorial coordinate system the angle measured round from the first point of Aries in the plane of the equator, in the sense NWSE.
- rising:** the moment when a celestial body crosses the horizon on the way up.
- Saros cycle:** the period of 18 years 11 days and 8 hours after which the pattern of lunar and solar eclipses tends to repeat.
- second (SI second):** the unit of time for the **international atomic time (TAI)** scale defined to be exactly 9 192 631 770 cycles of radiation corresponding to the transition between two hyperfine levels in the ground state of caesium 133.
- semi-major axis:** see **ellipse**.
- setting:** the moment when a celestial body crosses the horizon on the way down.
- solar elongation:** the angle between the lines of sight to the Sun and to the celestial body in question.
- Solar System:** the Sun and all the bodies, planets, comets and asteroids in closed orbits about it.
- solstice:** the points at which the apparent longitude of the Sun is 90° and 270° , or the moments at which the Sun is at either of these points. These occur around 21 June and 21 December.
- sub-Earth point:** the point on a celestial body (especially the Moon) where the line joining the centre of the Earth and the centre of the body intersects the surface of the body.

synodic period: the time between successive conjunctions in longitude.

terminator: the line marking the boundary between the dark and sunlit hemispheres of a member of the Solar System.

terrestrial dynamical time (TDT): see **time**.

terrestrial time (TT): see **time**.

time: (i) **atomic time:** time measured with respect to the natural period of oscillations of an atomic system. Caesium beam clocks currently constitute the most precise time-keepers available, and the SI unit of atomic time is defined in terms of the caesium 133 atom (see **second**). **International atomic time (TAI)** is the continuous scale resulting from analyses by the Bureau International des Poids et Mesures of atomic time standards in many countries, starting from the epoch 1958 January 1. **Coordinated universal time (UTC)** is the time scale distributed by standard time services and is tied to both TAI and UT in such a manner that (a) it differs from TAI by a whole number of seconds, and (b) it is never more than 0.9 s different from UT (strictly UT1). This is achieved by the introduction of leap seconds into UTC from time to time. UTC constitutes the basis for legal time keeping in most parts of the world. **Terrestrial time (TT)** (called **terrestrial dynamic time (TDT)** until 1991) is used as the argument in theories of celestial dynamics and in the compilation of the *Astronomical Almanac*. It is equal to TAI + 32.184 s. **TDT** had replaced **ephemeris time (ET)** in 1984, which was itself derived from analyses of the Moon's motion.

(ii) **solar time:** time measured with respect to the motion of the Sun or a fictitious body, no longer used, called the mean Sun (**mean solar time**). **Universal time (UT)** is, broadly speaking, the mean solar time as measured on the Greenwich meridian. It is formally defined by a mathematical formula as a function of **sidereal time** (see below), and is thus determined from observations of the stars. A direct application of the formula gives UT0; with a small correction for polar motion the scale UT1 is obtained. Whenever the term UT is used, UT1 is usually implied. **British summer time (BST)** is 1 hour ahead of UT and is an example of **daylight saving time** in which the time is adjusted to make the working day fit more conveniently into the daylight hours.

(iii) **sidereal time:** time measured with respect to the apparent motion of the stars. The **local sidereal time** at any place is equal to the hour angle of the first point of Aries; local sidereal time on the Greenwich meridian is called **Greenwich sidereal time**. The difference between **apparent sidereal time** and **mean sidereal time** is called the **equation of the equinoxes**, and takes account of **nutations**. It may be as much as 1.2 seconds.

time zone: a longitudinal strip on the surface of the Earth in which the **zone time**, usually a whole number of hours before or after UT, is adopted as the local civil time by national or international agreement.

transit: the moment at which a celestial body crosses the observer's meridian.

twilight: that period of semi-darkness after sunset or before sunrise during which the sun's zenith distance is more than 90° but less than some agreed figure. This figure is 108° for **astronomical twilight** and 102° for **nautical twilight**, while for **civil twilight** it is 96°.

umbra: the inner portion of a shadow where the light is completely obscured.

universal time: see **time**.

variation: a correction to the Moon's orbital motion about the Earth that takes account of the changing solar gravitational field.

vernal equinox: see **equinox**.

year: the interval between two successive passages of the Sun through a reference point. A particular point among the stars is used as reference in the **sidereal year**, equal to 365.2564 mean solar days. The **tropical year**, 365.242 191 mean solar days, uses the first point of Aries as its reference. When no qualifying adjective is used with the word ‘year’, it is usually the tropical year that is meant. Perturbations to the Earth’s orbit by the other planets cause small changes in the Earth’s orbital elements. The **anomalous year**, 365.2596 mean solar days, is the interval between two successive passages of the Sun through perigee. The **Besselian year**, not used since 1984, is the period of one complete revolution in right ascension of the fictitious mean Sun as defined by the astronomer Simon Newcomb (1835–1909). It is almost the same as the tropical year, but begins when the right ascension of the Sun is exactly 240° ; this instant falls very near the beginning of the **civil year**.

zenith: the point directly overhead at the observer. The **zenith angle** or **zenith distance** of a star is the angle between the star and the zenith.

zone correction: the number of hours that needs to be added to or subtracted from UT to get the **zone time**.

Symbols and abbreviations

☉	Sun	♃	Jupiter
●	midnight	♄	Saturn
☾	Moon	♅	Uranus
☿	Mercury	♆	Neptune
♀	Venus	♈	first point of Aries
♁	Earth	♊	ascending node
♂	Mars	♋	descending node

=	Begin a spreadsheet formula in a cell
*	Multiply two values together in a spreadsheet formula
'	Treat the contents of this spreadsheet cell as a label
####	Column width is too narrow in a spreadsheet
!	Use to link to a cell of another spreadsheet
,	Argument separator (delimiter) in <i>Excel</i>
;	Argument separator (delimiter) in <i>Calc</i>

α	right ascension
β	geocentric ecliptic latitude
δ	declination
Δ	difference; error
$\Delta\lambda$	Moon's hourly motion in ecliptic longitude
$\Delta\beta$	Moon's hourly motion in ecliptic latitude
ΔA	correction to azimuth
ΔT	ET – UT
Δt	value of equation of time
ε	elongation; obliquity of the ecliptic; longitude of planet at epoch
ε_g	geocentric longitude of Sun at epoch
ζ	apparent zenith angle
θ	angular diameter; displacement; general coordinate
θ_t	twilight zenith angle
λ	geocentric ecliptic longitude
μ	general coordinate
v	true anomaly; general coordinate
π	parallax; constant = 3.141 592 654
ϖ	heliocentric longitude of perihelion
ϖ_g	geocentric longitude of Sun's perigee
ρ	distance
τ	light-travel time
ϕ	geographical latitude
ϕ'	geocentric latitude
χ	position-angle
ψ	heliocentric ecliptic latitude; angle at the horizon; general coordinate
ϖ	longitude of ascending node
ω	to argument of perihelion
A	azimuth
A , etc.	matrix
a	altitude; semi-major axis
AD	Anno Domini
A_e	annual equation
A_3	third correction to Moon's mean anomaly
A_4	fourth correction to Moon's mean anomaly
AU	astronomical unit
B	heliographic latitude
b	galactic latitude
BC	Before Christ
BCE	Before the Common Era; Before the Christian Era
BST	British summer time
CRN	Carrington rotation number
CE	Common Era; Christian Era
DEC	declination
D	age of Moon or planet; number of days since an epoch
d	number of days; angle
E	eccentric anomaly
E	east point of horizon
e	eccentricity
E_c	correction applied in the equation of the centre
E_v	evection
EST	eastern standard time
ET	ephemeris time
F	phase

GBT	galactic barycentric time
GMT	Greenwich mean time
GST	Greenwich sidereal time
H	hour angle
I	inclination of Sun's equator
i	inclination
JD	Julian days
L	heliocentric longitude of Earth or heliographic longitude
l	galactic longitude; Moon's orbital longitude; heliocentric longitude of planet
LST	local sidereal time
M	mean anomaly
m	magnitude; precession constant
MJD	modified Julian date or day number
N	north point of horizon; longitude of ascending node
n	precession constant
NCP	north celestial pole
P	equatorial horizontal parallax; angle
p	(horizontal) parallax
q	perihelion distance
R	refraction angle; distance of Earth from Sun
r	radius vector
r_0	semi-major axis of orbit
RA	right ascension
S	south point on horizon
S_p	radius of Earth's penumbra
S_u	radius of Earth's umbra
SCP	south celestial pole
ST	sidereal time
T	period of orbit
t, t_0	epoch
TAI	International atomic time
TDT	terrestrial dynamic time
TT	terrestrial time
UT	universal time
V	variation
v	vertical shift
\mathbf{v} , etc.	column vector
V_0	planet's brightness factor
W	west point on horizon
Y	years
z	real zenith angle

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A useful website

Point your browser to

`www.cambridge.org/practicalastronomy`

to find all of the spreadsheets and spreadsheet functions mentioned in *Practical Astronomy with your Calculator or Spreadsheet*. You will need your copy of this book to access the software.

On our website we have also included links to other websites and sources of data that we hope will be of interest to you.

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